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SOME REMARKS ON LANDSAT MSS PICTURES

ABSTRACT

The lecture is carried out on investigating of possibilities of matching MSS pictures to a Hungarian projection system.

The MSS picture, issued in UTM at the scale of 1 : 1000 000 can be transformed to an other scale and other projection system, namely the Gauss-Krueger projection system. By this way a suitable photo base can be got for producing a photomap at the scale and in the projection system required.

An investigation was carried out on difference between the two projection systems from point of view of transforming possibilities. There is a difference between the datum surfaces of two systems, between the Hayford and Krasovsky ellipsoids. We followed prof. Hazay's method for carrying out transformations. We searched for taking a simple, fast method of transformation can be carried out by well-known photogrammetric method and by means of photogrammetric instruments.

INTRODUCTION

There is a LANDSAT MSS picture form issued at a scale of 1 : 1 000 000, with a format of 23x23 cm2. This picture is transformed to Universal Transversal Mercator projection system.

This space-born imagery can be employed to numerous tasks on the field of cartography and topography.

The topographic mapping in Hungary is carried out in Gauss-Krueger projection system.

We tried to investigate how to match MSS imagery to Gauss--Krueger projection system, and what is the difference between the two systems. We tried to find an analogue method to transform MSS imagery to Gauss-Krueger system from the point of view the difference existing between the UTM and G-K systems.

The Universal Transversal Mercator system's datum surface the ellipsoid of Hayford is, while Gauss-Krueger system is based on Krasovsky ellipsoid.

For transforming the most simple method is to use rectifiers. Working with parallel picture and object plane, the scale of object plane can be changed.

The parallel picture and object plane can be used if the deviation occured by difference of projection systems does not exceed half of nominal resolution at corners of picture. The nominal resolution 79 meters are in both direction on the surface of Earth.

From this point of view we tried to explain the deviation occured by difference of projection systems. In order to explain the deviation, we compared projection equations of projection systems. There is a difference between the datum surfaces of them, so we tried to find relation between the ellipsoids.

ON PARAMETERS OF ELLIPSOIDS

In surveying, geodesy, photogrammetry there is a plain used for representing results of measurements. This plain contains a coordinate system by means of which any point's location can be determined.

The measurements are carried out on physical surface of Earth. This surface is an irregular one and can not be managed by mathematical methods. For projecting results of measurements from this surface to plain of map, the surface must be replaced by a regular one which can be managed by mathematical methods.

This new surface is datum surface of a projection system. Generally it is an ellipsoid of rotation.

The ellipsoid is defined by two parameters, from one must define the size of ellipsoid.

Ellipsoidical parameters:

- a - half of longer axis
- b - polar radius
- e - first eccentricity
- e' - second eccentricity
- l - flattening

Radius of curvature in the meridian:

$$M = \frac{a \cdot (1-e^2)}{(1-e^2 \sin^2 \varphi)^{3/2}} \qquad / \ 1/$$

Radius of curvature in the prime vertical:

$$N = \frac{a}{(1-e^2 \sin^2 \varphi)^{1/2}} \qquad / \ 2/$$

Flattening: $$l = \frac{a - b}{a} \qquad /3/$$

Eccentricities:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} \qquad e' = \sqrt{\frac{a^2 - b^2}{b^2}} \qquad /4/$$

Introduce the following value:

$$\eta^2 = e'^2 \cdot \cos^2\varphi \qquad /5/$$

where φ = ellipsoidical latitude
λ = ellipsoidical longitude

THE GAUSS-KRUEGER PROJECTION SYSTEM

In geodesy generally conformal projection systems are used. The Gauss-Krueger conformal projection system's datum surface Krasovsky ellipsoid is.

The x abscissa is the same as one of the ellipsoid's Soldner coordinate system. The Soldner coordinate system is a rectangular one of ellipsoid, having a meridian as x axis. The y ordinate of Gauss-Krueger system differs from Soldner's one for providing conformal projection.

The Gauss-Krueger projection system is a transversal Mercator one of ellipsoid. The cylinder is tangential to ellipsoid in the central meridian. The zone is 6° wide.

The datum surface of projection system has the following parameters:

a =	6 378 245. 000 m
b =	6 356 863. 019 m
l =	1 / 298.3
e2=	0.006 693 4216
e'2=	0.006 738 5254

Plain coordinates can be got from ellipsoidical ones by using the following projection equations:

$$x = B + A_2\lambda^2 + A_4\lambda^4 + A_6\lambda^6 \qquad /6/$$

$$y = A_1\lambda^1 + A_3\lambda^3 + A_5\lambda^5 \qquad /7/$$

where:

$$A_1 = \frac{N}{\rho} \cos\varphi$$

$$A_2 = \frac{N}{2\rho^2} \operatorname{tg}\varphi \cos^2\varphi$$

$$A_3 = \frac{N}{6\rho^3} \cos^3\varphi (1 - \operatorname{tg}^2\varphi + \eta^2)$$

$$A_4 = \frac{N}{24\rho^4} \cdot \operatorname{tg}\varphi \cos^4\varphi (5 - \operatorname{tg}^2\varphi + 9\eta^2 + 4\eta^4)$$

$$A_5 = \frac{N}{120\,\rho^5}\cos^5\varphi\,(5 - 18\,tg^2\varphi + tg^4\varphi)$$

$$A_6 = \frac{N}{720\,\rho^6}\,tg\,\varphi\,\cos^6\varphi\,(61 - 58\,tg^2\varphi + tg^4\varphi)$$

$$B = \int_0^\varphi M\,d\varphi = a(1-e^2)\int_0^\varphi \frac{d\varphi}{(1-e^2\sin^2\varphi)^{3/2}}$$

THE UNIVERSAL TRANSVERSE MERCATOR PROJECTION SYSTEM

The UTM projection system is a conformal one. This system is a Gauss-Krueger type one. The central meridian is longitude of origin, while latitude of origin the Equator is. This system is not tangential, it is an intersecting one. The scale factor at the central meridian :

$$k_o = 0.999\ 6$$

The datum surface of this projection system the Hayford ellipsoid is having the following parameters:

a =	6 378 388. 000 m
b =	6 356 911. 946 m
l =	1 / 297
e2=	0.006 722 6700
ė2=	0.006 768 1702

Plain coordinates can be got from ellipsoidical ones by using the following projection equations:

$$x = /I/ + /II/p^2 + /III/p^4 + A_6 \qquad /\ 8/$$

$$y = /IV/p + /V/p^3 + B_5 \qquad /\ 9/$$

where:

$$(I) = S \cdot k_o$$

$$(II) = \frac{\nu \cdot \sin\varphi\,\cos\varphi \cdot \sin^2 1''}{2}\,k_o \cdot 10^8$$

$$(III) = \frac{\nu \cdot \sin\varphi\,\cos^3\varphi \cdot \sin^4 1''}{24}\,(5 - tg^2\varphi + 9\eta^2 + 4\eta^4) \cdot k_o \cdot 10^{16}$$

$$(IV) = \nu \cdot \cos\varphi \cdot \sin 1'' \cdot k_o\,10^4$$

$$(V) = \frac{\nu \cdot \cos^3\varphi \cdot \sin^3 1''}{6}\,(1 - tg^2\varphi + \eta^2)\,k_o \cdot 10^{12}$$

$$A_6 = \frac{\nu \cdot \sin\varphi\,\cos^5\varphi\,\sin^6 1''}{720}\,p^6(61 - 58\,tg^2\varphi + tg^4\varphi +$$

$$+ 270 \cdot e'^2\cos^2\varphi - 330\,e'^2\sin^2\varphi)\,k_o \cdot 10^{24}$$

$$B_5 = \frac{\nu \cdot \cos^5\varphi \ \sin^5 \ 1''}{120} p^5 (5 - 18\, tg^2\varphi + tg^4\varphi +$$

$$+ 14 e'^2 \cos^2\varphi - 58\, e'^2 \sin^2\varphi) \cdot k_o \, 10^{20}$$

$$S = \int_0^{\varphi} M d\varphi = a(1-e^2) \int_0^{\varphi} \frac{d\varphi}{(1 - e^2 \sin^2\varphi)^{3/2}}$$

$$p = 0,0001\, \lambda$$

$$\nu = \frac{a}{(1 - e^2 \sin^2\varphi)^{1/2}}$$

The factors of 10 take parts in expressions for calculating with 10 digit calculators /1/.

CONVERSION BETWEEN PROJECTION SYSTEMS

In first step of investigation we disregard of various datum surfaces of projection systems. In this manner the two systems can be regarded with a common, theoretical datum surface.

We suppose the projection equations of two systems are the same. Comparing expressions our supposition can be proved.

Let's compare the expressions B of Gauss-Krueger system, and /I/ of UTM.

$$B = \int_0^{\varphi} M d\varphi \qquad (I) = k_o S = k_o \int_0^{\varphi} M d\varphi$$

There is a difference existing between two terms, caused by k_o. This member of expression does not depend on position on ellipsoid, it is a constant.

Then compare the A_2 expression of G-K system with /II/ of UTM.

$$A_2 = \frac{N}{2\varrho^2}\, tg\,\varphi \cdot \cos^2\varphi$$

$$(II) = \frac{\nu \cdot \sin\varphi \cdot \cos\varphi \cdot \sin^2 1''}{2} \cdot k_o \cdot 10^8$$

$$N = \nu$$

CONSTANT TERMS: $1/2\varrho^2$

$\sin^2 1'' \cdot k_o \cdot 10^8/2$

$$N \cdot tg\,\varphi \cdot \cos^2\varphi = N \cdot \cos\varphi \cdot \sin\varphi$$

$$N \cdot \cos\varphi \cdot \sin\varphi = N \cdot \cos\varphi \cdot \sin\varphi$$

The existing difference is because of constant terms. The two expressions are fundamentally same.

Let's compare expressions A_4 of G-K, and /III/ of UTM.

$$A_4 = \frac{N}{24\varrho^4} tg\,\varphi \cdot \cos^4\varphi \cdot (5 - tg^2\varphi + 9\eta^2 + 4\eta^4)$$

$$(III) = \frac{\nu \cdot \sin\varphi \cdot \cos^3\varphi \cdot \sin^4 1''}{24} (5 - tg^2\varphi + 9\eta^2 + 4\eta^4) K_0 \cdot 10^{16}$$

The constant terms: $1/\varrho^4$; $\sin^4 1'' \cdot K_0 \cdot 10^{16}$

$$A_4 = \frac{N \cdot \sin\varphi \cdot \cos^3\varphi}{24} (5 - tg^2\varphi + 9\eta^2 + 4\eta^4)$$

$$(III) = \frac{N \cdot \sin\varphi \cdot \cos^3\varphi}{24} (5 - tg^2\varphi + 9\eta^2 + 4\eta^4)$$

The two expressions are the same.

Now, let's compare the expression A_6 of G-K, and A_6 one of UTM.

$$A_6 = \frac{N}{720\varrho^6} tg\,\varphi \cos^6\varphi (61 - 58\, tg^2\varphi + tg^4\varphi)$$

$$A_6 = p^6 \frac{\nu \sin\varphi \cos^5\varphi \sin^6 1''}{720} (61 - 58\, tg^2\varphi + tg^4\varphi + 270 e'^2 \cos^2\varphi - 330 e'^2 \sin^2\varphi) k_0 10^{24}$$

The two expressions are the same fundamentally. The p^6 term of A_6 of UTM does not disturb, because p = 0, 000 1 λ. The expression A_6 of G-K is multiplied by λ^6, but A_6 of UTM is not. The last two terms in parantheses of A_6 of UTM have no valuable influence to the value of y, so they are negligable.

Then compare G-K systems A_1 and UTM's /IV/ expressions:

$$A_1 = \frac{N}{\varrho} \cos\varphi \qquad (IV) = \nu \cdot \cos\varphi \cdot \sin 1'' \cdot 10^4$$

$$N = \nu$$

$$\frac{a \cdot \cos\varphi}{(1 - e^2 \sin^2\varphi)^{1/2}} = \frac{a \cdot \cos\varphi}{(1 - e^2 \sin^2\varphi)^{1/2}}$$

CONSTANTS: $\frac{1}{\varrho}$; $\sin 1'' \cdot 10^4$

They are fundamentally same.

A_3 expression of Gauss-Krueger projection system is same with /V/ expression of UTM:

$$A_3 = \frac{N}{6\varrho^3} \cos^3\varphi (1 - tg^2\varphi + \eta^2)$$

$$(V) = \frac{\nu \cdot \cos^3\varphi \cdot \sin^3 1''}{6} (1 - tg^2\varphi + \eta^2) k_0 10^{12}$$

Finally, the A_5 of G-K is same, as B_5 of UTM:

$$A_5 = \frac{N}{120\varrho^5} \cos^5\varphi (5 - 18\, tg^2\varphi + tg^4\varphi)$$

$$B_5 = p^5 \frac{\nu \cos^5\varphi \sin^5 1''}{120} (5 - 18\, tg^2\varphi + tg^4\varphi + 14 e'^2 \cos^2\varphi - 58 e'^2 \sin^2\varphi) K_0 10^{20}$$

CONSTANTS $1/120\,\varrho^5$

$\sin^5 1'' \cdot K_0 \cdot 10^{20}/120$

The last two terms in parantheses of B_5 of UTM can be negligable , having no valuable influence.

Our supposition was right. There is no fundamental difference between Gauss-Krueger projection system's and Universal Transverse Mercator projection system's projection equations.

The difference is caused by constants, but it does not cause difference in shape, cause difference only in size- which can be negliged by multiplying, or when rectifying, by changing scale.

It can be stated, that the difference between two projection systems will not cause a deviation, when enlarging MSS imagery.

RELATING ELLIPSOIDS

Above we disregard of various datum faces of projection systems. The same nature of projection systems will not cause deviations, when enlarging. The difference existing between the two ellipsoids may cause deviation.

When regarding one of ellipsoids as a datum surface a relation can be find to transform to the other one. The second one can be regarded as a picture surface.

We should carry out a projecting from datum surface to picture surface. The datum surface Hayford ellipsoid is, while the picture one Krasovsky ellipsoid is.

For conformal projecting, prof. Hazay stated a projection equation /5/ for projecting between two ellipsoids.

We have two conditions:

- the normal parallel of both ellipsoids should have the same ellipsoidical latitude

$$\varphi_0 = \varphi_{01} = \varphi_{02}$$

- after carrying out projection, the normal parallel should keep its lenght with no distorsion.

One of projection equations is:

$$\lambda_2 = n \cdot \lambda_1 \qquad /10/$$

where

λ_2 - longitude on picture surface

λ_1 - longitude on datum surface

$n = \frac{\lambda_2}{\lambda_1}$ - ratio of theirs

The other projection equation is given by the following formulae:

$$tg\left(45^\circ + \frac{\varphi_2}{2}\right)\left(\frac{1 - e_2 \sin \varphi_2}{1 + e_2 \sin \varphi_2}\right)^{\frac{e_2}{2}} = k \cdot \left[tg^n\left(45^\circ + \frac{\varphi_1}{2}\right)\left(\frac{1 - e_1 \sin \varphi_1}{1 + e_1 \sin \varphi_1}\right)^{\frac{n \cdot e_1}{2}}\right] \qquad /11/$$

After calculating values of n and k, the projecion can be done.

For finding values of n and k we should select $\varphi_o = 47^{\circ}$ as a latitude of normal parallel. It is normal parallel of Hungary.

$$\varphi_0 = \varphi_{01} = \varphi_{02}$$

The factor n can be determined:

$$n = \frac{N_{o1}}{N_{o2}} \qquad / 12/$$

where:

$$N_{o1} = \frac{a_1}{(1 - e_1' \sin^2 \varphi_0)^{1/2}} \quad \text{and} \quad N_{o2} = \frac{a_2}{(1 - e_2' \sin^2 \varphi_0)^{1/2}}$$

The factor k can be determined from equation /11/.

For $\varphi_o = 47^{\circ}$, $n = 1.\ 000\ 030\ 27$
$k = 0.\ 999\ 993\ 70$

By using these factors, a projection can be carried out from Hayford ellipsoid to Krasovsky ellipsoid.

When projecting the parallel of $= 49^{\circ}$, in latitude half a second change will occur. It's volume: 15,35 ms.

When calculating with the following equation: $\lambda_2 = n \cdot \lambda_1$
λ can be 3°.
$3 \times 1.\ 000\ 030\ 27 = 3,\ 000\ 090\ 81 = 3 - 00 - 00,326.$
It's less, than half a second.

SUMMARY

We investigated possibilities of transforming MSS imagery to Gauss-Krueger system. The difference between UTM and G-K systems will not cause deviations when enlarging the MSS imagery.

The conformal projection between their datum surfaces will cause in range of 3° in latitude 15 ms deviation in N-S direction and 9 ms in E-W direction.

As a summary, we can state that LANDSAT MSS imagery can be transformed to Gauss-Krueger projection system with smaller deviation caused by differences of projection systems than nominal terrain resolutions' half is. From point of view the projection systems, the enlarging can be done.

LITERATURE

/1/ Army Map Service: Universal Transverse Mercator Grid
U.S. Army Corps of Engineers, Washington D.C. 1955
pp 55.

/2/ Bahr, H.-P.: Geometrical Analysis and Rectification of LANDSAT MSS Imagery; Comparison of Different Methods
Presented Paper to the Symposium of Comm. III. of ISP. Moscow, 1978. pp25-46

/3/ Fister, F: A kozmikus felvételek anyagainak komplex felhasználása a térképészet céljaira.
Report, Budapest 1979. pp 135.

/4/ Hazay, I.: Földi vetületek
Budapest, 1954. pp 508.

/5/ Hazay, I.: Vizsgálatok a különböző ellipszoidok közötti vetités köréből
In: Geodézia és Kartográfia, vol 7 no. 1
Budapest 1955 pp 1 - 6.