# TRIAL TO DETERMINE THE COORDINATES IN X-RAY STEREOPHOTOGRAPHY USING SEVERAL TARGETS

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## 1. INTRODUCTION

In the study of cultural properties, there are some problems which must be known about the inner structure of the cultural property and there are the same problems in scoliosis; namely scoliotic deformities of the spine are frequently subjected to the studies which are aimed at assessing the magnitude of structural abnormalities. Especially, nowadays the Moiré technique has been applied in the field of scoliosis, because there is a correlation between Moiré patterns and scoliotic deformities of the spine. But this correlation has been derived from the relation between Moiré patterns and the deformities of the spine through X-ray photography. It is generally difficult to obtain three dimensional coordinates of the object from a mono-picture, so if one wants to obtain an accurate interrelation between Moiré Patterns and scoliotic deformities of the spine, Stereo X-ray photography must be applied. The method of Stereo X-ray photography has been well known for a long time, but it is mainly the stereo observation of the object in many cases. When the coordinates of the object in three dimensions are required, special machines, such as X-ray plotters or stereo-comparators have been used. In this method, there is a little difficult problem if one intends to obtain a high accuracy with the position of the anode. This is usually called Target or Focus in Radiography, because it is located inside an X-ray tube. Furthermore the accuracy of the coordinates of any object depends on the accuracy of the three dimensional coordinates of the focus.

The focal distance from the focus of the X-ray tube to the film is usually calculated from the three dimensional coordinates of the targets above the film and the coordinates of the target images, namely the observor obtains the coordinates of the intersection between two straight lines combined respectively. However these coordinates of the intersections distribute themselves in the space above the film, so one obtains slightly different values for the coordinates of the focus from each other in many cases, but sometimes a large difference and opposite sign occur. These facts depend on the procedure of the calculation.

In this report, the procedures of the calculation and the result obtained in this trial will be described.

## 2. DISCUSSION OF THE PROBLEM AND BASIC CONCEPTION

In analytical X-ray photogrammetry, one needs a pair of radiographs. But as it is difficult to fix the positions of the X-ray source and ensure a constant focus-film distance and constant length of the photogrammetric base, these elements must be determined indirectly with the aid of several targets which are embedded in the film cassette. Therefore, in this trial, it is assumed that the pair of photographs for an object are taken respectively at a constant position in a three dimensional space, namely the film cassette must be fixed to the constant position in a fundamental coordinate-system setting the space during the pair of photographs taken. Thus the image-position of the object in each radiograph can be measured respectively in a common coordinate-system which is given by fiducial marks on the film cassette. Then, it is not necessary to take account of the relative orientation between two radiographs in this trial.

The geometrical arrangement is shown in Fig. 1-a. There are two coordinate-systems; one is the external coordinate-system (O-XYZ system), such as the fundamental coordinate-system mentioned above.

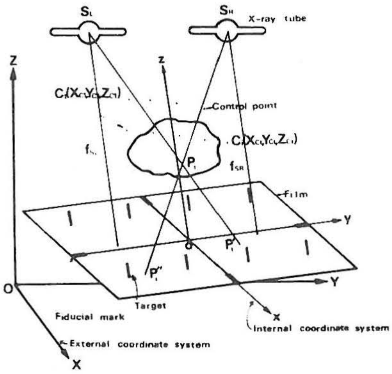

Fig. 1-a

Another one is the internal coordinate system (o-xyz system); sometimes this is called the Film-coordinate system. Since it may be seemed that the positions of the X-ray source belong to the film coordinate system, the interior orientation is given by the coordinates of the X-ray sources such as $x_{SL}$ , $y_{SL}$ , $z_{SL} = f_{SL}$ and $x_{SR}$ , $y_{SR}$ , $z_{SR} = f_{SR}$ . These values $x_{SL}$ , $y_{SL}$ , ...etc. are yielded from a series of the coordinates of the target's tips and their images measured in the film coordinate system. Eight targets are embedded in a special platform on the surface of the film-cassette and then this special film cassette is fixed to a film cassette holder in this trial.

Furthermore the object's position in the three dimensional space must be determined in a suitable coordinate system, such as the external coordinate system in which the object belongs so the original point of the coordinate-system may be set on the object when one needs to make the external-coordinate system set onto the object. To determine the relation between two coordinate-systems, several control points $c_1$ , $c_2$ , ... $c_n$ (as shown in Fig. 1-a) must be located in the three dimensional space close to the object.

The following is the procedure of this trial. Refer to Fig. 1-b.

(1) In the first place, the position of the X-ray source must be determined in each individual photograph. Refer to Fig. 2.

(2) The relation between the two coordinate-system, namely the coordinates of the original point of the film-coordinate system $X_0$, $Y_0$, $Z_0$ in the external coordinate-system and a series of elements of the rotation $\omega$ , $\phi$ , $\kappa$ , must be yielded by adopting

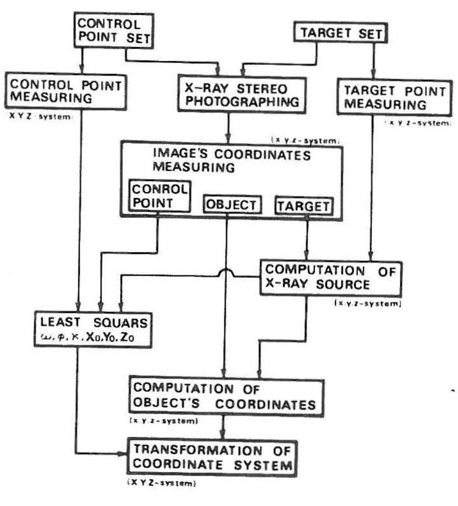


Fig 1-b Fundamental flow chart

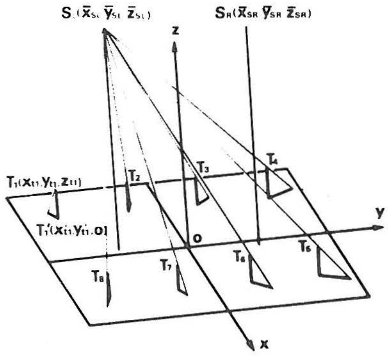

Fig. 2

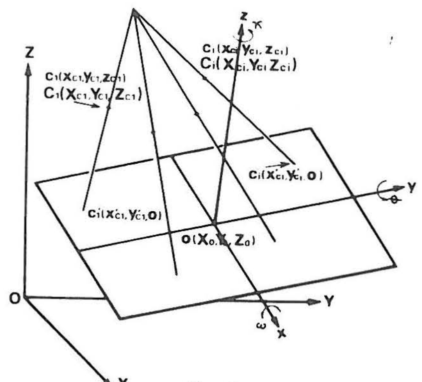

Fig. 3

the method of least squares. The following relationship is well known:

$$\begin{pmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \end{pmatrix} = R \begin{pmatrix} X_{ci} - X_0 \\ Y_{ci} - Y_0 \\ Z_{ci} - Z_0 \end{pmatrix} \text{---------} (1)$$

where R is as follows. Refer to Fig. 3.

$$R = \begin{pmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{pmatrix}$$

and $x_{ci}$, $y_{ci}$, $z_{ci}$ are the coordinates of the i-th-control point $c_i$ in the film coordinate system, $X_{ci}$, $Y_{ci}$, $Z_{ci}$ are the coordinates of the control point $C_i$ in the external coordinate-system. And then, the following relationships are given, Refer to Fig. 4

$$\begin{pmatrix} x'_{ci} \\ y'_{ci} \end{pmatrix} = \frac{f_{SL}}{f_{SL} - z_{ci}} \begin{bmatrix} x_{ci} - \bar{x}_{SL} \\ y_{ci} - \bar{y}_{SL} \end{bmatrix} + \begin{bmatrix} \bar{x}_{SL} \\ \bar{y}_{SL} \end{bmatrix} \text{--------} (3)$$

$$\begin{pmatrix} x''_{ci} \\ y''_{ci} \end{pmatrix} = \frac{f_{SR}}{f_{SR} - z_{ci}} \begin{bmatrix} x_{ci} - \bar{x}_{SR} \\ y_{ci} - \bar{y}_{SR} \end{bmatrix} + \begin{bmatrix} \bar{x}_{SR} \\ \bar{y}_{SR} \end{bmatrix} \text{---------} (3')$$

where $\bar{x}_{SL}$, $\bar{y}_{SL}$, $\bar{z}_{SL}$ are the mean values for $x_{SL}$, $y_{SL}$, $z_{SL}$ respectively and $\bar{x}_{SR}$, $\bar{y}_{SR}$, $\bar{z}_{SR}$ are the mean values for $x_{SR}$, $y_{SR}$, $z_{SR}$ .

The expressions (3), (3') must be equitably used when a series of observation equations are yielded, and then 6 normal equations are yielded from a series of observation equations, and

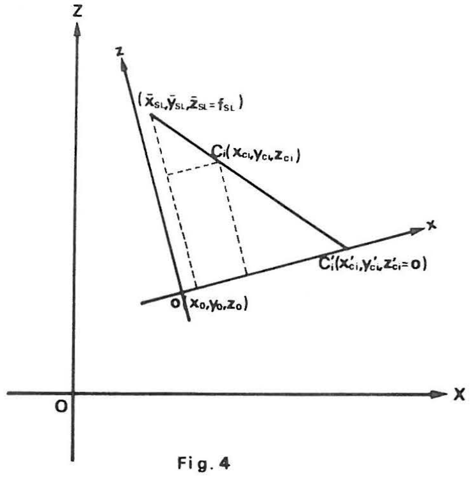

Fig.4

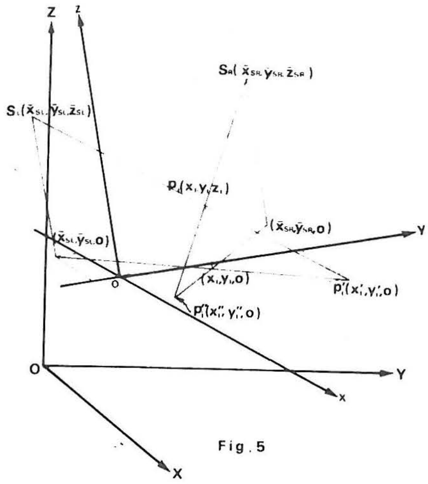

Fig.5

also as there are 6 unknown parameters $\omega$, $\phi$, $\kappa$, $X_0$, $Y_0$, $Z_0$ . Therefore, the most probable values for $\omega$, $\phi$ etc. are obtained by using the method of least squares.

(3) The coordinates of each object $x_i$, $y_i$, $z_i$ are calculated from a geometrical relation between the coordinates $x_i'$, $y_i'$, $x_i''$, $y_i''$ and the coordinates $\bar{x}_{SL}$, $\bar{y}_{SL}$, $\bar{z}_{SL}$, $\bar{x}_{SR}$, $\bar{y}_{SR}$, $\bar{z}_{SR}$ in Fig. 5. Here $x_i''$, $y_i''$ are the image's coordinates in the second film. Therefore, the coordinates of any object $P_i$ are given by the following relationships.

$$\left.\begin{aligned} \frac{x_i - \bar{x}_{SL}}{x_i' - \bar{x}_{SL}} &= \frac{y_i - \bar{y}_{SL}}{y_i' - \bar{y}_{SL}} = \frac{z_i - \bar{z}_{SL}}{z_i'=0 - \bar{z}_{SL}} \\ \frac{x_i - \bar{x}_{SR}}{x_i'' - \bar{x}_{SR}} &= \frac{y_i - \bar{y}_{SL}}{y_i'' - \bar{y}_{SR}} = \frac{z_i - \bar{z}_{SR}}{z_i''=0 - \bar{z}_{SR}} \end{aligned}\right\} \text{---------}(4)$$

(4) The position of the object in the external is determined from $x_i$, $y_i$, $z_i$ and the inverse matrix $R^{-1}$.

## 3. PROCEDURE OF COMPUTATION FOR THE FOCAL POINT'S COORDINATES

It is evident that the accuracy with regard to the position of an object depends on the positions of the X-ray source; therefore the aim of this trial is to find the best procedure of the computation for the position of the X-ray source.

Here, the straight line's equations are given as follows, refer to Fig. 6.

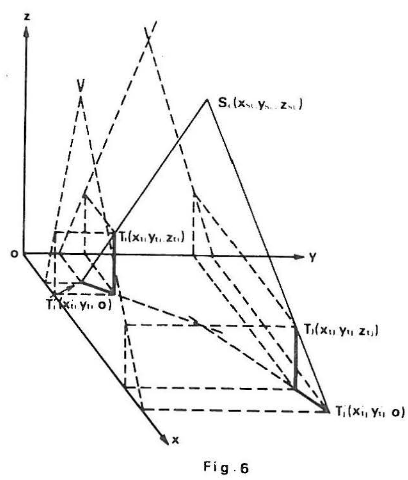

Fig.6

$$\left.\begin{aligned}\frac{x - x'_{ti}}{x_{ti} - x'_{ti}} &= \frac{y - y'_{ti}}{y_{ti} - y'_{ti}} = \frac{z}{z_{ti}} \\ \frac{x - x'_{tj}}{x_{tj} - x'_{tj}} &= \frac{y - y'_{tj}}{y_{tj} - y'_{tj}} = \frac{z}{z_{tj}}\end{aligned}\right\} \text{------------(5)}$$

where $x_{ti}$ , $y_{ti}$ , $z_{ti}$ are the coordinates corresponding to the i-th-target's tip, $x'_{ti}$ , $y'_{ti}$ , $z'_{ti}$ are the coordinates to the image of the i-th-target and $x_{tj}$ , $y_{tj}$ , $z_{tj}$ are the coordinates corresponding to the j-th-target, $x'_{tj}$ , $y'_{tj}$ , $z'_{tj}$ are the coordinates of the image of the j-th-target. The position of the intersection between the straight line $S_L T_i$ and the straight line $S_L T_j$ can be computed by a series of simultaneous equations which are yielded from the relationship (5). Namely, the coordinates of the intersection between two straight lines projected from the three dimensional space onto the xy plane can be given as $(x_{xy})_{ij}$ , $(y_{xy})_{ij}$ . The coordinates $(x_{xz})_{ij}$ , $(z_{xz})_{ij}$ , $(y_{yz})_{ij}$ $(z_{yz})_{ij}$ are also computed respectively in the xz-plane and the yz-plane. Here, the suffixes xy, xz, yz mean each plane of which the straight lines are projected, the combination between the i-th-target and the j-th-target is denoted by the suffixes ij. For one pair of targets, the coordinates of the X-ray source $S_{SL}$ are given by

$$\left.\begin{aligned}(x_{SL})_{ij} &= \frac{(x_{xy})_{ij} + (x_{xz})_{ij}}{2} \\ (y_{SL})_{ij} &= \frac{(y_{xy})_{ij} + (y_{yz})_{ij}}{2} \\ (z_{SL})_{ij} &= \frac{(z_{xz})_{ij} + (z_{yz})_{ij}}{2}\end{aligned}\right\} \text{-------(6)}$$

But, if the number of these targets is denoted by m, there occurs the combination of ${}_mC_2 = n$. Then the coordinates of the X-ray source must be decided by the mean value as follows:

$$\left.\begin{aligned}\bar{X}_{SL} &= \frac{1}{n}\sum_{k=1}^{n} \{(x_{SL})_{ij}\}_k \\ \bar{Y}_{SL} &= \frac{1}{n}\sum_{k=1}^{n} \{(y_{SL})_{ij}\}_k \\ \bar{Z}_{SL} &= \frac{1}{n}\sum_{k=1}^{n} \{(z_{SL})_{ij}\}_k\end{aligned}\right\} \text{----------------(7)}$$

When one wants to obtain the position of the X-ray source, there is another procedure of computation but the possibility must be avoided that the coordinate x or y of the intersection between two straight lines projected from the three dimensional space onto the xy-plane be substituted respectively into each of the straight line's equations on the xz-plane or the yz-plane, because there occurs a kind of accumulative error in these computations, and then this result is shown as compared with the former result in the latter section by their relative errors.

## 4. DISCUSSION ABOUT MEAN VALUES OF THE COORDINATES OF THE X-RAY SOURCE

When a mean value is required from a series of measured values or computed values, the special values containing a large error must be abandoned. For example, if the number of targets is 8, $n = {}_8C_2 = 28$ is yielded, therefore, $\bar{x}_{SL}$ is computed as follows:

$$\bar{x}_{SL} = \frac{1}{28}\left[\{(x_{SL})_{12}\}_1 + \{(x_{SL})_{13}\}_2 + \cdots + \{(x_{SL})_{ij}\}_k + \cdots + \{(x_{SL})_{78}\}_{28}\right] \qquad (8)$$

However, as is shown in Fig. 7, there occur numerous types of combinations such as the straight line $AT_1'$ and the straight line $AT_2'$ in the xy-plane, the straight line BC and the straight line BD in the yz-plane etc. The precision of the coordinates of each target's tip is constant for all targets embedded on the film cassette, because it can be measured by a vernier-gauge. But it is very difficult to measure the coordinates of the image of the target's tip with constant precision. Nevertheless they can be measured by a Stereocomparator. Here, the main problem factor is the generally limited quality of the image definition caused by the finite size of an X-ray focal spot substituted for an ideal projection center, and by the appreciable distance of the target's tip from the film-plane. These causes decrease the image

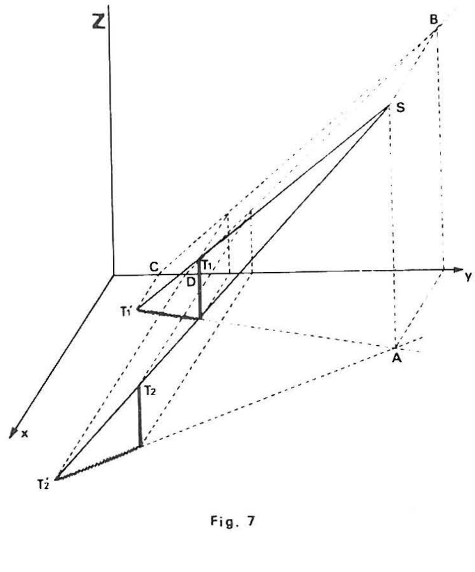


Fig. 7

sharpness and have an influence on the observer's decision which pertain in measuring the image's position. Therefore, the degree of the influence depends on any error in reading the image's position and on the angle between two straight lines, such as BC, BD and $AT_1'$ , $AT_2'$ , namely $\angle$ CBD, $\angle T_1'AT_2'$. Moreover, the magnitude of this angle depends on the combination of targets and on the question of which plane two straight lines passing through the X-ray source and each target's tips are projected. Therefore, the degree of the influence on the determination of position B is larger than the degree of the influence on position A. Then, in a procedure of computerizing the mean value of a series of $\{(x_{SL})_{ij}\}_k$ , unsuitable values such as an extreme value or an opposite sign, which is yielded from the situation in which two straight lines are almost parallel or the angle between two straight lines is less than a certain angle $\theta$, must be abandoned from the computation of the expression (8).

These procedure of computations are realized in the way of data processing by computer and some results are shown in this report.

## 5. APPARATUS AND EXPERIMENTAL RESULTS

Radiographs were taken by a standard X-ray machine in sequential mode involving a suitable shift of the X-ray tube, such as TOSHIBA ROTANODE, TYPE DRX-70. The focus-film distance was about 400mm$\sim$800mm. Since the film-cassette has two pairs of fiducial marks made of narrow lead wire, these marks are printed on the film even if it is inserted into the film-cassette with a slight rotation. These fiducial marks make the internal coordinate system, so the coordinates of each target's tip can be measured in the xyz-system. In this trial, 8 targets embedded in the special platform attached to the surface of the cassette are shown in Fig. 8-a. The height of these targets was about 30mm$\sim$120mm for the preliminary experiment, and was about 50mm for the final experiment. The focus-film distance was about 770 mm for the practical experiment.

In this practical experiment, a piano wire and a skeleton-model were used for the object. Five control points made of steel wire were also embedded in an even plate made of acrylic

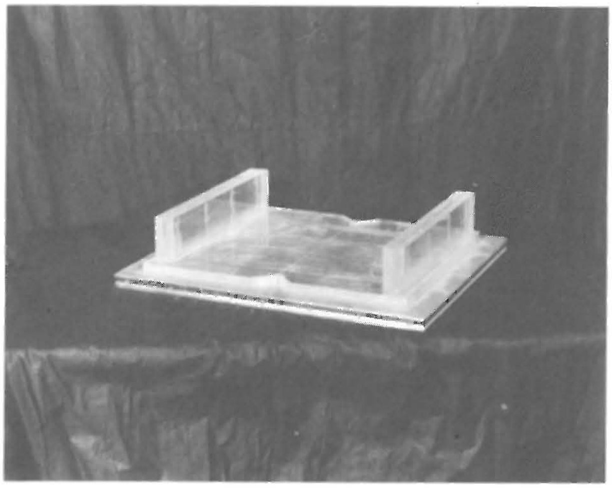

Fig. 8-a.

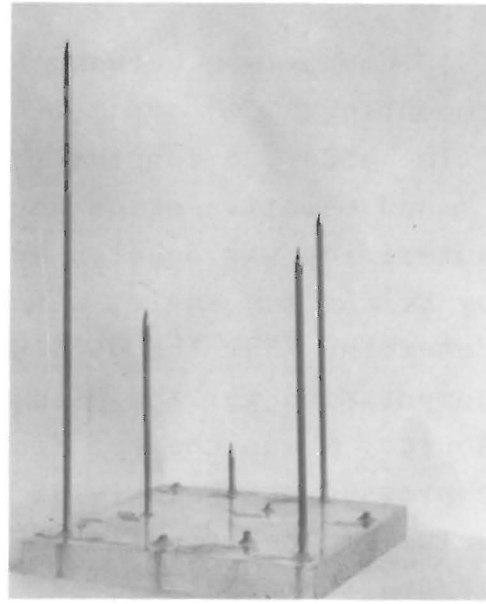

Fig. 8-b.

resin, and this acrylic plate was set on the surface of the special platform when a radiograph involving the relation between the interior orientation and the exterior orientation was taken by the X-ray machine. Refer to Fig. 8-b.

As seen, each top of the control-steel bar was pointed. There was considerable difference between the dispersion of the Z-coordinate of X-ray source and the dispersion of x or y-coordinate of the X-ray source. The standard deviation with regard to the x-coordinate or the y-coordinate was about 1mm$\sim$3mm, but the standard deviation of the z-coordinate was 1mm$\sim$70mm in the preliminary experiment.

(1) Relation between $\theta$ and standard deviation $\sigma$ in Z-direction. The relation between the angle $\theta$ and the standard deviation $\sigma$ in the Z-direction of the X-ray source was shown in Fig. 9, where, $\theta$ mentioned above, was changed from $2^\circ$ to $10^\circ$ . In this examination, the focus-film distance was 770.0mm. The target's height was used as a parameter and it was changed from 30mm to 120mm.

(2) Relation between $\theta$ and Standard deviation $\sigma$ in Z-direction as used as the focal length was the parameter.

This test was done in order to find out the influence on the standard deviation of the focus-film distance. The focus-film distance was expressed as a parameter and changed from 435 mm to 767mm and the angle $\theta$ was changed from $2^\circ$ to $10^\circ$. Refer Fig. 10.

(3) Comparison between First procedure and Second procedure of computing.

In order to compare the two procedures, the relation between $\theta$ and relative error were used. The first procedure of computerizing was denoted by A and the second procedure was denoted by B. As the angle, which was a limiting value in order to determine that the data should be contained or not in the computation for the mean value of the coordinates of the X-ray source, $\theta$ was changed from 2° to 15°. The relative error was expressed on the axis of ordinate by a percentage. The mean value of the target's height was about 50mm, and the focal length was about 770 mm. Refer to Fig. 11.

(4) Practical resultants by X-ray stereography:

(i) Piano-wires, NO1~NO5, were used for the object. The relative positions of each model were measured by a vernier-caliper. The relative error for a series of models were shown in Table 1.

(ii) Eight thoracic vertebras of a skeleton were used for the object. They were measured by the vernier-caliper. The relative error in the case of the former procedure A and the relative error in the case of the latter procedure B were shown in Table 2. Here, other data, i.e. a series of data for numerous angles, were spared. And then a pair of radiographs were also shown in Fig. 12.

6. CONCLUSION

The X-ray machine used in this trial has been used for a long time, i.e. about 20 years. During the former 10 years this machine had been used at a university hospital and afterwards this machine has been used for a basic experiment of radiography at one of the author's laboratory. Therefore, although the anode of the X-ray tube might have been damaged, comparatively high accuracy was obtained. From Fig. 9, 10, 11, it was evident that the procedure to determine the coodinates of the intersection beween two straight lines depends on the precision, namely the precision depended on the angle $\theta$ , and also that the precision of the computation depends on the procedure to solve the simultaneous equations from table 2.

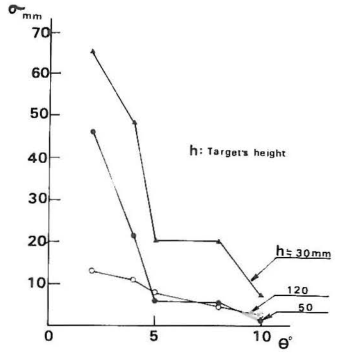


Fig. 9 Relation between θ and Standard deviation σ in z-direction

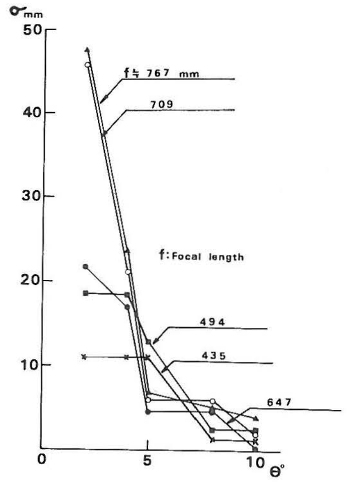


Fig. 10 Relation between θ and Standard deviation σ in z-direction

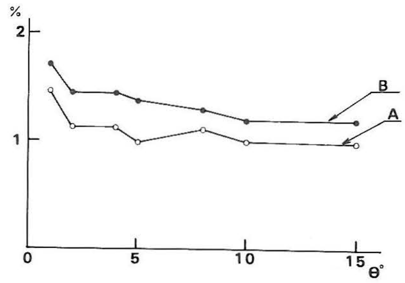


Fig. 11 Relation between θ and Relative error

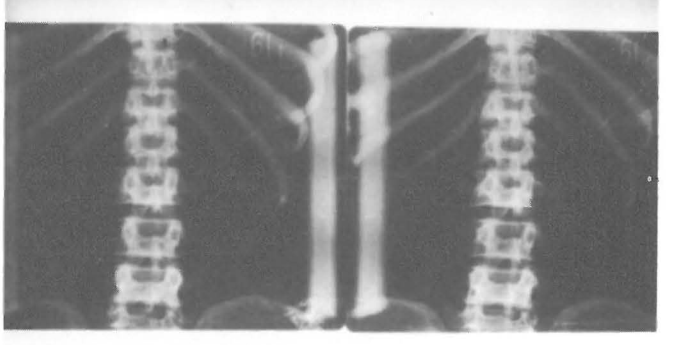

Fig.12

θ = 5°

| Model No | Measurement value in direct method (mm) | Calculated value (mm) | Relative error (%) |
|---|---|---|---|
| 1 | 42.20 | 42.23 | 0.07 |
| 2 | 40.90 | 40.94 | 0.09 |
| 3 | 41.65 | 41.56 | 0.21 |
| 4 | 38.65 | 38.53 | 0.31 |
| 5 | 35.80 | 35.71 | 0.25 |

Table. 1

θ = 10°

| Model No | Measurement value in direct method (mm) | Relative error A (%) | Relative error B (%) |
|---|---|---|---|
| 1 | 40.55 | 0.3 | 0.5 |
| 2 | 44.35 | 0.5 | 0.8 |
| 3 | 43.95 | 0.7 | 1.0 |
| 4 | 47.05 | 2.0 | 2.6 |
| 5 | 46.05 | 1.4 | 1.5 |
| 6 | 49.15 | 0 | 0.2 |
| 7 | 48.60 | 1.9 | 1.1 |
| 8 | 50.10 | 0 | 2.0 |

Table. 2

## 7. ACKNOWLEDGEMENT

An appreciation is extended to Dr. V. KRATKY of the National Research Council of Canada who initially suggested this study and was a good adviser while one of the authors was at the Photogrammetric Research section of NRC.

## 8. REFERENCE

KRATKY, V. 1975. Analytical X-ray photogrammetry in scoliosis. Presented at ASP/UI Symposium on Close Range Photogrammetric Systems, Champaign, Ill. (U.S.A.), July 1975.