Analytical Aerial Triangulation
with
Idealised Models

by

Gouri B. Das
North East London Polytechnic

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Abstract

In this paper a mathematical approach has been outlined for analytical aerial triangulation bypassing the difficulties involved in restitution of bundles to their original spatial situation. Instead of reconstructing the actual models, equivalent models are set up in an idealised way. Besides being simple in concept and principle, these idealised models are easy to form and work with.

One of the photographs in a stereographic pair, conventionally the left hand one, is treated as a perfectly vertical picture and the right hand photograph is then projectively transformed so that the principle of radial line intersection and derivation of heights by basic parallax formula become applicable.

Introduction

A photogrammetrist's idea of a perfect model is one with the two photos taken perfectly vertical from the same height so that they are coplanar. In this case the principles of radial line intersection are valid for planimetry and the simple parallax formula holds for altimetry. Such a model which may be set up in lieu of the actual spatial model is the Idealised Model and may be referred to as MOD for short.

The construction of such a MOD involves a transformation which has the semblance of relative orientation and will be called REL orientation. By this REL orientation the right hand bundle will be so projected that the planes of the two photographs become one and the same. With another conjugate transformation the MOD will be set in altimetry. This transformation will resemble absolute orientation and will be denoted as ABS orientation.

REL Orientation

As in normal analytical methods it will be presumed that the camera is calibrated and the focal length f and distortions dr are known. The image coordinates may then be corrected for all known discrepancies and finally the corrected values of the coordinates on the two photographs are say (x_1, y_1) and (x_2, y_2) . Besides observing the principal points and other pass and control points five good points of detail are observed preferably one in each corner of the model and the fifth in the centre for purposes of REL orientation.

The transformation

$$y_2' = y_1 = \frac{a_{21}x_2 + a_{22}y_2 + a_{23}}{a_{31}x_2 + a_{32}y_2 + a_{33}}, \quad a_{33} = 1,$$

will replace the usual relative orientation procedure. The basic idea is that the right hand bundle may be translated, rotated, scaled and projections and sections may be successively applied to form an Idealised Model with the left hand bundle treated as perfectly vertical.

ABS Orientation

The conjugate transformation

$$x_2' = \frac{a_{11}x_2 + a_{12}y_2 + a_{13}}{a_{31}x_2 + a_{32}y_2 + a_{33}}, \quad a_{33} = 1,$$

will replace the normal absolute orientation for purposes of altimetry. The parameters a_{31}, a_{32} are derived from the preceding REL Orientation. The remaining three parameters viz., a_{11}, a_{12}, a_{13} of ABS Orientation are to be derived from the heights of three known points.

x_2' for these three points may be derived from

$$p = x_1 - x_2' = \frac{fB}{H - h}$$

where p is x-parallax, f is focal length, B is airbase, H is height of aircraft and h is the height of the known point.

Airbase B and Height of Aircraft H

Accepting b as an approximate base and (x_1, y_1) and (x_2, y_2) as the image coordinates of a point the coordinates (X, Y) of the point of radial line intersection referred to the left hand principal point as origin and the base as the X axis are easily derived as

$$X = \frac{b x_1 y_2}{x_1 y_2 - x_2 y_1}$$

$$Y = \frac{b y_1 y_2}{x_1 y_2 - x_2 y_1}$$

If two points with known ground coordinates are radially intersected then the distance d between the points thus intersected with base b may be compared with the corresponding distance D on ground and the scale factor S is obtained as D/d and B the airbase is then S.b .

From $p = x_1 - x_2 = \frac{fB}{H - h}$, with two known height points

$$H = \frac{p_1 h_1 - p_2 h_2}{p_1 - p_2}$$

The focal length may also be calibrated as

$$f = \frac{p_i (H - h_i)}{B}$$

It will be noticed that x_2, y_2 have been used in these derivations. The values of p,f,B,H have to be improved by replacing x_2 y_2 by x_2' y_2'.

Coordination

Finally the known points, pass points and other points are radially intersected and their heights in the MOD are worked out. These three dimensional coordinates X_m Y_m Z_m are then transformed to the ground coordinates X_g Y_g Z_g by a similarity transformation

$$(X_g) = S\, R(X_m) + T(X)$$

where S is the dilation factor, R is the rotation matrix and T the translation vector. The transformation thus involves rotation, change of scale and translation. In case the model does not contain known ground control points then the coordinates may be formed on an arbitrary scale and three well conditioned points may be considered to form the datum plane. The MODs may then be treated as independent models and strips may be formed in the usual manner. The coordinates of the perspective centres are (0 0 H) and (B 0 H) to start with.

Principal Points

It might have been noticed that the principal points are points of reference for image coordinates and the x axis is in the direction of the principal point base line. This may seem to presume that these points are well defined and can be easily observed under stereoscopic fusion. But this is not usually the case. The photographic and the measurement subsystems and their limitations add to the problems of a precise image coordinate reference. However these difficulties are easily resolved as follows. Firstly the principal point has to be located in the respective photographs with reference to the fiducial marks and the available calibration data. If the measuring instrument can intersect only the right hand corners of the left photograph and the left hand corners of the rightphotograph then a similarity transformation

$$x' = a_{11}x + a_{12}y + a_{13}$$

$$y' = - a_{12}x + a_{11}y + a_{23}$$

may be used to derive the coordinates of the principal points.

If however the instrument is capable of intersecting the fiducial marks on the upper and lower sides of the principal points and also along the inner sides of the photographs then an affine transformation

$$x' = a_{11}x + a_{12}y + a_{13}$$

$$y' = a_{21}x + a_{22}y + a_{23}$$

may be used to derive the coordinates of the principal points from the calibration data. If the instrument is capable of intersecting all the fiducial marks then a least square solution will provide the adjusted coordinates of the principal points. To obtain the correct x axis even though the principal points may not be good points of detail the equations of REL and ABS orientations may be used with their approximate x parallax $x_1 - x_2$ to provide the coordinates of the corresponding images for the x parallax on the other photograph. The y coordinate thus obtained will subtend the angle of rotation of the improved axes with respect to the approximate ones.

Iteration

With the precise coordinates of the principal points and the angles of rotation of axes it now remains to transform the approximate coordinates of all the points to the corrected reference system using a metric transformation

$$x' = x \cos \theta + y \sin \theta + t_x$$

$$y' = -x \sin \theta + y \cos \theta + t_y$$

where θ is the angle of rotation and t_x t_y are the translations between the two reference systems.

Having transformed the coordinates of all the points f B H and REL and ABS orientations have to be iterated with the fresh coordinates. This iteration is to be continued till the y coordinates of all corresponding image points are within a prescribed limit say 5 micrometers.

Conclusion

This analytical approach avoids explicit determination of orientation elements and the complexities involved therein. The basic concepts are simple and possibilities of suitable instrumentation are immense specially with chips. The transformations can be utilised for orthophotography and preparation of idealised stereomates.

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