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The internal reliability in a 3-dimensional strip adjustment with independent models

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Abstract

In this paper the author will give a study of the reliability of testing the photogrammetric observations in a 3-dimensional strip adjustment with independent models, using the 'B-method of testing'.
Several strip parameters will be changed, for example strip length, position of the tie points, covariance matrix, height differences in the model.
Finally an example is given of testing the observations.

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1 Introduction

The reliability of testing,which is in the terrestrial measurement already a well-known concept,can also be used in the photogrammetric measurements.It concerns a magnitude of error which under certain conditions just can be detected.The benefit of this test value has already been prooven in terrestrial networks,where the measurements are done in such a way that they satisfy to the specified demands of precision and reliability.In every adjustment the reliability of testing the observations can be computed.This reliability can be expressed in boundary values of the observations.Every observation,which is checked, has a boundary value,while a observation with an infinite boundary value is unchecked.These values,coming up for discussion in this paper,belong to the 'B-method of testing' of Baarda[1968].Testing of observations is done by computing variates,which are stochastically independent and have a standard normal probability distribution.

An important adjustment in the photogrammetric process is the block adjustment with photos or models.One of the possibilities is now to examine the reliability of testing the observations of a triangulation with models.In order to check the photogrammetric measurements only it is necessary to connect the block or strip to a minimum number of control points.This means for a 2- or 3-dimensional measurement a transformation to 4 respectively 7 control point coordinates.Only in this way the photogrammetric measurements can be checked in the right way on gross errors.After detection of all the errors,the block or strip can be connected to all the control point coordinates.The investigation, concerning the reliability in a 2-dimensional block,has been published in Förstner[1978] and Neleman[1978].

In this paper is treated the 3-dimensional strip adjustment and not the block adjustment,because a block,with 20% sidelap of the photos, cannot be connected to the minimum number of 7 control point coordinates.

Since the boundary value is among others depending on the number of conditions it is possible to do the measurements in such a way,that an accepted error detection arises.

The 3-dimensional block adjustment has been programmed according to the theory in chapter 2.The boundary values are strongly correlated to the covariance matrix of the observations.That's why all the parameters will be computed with two different covariance matrices. Which matrix will be choosen,depends on the user.Finally a practical example will be given of the 'B-method of testing' with real observations of 2 models.

2 Theory

2.1 Adjustment

A condition model for the 3-dimensional block adjustment is the orthogonal transformation.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda (R_{\omega\phi\kappa}) \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \qquad (2.1)$$

X,Y,Z : terrain coordinates
λ : scale factor
$R_{\omega\phi\kappa}$: orthogonal matrix
x,y,z : observations in the model
$\delta X, \delta Y, \delta Z$: translational parameters

The adjustment is carried out according to the 2nd standard problem for non-linear problems in the form

$$(\underline{\Delta x}^i + \underline{\varepsilon}^i) = (a^i_\alpha)(\underline{\Delta Y}^\alpha) \qquad (2.2)$$

Planimetry and heigt are not adjusted separately.Observations are the terrain coordinates of the control points and the observations in the model(perspective centre included).The unknown variates are all the terrain coordinates and per model the 7 transformation parameters. Linearisation of formula (2.1) gives the normal equations.The program, devoleped for research at the Department for Geodesy in Delft,computes also the approximate values of the unknowns.The way which these approximate values are computed and also the testing before the adjustment makes it not necessary to iterate.Apart of the computation of the terrain coordinates,the program can also compute the corrections to the observations,test the observations by the 'B-method of testing' and make precision and/or reliability analises.The corrections to the obsevations are computed as follows:

$$(\underline{\varepsilon}^i) = (\underline{\Delta \overset{\cdot}{X}}^i) - (\underline{\Delta x}^i) \qquad (2.3)$$

$(\underline{\Delta x}^i)$ = derivated observation
(≡misclosure of formula (2.1) if filled with approximate values for the unknows and the measured values for the observations)

$(\underline{\Delta \overset{\cdot}{X}}^i)$ = adjusted derivated observation

In the adjustment the following assumptions are made :

1. the photogrammetric model is similar to the terrain
2. the observations in the model don't correlate with the observations of the perspective centre
3. the observations in the 'whole' model don't correlate with the coordinates in the terrain.

2.2 Internal reliability and testing

The reliability of testing the observations is computed according to the theory of the 'B-method of testing' of W.Baarda[1968].In this paragraph the most important formulas of this theory are given.
The internal reliability is described by the boundary values of the observations.The boundary value is the error in an observation,which just can be detected by using a certain alternative hypothesis with a power of $100\beta_o$% and a significance level of $100\alpha_o$%.

$$(\nabla_p \tilde{x}^i/\sigma) = (c^i_p)\sqrt{\lambda_o / N_p} \qquad (2.4)$$

$\nabla_p \tilde{x}^i$ = boundary value of the i^{th} observation and the p^{th} alternative hypothesis.

σ = square root of the variance factor

(c^i_p) = vector,defining the alternative hypothesis

λ_o = level,computed from the numerical function $\lambda_o = \lambda(\alpha_o, \beta_o, 1, \infty)$

$$N_p = (c^j_p)^*(\bar{g}_{ji})(g^{ij} - G^{ij})(\bar{g}_{ji})(c^i_p) \qquad (2.5)$$

(g^{ij})= matrix of weight coefficients

$(\bar{g}_{ji})$= $(g^{ij})^{-1}$ = weightmatrix

$\sigma^2(G^{ij})$= covariance matrix of the adjusted observations

From (2.4) can be derived that the internal reliability is dependent on
1. the covariance matrix
2. the alternative hypothesis
3. the values of α_o and β_o
4. the condition model and the design of the measurement

Special attention must be devoted to the (c_p^i) vector.This vector defines the alternative hypothesis.The easiest alternative hypothesis is the conventional alternative hypothesis(=H_a). This one defines that one observation has an error and all the others are 'good'. The (c_p^i) vector is then a unit vector with the element '1' at the place of the wrong suggested observation.

The testing of the observations occurs in two steps
1. testing of the shifting variate.
2. testing of the observations with the conventional alternative hypothesis ('data-snooping').

The shifting variate $\underline{E}$ is computed as follows:

$$\underline{E} = (\underline{\varepsilon}^j)^*(\bar{g}_{ji})(\underline{\varepsilon}^i) \tag{2.6}$$

From this $\underline{E}$ we can make the estimator of σ^2 with

$$\underline{\hat{\sigma}}^2 = \underline{E} / b \tag{2.7}$$

(b = number of conditions)

The shifting variate $\underline{E}$ will be accepted or the null hypothesis (H_o) will be accepted if

$$\underline{\hat{\sigma}}^2/\sigma^2 \leq F_{1-\alpha,b,\infty}$$

where α is the significance level of the multi-dimensional test. When H_o of the multi-dimensional test will be accepted it is still necessary to continue with 'data-snooping', because the H_o can be accepted wrongly. If H_o is rejected, there are 3 possibilities or a combination of them:
1. condition model is wrong
2. variance model of the observations is wrong
3. one or more observations don't fit to the condition model

Testing of the observations occurs with the help of the conventional alternative hypothesis.The test variates are the $\underline{w}_p$ variates, with a standard normal distribution.

$$\underline{w}_p = 1/\sqrt{N_p}(c_p^j)^*(\bar{g}_{ji})(-\underline{\varepsilon}^i/\sigma) \tag{2.8}$$

$$\text{Suppose } (\underline{\varepsilon}_j) = (\bar{g}_{ji})(\underline{\varepsilon}^i) \tag{2.9}$$

Then (2.8) together with (2.5) and (2.9) :

$$\underline{w}_p = 1/\sqrt{N_p}(c_p^j)^*(-\underline{\varepsilon}_j/\sigma) \tag{2.10}$$

$$N_p = (c_p^j)^* \overline{(\varepsilon_j),(\varepsilon_i)^*}(c_p^i)$$

If (g^{ij}) = unit matrix and (c_p^i) = unit vector (2.10) becomes easier to work with namely

$$\underline{w}_p = -\underline{\varepsilon}^i/\sigma_{\varepsilon^i}$$

The observations will be accepted or H_a will be rejected if

$$\underline{w}_p \cdot \underline{w}_p \leq F_{1-\alpha_o,1,\infty}$$

where α_0 is the significance level of the one-dimensional test.
H_a will be accepted if

$$\underline{w}_p \cdot \underline{w}_p > F_{1-\alpha_0,1,\infty}$$

Accepting of H_a doesn't mean always that the observation is wrong.
The testing parameters are the significance level α or α_0 and a power β_0. The relation between α and α_0 is expressed in the numerical function

$$\lambda_0 = \lambda(\alpha_0,\beta_0,1,\infty) = \lambda(\alpha,\beta_0,b,\infty) \qquad (2.11)$$

The $\underline{w}_p$ variate is depending on the :
1. covariance matrix of the observations
2. alternative hypothesis
3. condition model and design of the measurement

3 The boundary values in a 3-dimensional strip with independent models

From (2.4) follows that the internal reliability is depending on the number of conditions and the design of the measurement. Because there is a relation between these two it is now possible to make rules for the survey. First of all we have to know to which demands the boundary values have to satisfy or in other words which size of error we want to detect. An answer can be: make the boundary values equal; this means that the error detection is equal in every observation. The size of the error depends then on the user.
In the next paragraphs we will see what the influence is of different strip parameters on the internal reliability of the photogrammetric observations. For the examination of this we use a theoretical strip and if not mentioned otherwise the next data are used:
- testing parameters : α_0 = 0.001 and β_0 = 0.8
- alternative hypothesis : conventional alternative hypothesis per observation
- all the coordinates of the perspective centres are measured
- the strip will be connected to 7 terrain coordinates
- the covariance matrix of the model observations is a unit matrix with $\sigma_x = \sigma_y = \sigma_z = 10\mu m$
- terrain coordinates have a zero matrix as covariance matrix
- size of the photo : 23 x 23 cm
- principal distance : 150 mm
- scale of the photo : 1 : 5000
- scale of the model : 1 : 2500
- overlap photos : 60 %
- number of models : 6
- number of tie points : 4 (regular divided in the overlap)
- maximum height differences in the terrain : 50 m

Before we are going to look to the different parameters a few remarks:
- all the numbers are given in micrometers of model scale
- an observation is not checked if the boundary value is greater than 100 x standard deviation (assumption)
- as characteristic points are choosen:
  PB : tie point at the border of the strip
  PM : tie point in the middle of the strip
  PPC: perspective centre as tie point
- 'ground-model' observations are the x,y and z observations in the model
  model observations are the 'ground-model' observations and the observations of perspective centre

- For the interpretation of the boundary values the value $(\nabla_p \tilde{x}^i/\sigma_{x^i})$ is a good approximation, if the covariance matrix of the model observations isn't equal to a unit matrix.

3.1 Strip length

From several computations can be concluded that, by using a unit matrix as a 'diagonal' matrix (see par.3.3) for the model observations the boundary values are independent of the strip length.This conclusion is important when the measurements are done with a stereo plotter,which is coupled to a computer.All kind of measurements can be checked by software.The connection of the last measured model to the previous one is then also possible.
According to this conclusion it is admitted now to do computations with a strip of 6 models.

3.2 Determination of the perspective centre

The perspective centre coordinates can be determined in several ways:
1. direct method with instrumental aids
2. indirect method by seperate measurements
   a. spatial inter-section in 2 or more levels in the model space
   b. spatial resection in 1 level in the model space
   c. spatial resection in 2 or more levels in the model space

In this case of interest is the covariance matrix of the coordinates of the perspective centre.For the direct method can be used,for example the matrix of variant I (see table 1).This matrix can be determined empirically.Variant II (see table 1) is a covariance matrix of coordinates of the perspective centre,obtained by an intersection or resection in two levels,while variant III is one of the resection in 1 level.The best way to measure in the levels in the model space is to choose 6 good devided points,see Ligterink[1969],Neeft[1973] and Neleman[1977].The choosen levels in this case are 225mm and 375mm. For variant III is this the level 375mm.

Variant I

	X	Y	Z
X	100		
Y	0	100	
Z	0	0	100

Variant II

	X	Y	Z
X	113		
Y	0	151	
Z	-67	0	303

Variant III

	X	Y	Z
X	1322		
Y	0	1182	
Z	132	0	104

Table 1 : The covariance matrices of the coordinates of the left perspective centre in $(\mu m)^2$

The influence of these three variants on the bounadry values of the observations in the model is shown in table 2.

	PB			PM			PPC		
	x	y	z	x	y	z	x	y	z
variant I	107	79	79	70	67	67	∞	91	91
variant II	107	88	83	70	68	68	∞	100	123
variant III	107	81	97	70	68	68	∞	213	92

Table 2 : the influence of the determination of the coordinates of the perspective centre on the boundary values in µm.
$\sigma_x = \sigma_y = \sigma_z = 10$ µm for PB and PM

The conclusion is that variant III,having as seperate measurement already a bad reliability (see Neleman[1977]),has an adverse effect on the error deterction of the z observation of point PB,while the x observation of point PB and all the observations of point PM are independent of the determination of the coordinates of the perspective centre.

3.3 The covariance matrix of the 'ground-model' observations

The assumption that the x,y and z observations don't correlate has many advantages concerning the computer program,but cannot be defending theoretically.The measurement of a point in the 'ground-model' is a complex one.It is a combination of three actions namely:putting the measuring mark on the 'ground' (eliminating x-parallax) and the x and y setting of the measuring mark.This kind of measurement cannot be expressed in a unit matrix of the 'ground-model' observations. Ligterink[1972] describes in chapter 1 a covariance matrix linked to the measurement and in formula:

$$\begin{bmatrix} \Delta x_m \\ \Delta y_m \\ \Delta z_m \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x/b \\ 0 & 1 & -y/b \\ 0 & 0 & -z/b \end{bmatrix} \begin{bmatrix} \Delta x' \\ \Delta y' \\ \Delta px \end{bmatrix}$$

After application of the law of propagation of errors:

$$\sigma^2(g^{ij}) = \begin{bmatrix} \sigma^2_{x'} + \frac{x^2}{b^2}\sigma^2_{px} & & \\ \frac{xy}{b^2}\sigma^2_{px} & \sigma^2_{y'} + \frac{y^2}{b^2}\sigma^2_{px} & \\ \frac{xz}{b^2}\sigma^2_{px} & \frac{yz}{b^2}\sigma^2_{px} & \frac{z^2}{b^2}\sigma^2_{px} \end{bmatrix} \qquad (3.1)$$

x,y,z : model coordinates with respect to the centre of the model (between the perspective centres)
b : model base
$\sigma_{x'}$: standard deviation of the x setting
$\sigma_{y'}$: standard deviation of the y setting
σ_{px} : standard deviation of the elimination of the x parallax

The covariance matrix of the observations is according to (3.1) a 'diagonal' matrix with correlation between the coordinates of a point and no correlation between the coordinates of different points.In my opinion, the correlation between the points can be neglected,if all the earlier processes are checked in one or other way before.Potential systematic errors must be measured separately or detected in the corrections to the observations in the strip or block adjustment.
The influence of (3.1) is shown in table 3.First of all two remarks:
- the standard deviation of the y setting is greater than the one of the x setting.This difference appears from many experiments.
- if a 'diagonal' matrix for the 'ground-model' observations is used the boundary values of the z observations of the 'ground-model' differ 1 à 2μm.These differences are caused by the existing height differences in the terrain.

Conclusions are difficult to give because every variance model will give another internal reliability.Which model will be used depends on the user.
Choosing for the 'simple' variance model the standard deviation of the z observation will be greater than those of the x and y observations. The influence of this model is shown in table 3.
Remarkably is the fact that the error detection is getting worse in the y observation of the perspective centre.

covariance matrix	standard deviations	PB x	PB y	PB z	PM x	PM y	PM z	PPC x	PPC y	PPC z
unit	σ_x=10µm, σ_y=10µm, σ_z=10µm	107	79	79	70	67	67	∞	91	91
diagonal	σ_x=10µm, σ_y=10µm, σ_z=20µm	106	84	136	70	69	129	∞	143	104
'diagonal'	σ_x=10µm, σ_y=12µm, σ_{px}=10µm	117	95	102	79	79	109	∞	134	96

Table 3 : the boundary values in µm in the characteristic points

3.4 The number of tie points

The number of tie points influences the number of conditions in the adjustment.The points are equally divided in the model overlap.The tables 4 and 5 show the boundary values for different number of tie and for two covariance matrices for the 'ground-model' observations.

- tables 4 and 5 -

Some remarks:

- the boundary values of the x observation of PPC keeps an infinite value (at least : >100 x σ)
- the x observation of PB is the most sensible observation
- the connection with less than 3 points in the 'ground-model' is not recommended
- the demand of the equal boundary values is fullfilled with 7 to 8 tie points in the 'ground-model'

3.5 Distribution of the tie points

In paragraph 3.4 the tie points are regular divided in the model overlap.It is also possible to make 'double' points,which lying close together,see figure 1.

By choosing these double points,there are two variables namely the distance between the points and the orientation of the two points.In table 6 and 7 the boundary values are given with the variables Δx and Δy.Δx and Δy are the differences in the x resp. y observations of the double points.

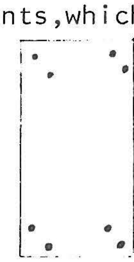

figure 1 : principle of double points

- tables 6 and 7 -

The conclusion is that the double points have a more uniform reliability of testing the observations than the equally divided points in the model overlap.A danger by using this method is, that the error is not detectable,if the points are choosen too cloose to each other. Table 8 shows such an example.In point 1 is an error introduced of the size of the boundary value.The distance between the points is the variable Δl and Δx is equal to Δy in this example.

- table 8 -

Another advantage of the double points is that the determination of the coordinates of the perspective centre (see par. 3.2) almost not influences the boundary values of the observations in the 'ground-model'.

Important in this whole concept is that the double points must be measured independently,so that there will be no correlation between the observations of the double points.

3.6 Height differences in the model

One of the strip parameters are the height differences between the tie points. In contrary to the other parameters, this one cannot be changed. It is important to examine if a height difference has any influence on the error detection. It could be possible that in flat terrain more tie points are needed. To examine this computations are executed with changing height differences. They change from 1% to 10% of the flying height. The boundary value which changes, is that one of the x observation of PPC, but keeps an infinite value. If using a 'diagonal' matrix as covariance matrix for the 'ground-model' observations also the internal reliability of the z observations in the 'ground-model' varies until 10µm by a height difference of 10% of the flying height.

3.7 The design of a measurement

Using the results of the previous paragraphs it is possible now to make a design of the measurement.
The general data of the theoretical strip is used, with 3 exceptions namely:

- Every PPC is measured by spatial intersection in two levels in the model space
- Double points with $\Delta x = \Delta y = 21$mm in the model are used
- Covariance matrix of the 'ground-model' observations.

Table 9 gives the boundary values in this type of measurement.

	point 1			point 2			PPC		
	x	y	z	x	y	z	x	y	z
$\sigma_x=10\mu m, \sigma_y=12\mu m, \sigma_z=20\mu m$	85	95	141	80	91	138	∞	125	131
$\sigma_x=10\mu m, \sigma_y=12\mu m, \sigma_{px}=10\mu m$	95	95	98	90	90	96	∞	134	117

Table 9 : The boundary values in µm in the model if using double points and measured PPC by inter section. Points 1 and 2 are the double points.

4 A practical example of the 'B-method of testing'

As an example for error detection two stereo models are measured. Figure 2 shows the position of the measured points.

Measured are:
in model I : points 1,2,3,4 and 100
in model II : points 1,2,3,4 and 100

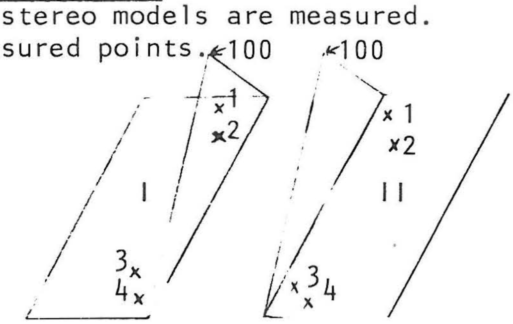

figure 2 : the two stereo models with the points

General information:
size of the photos : 23x23 cm
photo scale : 1:6000
principal distance : 152.93mm
enlargment photo→model : 2.5 x
σ_x= 5µm in the photo
σ_y= 6µm in the photo
σ_{px} (=standard deviation of the eliminating the x parallax) = 5µm in the photo
Standard deviation of the observations of the perspective centre
$\sigma_x = \sigma_y = \sigma_z = 7\mu m$
Testing parameters : $\alpha_0 = 0.001$ and $\beta_0 = 0.8$

1st Step

The multi-dimensional test is rejected because

$$\underline{\hat{\sigma}}^2/\sigma^2 = 4.0451 > F_{1-\alpha,b,\infty} = 2.1464$$

See formula (2.11) : $\left.\begin{array}{l}\alpha_o = 0.001 \\ \beta_o = 0.8\end{array}\right] \rightarrow \left.\begin{array}{l}\lambda_o = 17.0749 \\ \beta_o = 0.8 \\ b = 8\end{array}\right] \rightarrow \alpha = 0.0284$

In table 10 the following data are listed:

Per observation: point number, observation, boundary value ($\nabla_p \tilde{x}^i$), $\underline{w}_p$-variate and correction to the observation ($\underline{\varepsilon}^i$)

Looking only to the corrections of the observations, the error could be present in the z observation of point 2 in model I or in the z observation of point 4 in model II or in both.

With 'data-snooping' :

acception of the observation if $|\underline{w}_p| \leq \sqrt{F}_{1-\alpha_o,1,\infty}$

rejection of the observation if $|\underline{w}_p| > \sqrt{F}_{1-\alpha_o,1,\infty}$

$$\alpha_o = 0.001 \rightarrow \sqrt{F}_{1-\alpha_o,1,\infty} = 3.2906$$

The $\underline{w}_p$ variates of the y observation of PPC in model I and II and the z observation of point 2 in both models are larger than the critical value. The last one however is smaller than the $\underline{w}_p$ variate of the y observation of PPC. The conclusion can be that the y observation of PPC in model I or II must have an error in spite of its small correstion.

2nd Step

Elimination of the perspective centre (PPC) is not possible and it should be measured again. As a check we do the adjustment again with a corrected y observation in model I. If this is the real error, then this correction must give better results.

The null hypothesis of the shifting variate is now accepted:

$$\underline{\hat{\sigma}}^2/\sigma^2 = 1.7213 < F_{1-\alpha,b,\infty} = 2.1464$$

The data per observation of this step are listed in table 11. With 'data-snooping' no observation is rejected.

Notice that the error cannot be located if the point lies in 2 models.

5 Conclusions

Concerning the internal reliability of the observations in a 3-dimensional strip the following survey rules can be given:

- The strip length has no influence on the error detection
- Double points, as tie points, give a more uniform reliability of testing than regular divided points.
- When using double points, the determination of the coordinates of the perspective centre has no influence on the error detection of the observations in the 'ground-model'.
- The error detection is almost independent of the existing height differences in the terrain if the maximum is 10% of the flying height.

Literature

Baarda, W. 1968: A testing procedure for use in geodetic networks. Netherlands geodetic commission volume 2 number 5

Förstner, W. 1978: Die Suche nach groben Fehlern in photogrammetrischen Lageblöcken. Deutsche Geodätische Kommission, Reihe C Heft nr. 240

Ligterink, G.H. 1969: The coordinates of the perspective centre and their accuracy. Delft University of Technology.

Ligterink, G.H. 1972 The precision of photogrammetric models. Netherlands geodetic commission volume 4 number 3

Neeft, H. 1973 Projektiecentrumkoordinaten bij onafhankelijke modellen Ingenieursscriptie Technische Hogeschool Delft.

Neleman,R.C. 1977: De invloed van de koordinaten van projektiecentra op strookkoordinaten. Ingenieursscriptie Technische Hogeschool Delft.

Neleman,R.C. 1978: Het opsporen van foute waarnemingen in een 2-dimensionale blokvereffening (met onafhankelijke modellen). Nederlands geodetisch tijdschrift. 8e jaargang no 8, blz. 115-123.

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number of tie points	boundary values of the x observations of tie points 'ground-model'	PPC	boundary values of the y and z observations of tie points 'ground-model'	PPC
2	∞ ∞	∞	103μm 103	98
3	145μm 71 145	∞	86 69 86	95
4	107 70 70 107	∞	79 68 68 79	91
5	93 70 65 70 93	∞	75 67 65 67 75	88
6	85 69 65 65 69 85	∞	72 66 64 64 66 72	85
7	80 69 65 63 65 69 80	∞	71 66 64 63 64 66 71	82
8	76 68 65 63 63 65 68 76	∞	69 65 63 62 62 63 65 69	80

Table 4 : The boundary values of the model observations if $\sigma_x = \sigma_y = \sigma_z = 10\mu m$ in the model.

number of tie points	boundary values of the x observations of tie points 'ground-model'	PPC	boundary values of the y observations of tie points 'ground-model'	PPC
2	∞ ∞	∞	110μm 110	170
3	155μm 80 155	∞	101 80 101	142
4	117 79 79 117	∞	95 79 79 95	134
5	103 79 74 79 103	∞	91 79 77 79 91	126
6	94 78 73 73 78 94	∞	88 79 76 76 79 88	118
7	90 77 72 70 72 77 90	∞	86 76 75 74 75 78 86	113
8	85 77 72 70 70 72 77 85	∞	84 78 75 74 74 75 78 84	108

number of tie points	boundary values of the z observations x) of tie points 'ground-model'	PPC
2	146 - 160μm	101
3	112 - 120 - 112	99
4	100 - 110 - 100	96
5	95 - 108 - 95	93
6	91 - 106 - 91	89
7	88 - 103 - 88	87
8	86 - 100 - 86	85

Table 5 : The boundary values of the model observations if $\sigma_x = 10\mu m$, $\sigma_y = 12\mu m$ and $\sigma_{px} = 10\mu m$ in the model

x) The boundary values of the z-observations are dependent on the height differences. The region is given, laying the boundary values in.

		point 1			point 2			PPC		
		x	y	z	x	y	z	x	y	z
Δx=0mm	Δy=10mm	82	74	74	84	75	75	∞	81	81
	Δy=20mm	81	74	74	84	75	75	∞	81	81
	Δy=30mm	80	73	73	85	76	76	∞	81	81
Δx=Δy=7mm		82	74	74	83	75	75	∞	81	81
Δx=Δy=14mm		81	74	74	84	75	75	∞	81	81
Δx=Δy=21mm		81	74	74	85	75	75	1000	81	81
Δx=10mm	Δy=0mm	83	74	74	83	74	74	∞	81	81
Δx=20mm		83	74	74	83	74	74	1000	81	81
Δx=30mm		83	74	74	83	74	74	740	81	81

Table 6 : The boundary values of the observations in μm in the model if for all points $\sigma_x=\sigma_y=\sigma_z=10\mu m$. Δx and Δy are distances in the model. Point 1 + point 2 = double point.

		point 1			point 2			PPC		
		x	y	z	x	y	z	x	y	z
Δx=0mm	Δy=10mm	91	86	95	94	88	95	∞	128	82
	Δy=20mm	90	86	95	95	89	95	∞	128	82
	Δy=30mm	89	85	95	96	89	95	∞	128	82
Δx=Δy=7mm		92	87	95	93	88	95	∞	128	82
Δx=Δy=14mm		91	86	95	94	88	95	∞	128	82
Δx=Δy=21mm		90	86	95	95	88	95	∞	128	82
Δx=10mm	Δy=0mm	93	87	95	91	86	95	∞	128	82
Δx=20mm		92	87	95	92	87	95	∞	128	82
Δx=30mm		91	87	95	93	87	95	800	128	82

Table 7 : The boundary values of the observations in μm in the model if for all points in the 'ground-model' $\sigma_x=10\mu m, \sigma_y=12\mu m$ and $\sigma_{px}=10\mu m$ and if for PPC : $\sigma_x=\sigma_y=\sigma_z=10\mu m$.
Δx and Δy are distances in the model.
Point 1 + point 2 = double point

	point 1		point 2	
	$\underline{w}_p$	$\underline{\varepsilon}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$
ΔL = 5 mm	4.234	-21μm	-4.233	21μm
10mm	4.217	-21	-4.216	21
15mm	4.201	-21	-4.196	21
20mm	4.185	-21	-4.178	21
30mm	4.153	-20	-4.137	21

Table 8 : the influence of the distance ΔL in the model between double points on the error detection.

model	point	x	$\nabla_p \tilde{x}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$	y	$\nabla_p \tilde{x}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$	z	$\nabla_p \tilde{x}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$
I	1	106.492mm	116μm	-.773	3μm	254.558mm	104μm	2.006	-20μm	-362.491mm	103μm	.834	4μm
	2	148.956	106	.909	-24	209.602	100	-.558	-31	-364.118	104	-3.771	47
	3	94.326	106	-1.112	10	-211.998	100	1.270	-20	-377.512	109	.105	-15
	4	128.002	116	1.296	6	-257.032	105	.701	-22	-378.739	107	2.816	-38
	100	115.000	798	-2.031	1	-2.535	131	-4.312	7	5.530	80	.062	0
II	1	-123.509	116	.773	-7	259.591	104	-2.006	24	-373.507	103	-.834	-10
	2	-81.076	106	-.909	-4	214.641	100	.558	18	-375.063	104	3.771	-41
	3	-135.657	106	1.112	-4	-206.941	100	-1.270	14	-388.597	109	-.105	6
	4	-102.008	116	-1.296	19	-252.007	105	-.701	37	-389.893	107	-2.816	47
	100	-115.000	798	2.031	-1	2.535	131	4.312	-7	-5.530	80	-.062	0

Table 10 : listed data of an analyses of a measurement with a rejected null hypothesis of the multi-dimensional test.

model	point	x	$\nabla_p \tilde{x}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$	y	$\nabla_p \tilde{x}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$	z	$\nabla_p \tilde{x}^i$	$\underline{w}_p$	$\underline{\varepsilon}^i$
I	1	106.492mm	116μm	-1.015	12μm	254.558mm	104μm	1.451	3μm	-362.491mm	103μm	2.496	-22μm
	2	148.956	106	1.067	-17	209.602	100	-1.726	0	-364.118	104	-2.772	27
	3	94.326	106	-.969	4	-211.998	100	.167	3	-377.512	109	-.958	8
	4	128.002	116	1.150	-2	-257.032	105	.216	-10	-378.739	107	1.108	-12
	100	115.000	798	-1.633	0	-2.400	131	-.043	0	5.530	80	.051	0
II	1	-123.509	116	1.015	0	259.591	104	-1.451	2	-373.507	103	-2.496	15
	2	-81.076	106	-1.067	2	214.641	100	1.726	-5	-375.063	104	2.772	-19
	3	-135.657	106	.969	-11	-206.941	100	-.167	-7	-388.597	109	.958	-16
	4	-102.008	116	-1.150	11	-252.007	105	-.216	15	-389.893	107	-1.108	20
	100	-115.000	798	1.633	0	2.535	131	.043	2	-5.530	80	-.051	0

Table 11 : listed data of an analyses of a measurement with an accepted null hypothesis of the multi-dimensional test.