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DIFFERENTIAL RECTIFICATION OF DIGITAL OR DIGITIZED IMAGERIES

ABSTRACT

After a short introduction to the problem, firstly the direct and the indirect method for differential rectification of digital or digitized imageries are described. Ensuing the application of these two methods to the rectification of digital satellite images, digital aircraft images and to digitized two dimensional images is discussed with regards to various aspects connected with the problem. Some aspects are: restitution of the exterior orientation, implementation of digital height models, post-processing of the output image. The last chapter deals with data handling problems.

## INTRODUCTION

With the beginning of space travel the development of digital image recording, transmission and processing was pushed essentially. Naturally firstly devices for digital image recording the so called scanners and devices for writing digital images were developed. Soon the last ones were used to digitize ordinary aerial or other photography in order to apply the possibilities of digital image processing to them too. The development of the scanner went on from those with rotating mirror optics to the so called linear solid state scanners. In these scanners the scanning line is sampled in small, discrete adjacent sensor elements. It allows scanning without any moving parts. The development tendencies aim to two dimensional arrays, but the current state of technology allows only to produce small areas of high resolution. Up to now matrices of 380 by 438 detectors are available (/1/).

The techniques and methods in digital image processing have been pushed to high standards too. This is true in regard to hardware aspects as well as in regard to software developments. But for a long time it was not payed attention enough to the first step of image processing, the geometric rectification. Only since the other image processing methods meet high standards, the necessity for high efficient differential rectification methods became evident in general. The rectification itself is performed in two steps. In the first step the transformation parameters between image and terrain coordinates have to be calculated and in the second step the rectification has to be performed according to these parameters. Firstly the transformation parameters between the distorted image and the ground control have to be determined. There are existing different methods, which can be divided in two groups (/2/, /3/). The most simple methods determine one set of transformation parameters for the whole image according to a certain interpolation model (similarity transformations, affin transformations, etc.) from available control at control points (/4/, /5/). This kind of methods can be sufficient for relatively undistorted images (e.g.: satellite imagery). In general they are called non parametric methods. In opposite to them are the parametric methods, which rectificate the image data by rigorous relations between image and terrain coordinates (collinearity equations). The variation of the unknown exterior orientation parameters from line to line is described by certain mathematical models (e.g.: polynomials, Gauss Marcov processes). Usually the necessary input information are control point coordinates. These rectification methods are able to consider recorded exterior orientation parameters, but in general they do not because of the low accuracy level of the recordings. That shall be enough for this topic. It will be treated in detail in another paper of WG III/1 (/6/).

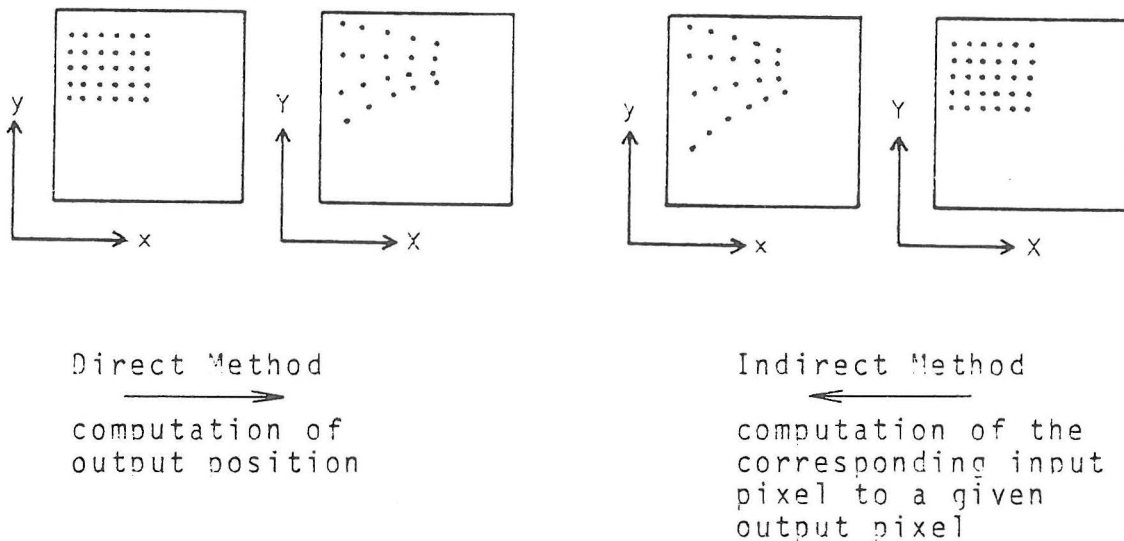
This paper deals with the second step of digital rectification the transfer of every single picture element (pixel) from its

actual position to the corrected location. For this task there are existing only two methods, the direct and the indirect method.

### RECTIFICATION METHODS

Both methods are using as input information the digital or digitized image, which means a rectangular matrix of  $m \times n$  numbers representing the grey shades  $q_{ij}(x_i, y_j)$  ( $i = 1 \dots m$ ,  $j = 1 \dots n$ ) of the picture elements. Row and column number define the location of each pixel within the image.

The direct method calculates for each pixel of the rectangular matrix an output position in a rectangular coordinate system. The grid structure however will be destroyed in the output image. The indirect method goes the reverse way. For each pixel of a rectangular output grid this method calculates the corresponding input position for the transfer of grey shades.



### RECTIFICATION OF DIGITAL SATELLITE IMAGES

Digital images recorded from satellites are characterized by a relatively good geometric quality, which is caused by the quiet and stable flight behaviour of satellites. This means that also simple or less rigorous rectification methods (non-parametric methods) may supply with good rectified imagery. Using for example a two dimensional polynomial of second order we get the following formulas for the direct (1) and the indirect (2) rectification method.

Direct method:

$$X = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 \quad (1a)$$

$$Y = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2 \quad (1b)$$

Indirect method:

$$x = c_0 + c_1X + c_2Y + c_3XY + c_4X^2 + c_5Y^2 \quad (2a)$$

$$y = d_0 + d_1X + d_2Y + d_3XY + d_4X^2 + d_5Y^2 \quad (2b)$$

X,Y: terrain coordinates

x,y: image coordinates

The parameters a, b or c, d have to be determined by a least squares adjustment. The necessary observation equations can be derived from equation (1) resp. (2), which lead to two equations for each control point. The disadvantage of the direct method is, that it does not result in a regular output grid. Apart from the non regular grid, which will be obtained, the output image has gaps and overlaps. That means that from this image a regular grid must be computed for the digital image output device. This has to be done off-line using the methods of digital image processing. In contrast to it the indirect method has the big advantage that a regularly formed output grid will be obtained immediately.

However, the more efficient the sensors become (which firstly means higher resolution) the better the rectification methods (parametric methods) have to be. Especially relief displacement has to be taken into consideration. For images taken from aircraft this is absolutely necessary, if good rectification results shall be obtained.

## RECTIFICATION OF DIGITAL AIRCRAFT IMAGES

Digital aircraft images are characterized by relatively large distortions and big relief displacements caused by wide scan angles, if the surface has not been too flat. For the rectification of aircraft scanner images therefore high efficient methods (parametric methods) using rigorous formulas describing the relation between image and ground coordinates have to be applied. The collinearity equations are those rigorous relations between image and terrain. Certain inaccuracies caused by the exterior orientation parameters are unavoidable, because it is not possible to determine them for each image line (recordings are too inaccurate up to now). The variations from line to line have to be modeled adequately in order to bridge over controlless areas. Thus the various methods differ from each other. Such methods can use polynomials, Gauss-Markov processes (random processes), Fourier Functions

or something else. Apart from that unavoidable inaccuracy the following equations are the rigorous relations between image and terrain.

Direct method:

$$X = (Z - Z_{oj}) \frac{a_{11j}x + a_{12j}y - a_{13j}c}{a_{31j}x + a_{32j}y - a_{33j}c} + X_{oj} \quad (3a)$$

$$Y = (Z - Z_{oj}) \frac{a_{21j}x + a_{22j}y - a_{23j}c}{a_{31j}x + a_{32j}y - a_{33j}c} + Y_{oj} \quad (3b)$$

$Z = f(x,y)$  from a digital height model (DHM)

Indirect method:

$$x = -c \frac{a_{11j}(X-X_{oj}) + a_{21j}(Y-Y_{oj}) + a_{31j}(Z-Z_{oj})}{a_{13j}(X-X_{oj}) + a_{23j}(Y-Y_{oj}) + a_{33j}(Z-Z_{oj})} \quad (4a)$$

$$y = -c \frac{a_{12j}(X-X_{oj}) + a_{22j}(Y-Y_{oj}) + a_{32j}(Z-Z_{oj})}{a_{13j}(X-X_{oj}) + a_{23j}(Y-Y_{oj}) + a_{33j}(Z-Z_{oj})} \quad (4b)$$

$Z = f(X,Y)$  from a DHM

With

$c$ : calibrated focal length

$a_{ikj} = f(\omega_j, \phi_j, \kappa_j) \quad i = 1...3, \quad k = 1...3$

$j$ : index of the image line

The exterior orientation parameters  $X_{oj}, Y_{oj}, Z_{oj}, \omega_j, \phi_j, \kappa_j$  may be functions of the image coordinates  $x, y$  as used for instance for polynomials (/7/). A more adequate way to model the time dependend courses of the exterior orientation parameters can be the use of stochastic (random) processes as it is shown e.g. in /8/ and /9/.

Both methods (direct, indirect) need for a complete rectification height information. The direct method needs a DHM based on image coordinates. But usually the DHM's exist in rectangular grids based on ground coordinates. Therefore the height must be calculated by intersecting the projection beam with the surface of the DHM. This surface has to be modeled artificially by using the surrounding discrete height infor-

mation of several grid points. Usually the interpolation between the discrete heights goes linearly in X and Y if the grid is dense enough. The intersection has to be calculated iteratively. The number of iterations strongly depends on the ground form and on the approximate value Z of the height which has to be calculated. It is easy to obtain good approximate values Z. If firstly the ground coordinates of the first and the last pixel of each line are determined by intersecting the projection beams with the DHM, the approximate values X, Y and Z of the other picture elements can be calculated very easily from the known pixel size and the known general direction of the projected line on the ground (the projection is not a straight line). Usually one iteration of the intersection will be enough, because the deviations of the new values X, Y, Z from X, Y, Z will be within acceptable tolerances. Connected with this topic is the problem of hidden surfaces and multiple solutions. A definit solution may be found as explained for instance in /10/. Subsequently the output image has to be processed by means of digital image processing too, because of the gaps and overlaps.

The indirect way needs the height information based on a rectangular ground coordinate grid. Usually DHM's do have this configuration, eventually they have to be densified. From DHM's based on triangle meshes, the rectangular grid has to be interpolated firstly.

When calculating the position x, y of a pixel in the image space the problem appears, that we do not know, which projection center (we have one for each line) belongs to the ground pixel. A solution for this problem can be found by calculating the correct projection center iteratively, if the orientation parameters depend on x and y (/11/). If they do not (/9/) the solution can be very complicated. In this case the direct method should be preferred.

## RECTIFICATION OF DIGITIZED TWO DIMENSIONAL IMAGES

Digital image processing and digital rectification respectively mostly have been used for remote sensing applications, but can be applied to conventional imagery too, if they have been digitized before. For aerial photography then we have the well known collinearity equations, which give us the relation between ground and image coordinates.

$$x = -c \frac{a_{11}(X-X_0) + a_{21}(Y-Y_0) + a_{31}(Z-Z_0)}{a_{13}(X-X_0) + a_{23}(Y-Y_0) + a_{33}(Z-Z_0)} \quad (5a)$$

$$y = -c \frac{a_{12}(X-X_0) + a_{22}(Y-Y_0) + a_{32}(Z-Z_0)}{a_{13}(X-X_0) + a_{23}(Y-Y_0) + a_{33}(Z-Z_0)} \quad (5b)$$

Z = f(X, Y) from a DHM

The set of the six orientation parameters  $X_0, Y_0, Z_0$  and  $\omega, \phi, \kappa$  ( $a_{ijk} = f(\omega, \phi, \kappa)$ ) can be calculated for instance by a resection in space. Because we do have a two dimensional image with only one corresponding projection center we do not have coordination problems as explained above. Therefore we can apply both methods (direct, indirect) without difficulties. In this case the indirect method (formulas (5a), (5b)) should be preferred, because then we can avoid to calculate a rectangular output grid afterwards. The height information must be based on a rectangular ground coordinate grid. Existing DHM's eventually have to be densified or interpolated again if they do not have rectangular meshes.

There are interesting applications in terrestrial photogrammetry too, especially in the field of architecture photogrammetry, where often unwindable facades have to be reeled off onto a plane, which can be done very comfortable by digital image processing methods as explained in /12/. For this task an analytical formulation of the object to be rectified is necessary in advance (the corresponding task in aerial photography would be the implementation of a DHM). Thus we are able to calculate rectified object coordinates of each picture element and its position in the projection. In this case the indirect method has the same advantages as explained above for aerial photography and can be used very successfully for digital rectification.

#### DATA HANDLING ASPECTS

Data handling is the biggest problem of digital differential rectification. Because of the very large quantity of data, which has to be processed, an optimal data handling concept will help to save time and costs. Operational programs should find a reasonable middle course between storage and computing time requirements. Roughly said time and storage are reciprocal to each other. As storage in core usually is a limiting factor the rectification has to be done in blocks.

Considering the direct rectification method using collinearity equations, we are able to estimate the extent of an output matrix, which is necessary in order to rectify one single image line on-line by directly addressing in core. The extent of the matrix we get from simplified differential equations (6). They are true for a central perspective linear array scanner.

$$\Delta X_{\max} = -h\Delta\phi - h \tan \frac{\theta}{2} \Delta\kappa \quad (6a)$$

$$\Delta Y_{\max} = \tan \frac{\theta}{2} \Delta Z_0 + h(1 + \tan^2 \frac{\theta}{2})\Delta\omega \quad (6b)$$

The scanner characteristics are assumed to be: resolution 13  $\mu\text{m}$ ,  $c = 24 \text{ mm}$ , scan angle  $\theta = 50^\circ$  and 1728 pic-



ture elements per image line. This corresponds with the electro optical (linear array) scanner of MBR (compare /14/). The deviations of the orientation parameters are assumed to be within the following values (compare /13/):

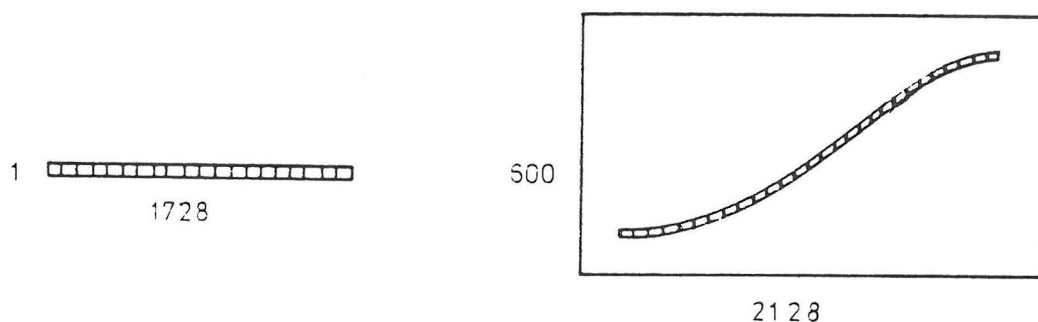
$$\Delta Z_0 < 0.02h, \quad \Delta \omega < 5^{\circ}, \quad \Delta \phi < 5^{\circ}, \quad \Delta \kappa < 10^{\circ}$$

In the worst case, if all the deviations have the same sign, we get, expressed in pixels of ground resolution:

$$|\Delta X_{\max}| \approx 300 \text{ [rsu]}$$

$$|\Delta Y_{\max}| \approx 200 \text{ [rsu]}$$

Therefore we should have an output matrix of not less than 600 x 2128 elements, if we would like to address the output pixels directly in central memory.

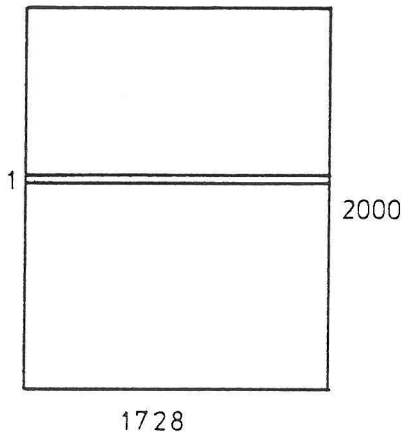


Input Image Line

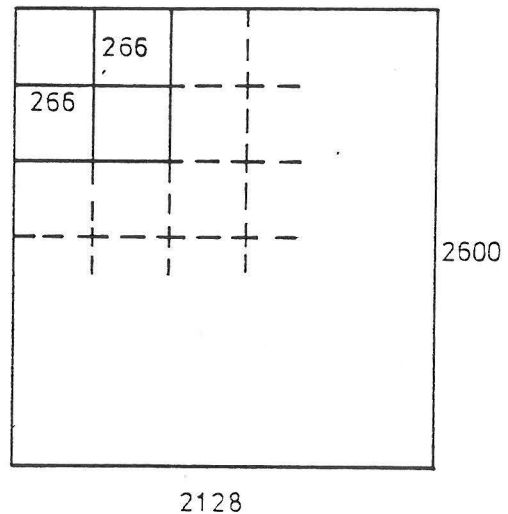
Output Image Matrix

But even this output matrix of 1.28 Mbyte, which is only able to allow direct addressing for one single image line, is too large for the core storage. Therefore it is necessary to divide the output matrix into submatrices or blocks both by the line and by the column. The best way to do it, is to create a direct access file of some records, whose lengths correspond with the submatrix size. In this way the elements of an input matrix of for example 2000 x 1728 pixels can be transferred line by line to the output matrix of 2600 x 2128 elements which may be divided into 80 submatrices of 266 x 266 elements.





Input Matrix  
(actual required  
storage: 1 line)



Output Matrix  
(actual required storage:  
submatrix of 266 x 266  
elements)

This direct access technique has the big advantage, that the central memory demand, which is the main limiting factor of a computer, can be lowered down to the absolute minimum if the submatrix size is made very small. Therefore such digital rectifications even can be done on minicomputers, which usually have a core memory capacity from 32 K-bytes, up to 256 K-bytes and direct addressable disk cartridges of several M-bytes. By it computing times will rise, but that is no limiting factor, because minicomputers usually belong to a small circle, which does not employ the computer to capacity.

Equal considerations are true for the indirect rectification method with changed signs. In this case the output image is handled sequentially line by line and the input image has to be stored divided in submatrices and direct accessible.

Computing times can be lowered down essentially, if only some pixels are transferred using collinearity equations and digital height information. To the picture elements located within those corner points simple rectification methods (e.g. interpolation models) can be applied to. But reducing computing times by this way could be paid by a certain loss of accuracy.

An interesting possibility for digital differential rectification of aerial photography in connection with a microdensitometer was shown in /15/. Using the indirect rectification method for each pixel of the output matrix the corresponding input position is calculated using the rigorous equations and height information from a DHM. Then the densitometer is moved by servomotors into this position in order to take the grey shade of the pixel. The grey shades are stored on a magnetic tape. This method can be done with a relatively small computer, because there are only low demands on storage capacity.

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