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PITFALLS IN BLUNDER DETECTION TECHNIQUES

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Pitfalls in blunder detection techniques

The blunder detection and location techniques, although always being of essential importance for quality control of adjustment results, gained more general scientific attention for the past few years only. In numerous recent publications, the problem of blunder detection has been approached from various standpoints and many experiments have been completed in an effort to achieve an optimum procedure. The paper represents a trial to scrutinize the published concepts, to analyze where and when the failures may be expected and finally to discuss the prerequisites, potentials and limitations of blunder location in practical applications in aerial triangulation.

Die Verfahren der Prüfung und Lokalisierung groben Fehler, obschon sie immer von wichtiger Bedeutung waren für die Qualitätskontrolle ausgeglichener Resultate, haben erst in den letzten Jahren eine mehr wissenschaftliche Aufmerksamkeit gewonnen. In vielen kürzlich erschienenen Veröffentlichungen wurde das Problem der Prüfung auf grobe Fehler von verschiedenen Standpunkten angefasst und viele Versuche zu einem optimalen Verfahren zu gelangen wurden ausgeführt. Der vorliegende Beitrag versucht die veröffentlichten Begriffe eingehend zu untersuchen, zu analysieren wo und wann Fehlentscheidungen zu erwarten sind und schliesslich die Voraussetzungen, die Anwendungsmöglichkeiten und Beschränkungen der groben Fehlerlokalisierung in Anwendungen der Aerotriangulation zu erörtern.

Les techniques de recherche et de localisation des fautes, bien qu'ayant toujours été d'une importance essentielle pour le contrôle de la qualité des résultats de la compensation, ont gagné une attention scientifique plus générale depuis quelques années seulement. Dans beaucoup de publications récentes, le problème de la détection de fautes a été appréhendé de divers points de vue et plusieurs essais ont été réalisés en vue d'atteindre une procédure optimum. Le présent article tente d'examiner minutieusement les concepts publiés, d'analyser où et quand des echecs sont à attendre et finalement de discuter les conditions préalables, les possibilités et les limites de localisation de fautes dans les applications pratiques d'aéro triangulation.

1. Introduction

The paper deals with the numerical treatment of blunders (gross errors) amongst the observations to be adjusted. The present mass application of photogrammetric numerical procedures (e.g. analytical plotters, on-line analogue plotters, aerial triangulation, etc.) requires a practical solution for location of blunders as quickly as possible. Otherwise one of two evils has to be tolerated, i.e. either the smooth and quick throughput of numerical data shall be seriously jeopardized by inadequate manual location methods, or the expected accuracy of adjustment results will be jeopardized by the presence of blunders. While the latter evil is absolutely unacceptable, the former one is highly undesirable. We intend therefore not to dwell so much upon theoretical aspects, and mathematical and statistical proofs (which have already been treated extensively) but to discuss the actual practical applications of blunder detection techniques and its dangers.

2. Present needs for blunder detection

The elimination of blunders from observations is a complex problem, which may be solved in various ways and at various stages of observation treatment, i.e. by using reliable measuring instruments; by application of observational methods designed to prevent the occurrence of blunders; by safeguarding against erroneous data transfer; by planning such a pattern and arrangement of quantities to be observed, which makes occurrence of blunders transparent; by testing the functional conditions which the observations in question must satisfy (e.g. sum of angles in a triangle must be 200^{G}); and finally by detection of blunders through analysis of adjustment results (residuals). The ideal situation would be, naturally, that preventive measures would guarantee the absence of blunders. Such a guarantee is at present practice hardly possible. In spite of that, all preventive measures are extremely important, and should, by all means, be taken in order to reduce the number of blunders to an absolute minimum. It follows that the last mentioned method for detection of blunders, i.e. analysis of residuals, is then always required as a final safeguard immediately before the adjustment results would be accepted. Practically, it means that together with each adjustment algorithm there has to be a corresponding blunder detection algorithm.

3. Specifications for algorithm

In order to specify such an algorithm in more detail, we have to take into account economical aspects as well as present technical potentials and constraints. It seems that whenever the numerical photogrammetric methods are applied, a modern high speed and high capacity computer is an inevitable prerequisite. In such an environment we shall assume that, in general, the following statement is true : "The computations are quicker, more user friendly and less expensive than the completion of observations". It follows that the mentioned algorithm should be designed to locate the blunders by computational methods and not by reobservations. The location has to be possible without an excessive number of observations. From the practical point of view, it is desirable that the detection algorithm is designed to treat blunders of a large magnitude equally well as blunders of the smallest possible magnitude. For the same reason it is unacceptable that the algorithm is based on an apriori knowledge of the number of blunders which may occur. Further, the detection algorithm should not be restricted to standard cases only. It is also of importance that the algorithm provides information on sensitivity of detection and issues a warning if this sensitivity is too low.

4. Basic formulae

In order to enable references for further discussions, we shall give the basic relations relevant to blunder location. In order to make it as short as possible, we shall avoid any proofs and derivations (which may be found in Mikhail (11) and Stefanovic (16)). The relation between the vector of residuals $\underline{v}$ and true observational errors $\underline{\varepsilon}$ is given by

$$(4.1) \qquad \underline{v} = - QW\underline{\varepsilon} = \overline{W}\underline{\varepsilon}$$

which is valid generally for any type of the least squares adjustment. The matrix Q represents the cofactor matrix of residuals and W is the weight coefficient matrix of observations. All Q, W and $\underline{v}$ are known or may be computed after adjustment. In order to enable easier general treatment of observational errors, the true errors $\underline{\varepsilon}$ shall be replaced by uncorrelated, equally accurate errors $\underline{\bar{\varepsilon}}$. This may be achieved by the following transformation, $\underline{\varepsilon} = V\underline{\bar{\varepsilon}}$, where V may be obtained by decomposition $W = V^{T}V$, which is always possible if W is a symmetric positive definite matrix. So the residuals can be written as a function of uncorrelated, equally accurate observational true errors :

$$(4.2) \qquad \underline{v} = - QV^{T}\underline{\bar{\varepsilon}} = \overline{V}\underline{\varepsilon}$$

The matrix $\overline{W} = - QW$ is idempotent : $QWQW = QW$, which for the case where W is non-singular gives the following relation $QWQ = Q$ or

$$(4.3) \qquad \overline{V}\overline{V}^{T} = Q$$

The relations (4.1) and (4.2) are of basic importance for the understanding of blunder detection problem.

4.1. Acceptance interval

We assume, that the errors $\bar{\varepsilon}$ have the following characteristics :

(a) errors of a smaller magnitude have a larger probability; (b) no error can assume a magnitude larger than t_1 by normal observational procedures. With the help of error propagation laws the acceptance interval for v_i may be estimated using (4.2) and (4.3)

$$(4.4) \qquad |v_i| \leqslant \sqrt{q_{ii}} \quad t_1$$

The inequality (4.4) may be divided by $\sqrt{q_{ii}}$, what brings it into the form usually applied :

$$(4.5) \qquad d_1 = |v_i| / \sqrt{q_{ii}} \leqslant t_1$$

and where the left hand side is completely defined after adjustment is completed. For the expression on left hand side the term standardized residual is used (Mikhail (11)). The symbol t_1 stands for the so-called critical value. In place of this statistical term we shall use the technical term tolerance in the further text, meaning the maximum value an individual error $\bar{\varepsilon}$ may assume. It is not difficult, by application of similar reasoning, to estimate the acceptance interval for more than one residual simultaneously. For this purpose we select several rows, i, j ... (say p in number) from expression (4.2) and obtain the condition :

$$(4.6) \qquad d_p \leqslant t_p$$

where t_p represents the upper bound for the norm of a vector containing p errors $\bar{\varepsilon}$:

$$(4.7) \qquad t_p = \|\underline{\bar{\varepsilon}}\| = \sqrt{\bar{\varepsilon}_1^2 + \bar{\varepsilon}_2^2 + \ldots + \bar{\varepsilon}_p^2}$$

$$t_p = \max \bar{t}_p$$

The quantity d_p may be computed as :

$$(4.8) \qquad d_p^2 = \underline{v}_p^T Q_p^{-1} \underline{v}_p$$

where $\underline{v}_p$ is a vector containing a selected combination of p residuals and Q_p is the corresponding submatrix of Q. For the sake of completeness, we shall also add the well-known acceptance interval for the sum of weighted squares of the entire set of residuals :

$$(4.9) \qquad s = \sqrt{\underline{v}^T W \underline{v}} \leqslant t_r$$

where r is the number of redundant observations.

4.2. Sensitivity indicators

Obviously, even if the inequality (4.5) is satisfied, in general, we cannot say that the individual values ε on the right hand side of equation (4.1) are all inside the pre-specified bounds. Simple analysis reveals, that if the left hand side of a row (4.1) is known (i.e. v_i), there are n-1 free unknowns ε on the right hand side, which may assume any arbitrary values. In order to overcome this difficulty we shall suppose that n-1 errors are acceptable (i.e. compatible with assumed stochastical characteristics). Then we are interested in the maximum magnitude of a true error ε_j selected from all n true errors. With the help of row i of (4.1) its estimation may be completed using the known error characteristics and Cauchy-Schwarz inequality. Such a maximum magnitude (maximum undetectable blunder) shall be denoted by e_{ij} :

$$(4.10) \qquad e_{ij} = (\bar{d}_1 + t_1 \sqrt{q_{ii} (1-(\bar{w}_{ij}\bar{w}_{ji} / \bar{w}_{ii}))}) / \bar{w}_{ii}$$

where $\bar{d}_1 = |v_i|$. The maximum magnitude of error ε_i when v_i is known, has a special significance :

$$(4.11) \qquad e_{ii} = (\bar{d}_1 + t_1 \sqrt{q_{ii} (1-\bar{w}_{ii})}) / \bar{w}_{ii}$$

Of course, we may select several rows in expression (4.1): i, j, ... (say p in number). Under the same assumptions as above we are now able to compute the maximum magnitude (norm) of the p-dimensional vector containing any combination of errors ε. However, the formula shall be given only for the case where the indices for the ε in questionm and v are equal, which corresponds to the one dimensional case (4.11). Using the same symbology as in (4.11) the maximum magnitude in the three-dimensional case may be denoted by, for example, e_{ijkijk} ; but for the sake of simplicity the indices shall be neglected. In such a way we may write a general formula :

$$(4.12) \qquad e = (\bar{d}_p + t_p \sqrt{\lambda_{max} (1 - \bar{\lambda}_{max})}) / \bar{\lambda}_{min}$$

where $\bar{d}_p^2 = v_i^2 + v_j^2 + \ldots$ $\bar{\lambda}_{min}$ and $\bar{\lambda}_{max}$ are minimum and maximum latent roots of the relevant submatrix of $\bar{W}$ and λ_{max} is the maximum latent root of submatrix Q_p (see 4.8)). Naturally, the expression (4.10)(4.11) and (4.12) can be evaluated only after adjustment and are not dependent on the procedure for blunder detection.

4.3. Apriori sensitivity indicators (blank range)

However, for planning purposes we would like to have the mentioned information in advance. Then, of course, we have no residuals at our disposal. But the matrix $\bar{W}$ may be simulated, often with high accuracy, and for re-

siduals (or values d_p) the most conservative estimation may be introduced i.e. the maximum permissible value. Naturally, such estimates are, then, dependent on the blunder detection method. We shall assume, that tests (4.6) or (4.8) are to be applied. The expressions corresponding to (4.10) (4.11) and (4.12) may now be directly written. In order to distinguish these new values we shall denote them by $\bar{e}$:

$$(4.13) \qquad \bar{e}_{ij} = (1 + \sqrt{1 - (\bar{w}_{ij}\,\bar{w}_{ji} / \bar{w}_{ii})})\, t_1 \sqrt{q_{ii}} / \bar{w}_{ii}$$

$$(4.14) \qquad \bar{e}_{ii} = (1 + \sqrt{1 - \bar{w}_{ii}})\, t_1 \sqrt{q_{ii}} / \bar{w}_{ii}$$

$$(4.15) \qquad \bar{e} = (1 + \sqrt{1 - \bar{\lambda}_{max}})\, t_p \sqrt{\lambda_{max}} / \bar{\lambda}_{min}$$

The quantities $\bar{e}$ may be suitably named blank range. They represent the maximum magnitude of a blunder, which can stay undetected if the given blunder detection procedure is applied. The sum of squared residuals s^2 may also be expressed as a function of true errors :

$s^2 = \underline{\varepsilon}^T WQW\underline{\varepsilon} = \underline{\varepsilon}^T\tilde{W}\underline{\varepsilon}$ or in approximation $s^2 = \tilde{w}_{11}\varepsilon_1^2 + \tilde{w}_{22}\varepsilon_2^2 + \ldots + \tilde{w}_{nn}\varepsilon_n^2$.

Assuming that all other true errors except ε_i are acceptable, we may obtain an approximate estimation of the maximum magnitude of the true error ε_i when the inequality (4.9) is satisfied :

$$(4.16) \qquad e_i = (t_r - m_r)/\tilde{w}_{ii}$$

where m_r is the minimum which a norm of an r-dimensional random vector can achieve.

4.4. Contribution of individual observations (or groups of them) to the total sum of squares residuals

Consider the following problem : we have an adjustment with n observations, from which r are redundant. After the adjustment, the total sum of squared residuals $s^2 = \underline{v}^T W \underline{v}$ may be computed. Assume that the number of observations is now reduced (some observations may be excluded, say, p in number). After completing the adjustment with the new reduced set of observations, the new (n-p)-dimensional vector of residuals would have been obtained $\dot{\underline{v}}$. Also the new sum of squared residuals $\dot{s}^2 = \dot{\underline{v}}^T W \dot{\underline{v}}$ would have been computed where W is in this case (n-p)-dimensional submatrix of the original weight matrix. The question is : what is the relation between s^2 and $\dot{s}^2$? It may be shown that the following formula is valid (see Stefanovic (16)) :

$$(4.17) \qquad \dot{s}^2 = s^2 - d_p^2$$

This means that the new sum of squared residuals may be computed without actually executing the new partial adjustment, because the right hand side is completely known directly after adjustment of the entire set of observations. While all other formulae in this chapter are valid with no restrictions for any type of the least squares adjustment with non-singular observational weight matrix, the formula (4.17) is strictly valid only for uncorrelated observations and non-iterative adjustment procedures.

4.5. The limits of blunder location

The given formulae may be used to analyse the limiting factors for blunder location. At first we shall take the case, when p = r , i.e. the number of observations to be tested is equal to the redundancy of the system. If r observations are excluded, we always get a perfect fit of residual system

comprising only of the necessary number of observations. Consequently, introducing $\dot{s} = 0$ in equation (4.17) we obtain $d_r^2 = s^2$, what means that all possible d_r have the same value equal as the sum of squared residuals. The tests on all d_r give therefore exactly the same result, what makes location impossible. If the size of the test group is increased further, i.e. $p > r$, then even detection is not possible. For computation of d_p an inversion of a submatrix of Q of size p is required. Because Q has rank r, every submatrix of Q, which has a size larger than r, is singular, making the calculation impossible. But even submatrices of Q, which have a size smaller than r, may become singular. So, obviously, in such a case computation of d_p is not possible even if p is smaller than r. Concluding we may say : in order to enable blunder location the number of blunders has to be smaller than the smallest size of a non-singular submatrix of Q corresponding to the observations in question. The analysis here is concerning absolute limits only. But the question of sensitivity is equally important.

4.6. Iterative adjustment

The question of blunder location in iterative procedures requires special attention because most photogrammetric adjustment problems are non-linear and therefore require iterative solutions. If all necessary iterations are completed and the final results used, all given formulae can be employed without any changes except (4.8). Moreover, the validity of this formula comes into question only if used in connection with (4.17), when blunders are actually present. The rigorous value for d_p in such a case (to be denoted by $\tilde{d}_p$) may be computed as follows $\tilde{d}_p^2 = d_p^2 + c$, where c is the correction which can be computed according to formulae given in Stefanovic (17).

5. Definition of blunder

All variables in equations or in equalities given in Chapter 4 may be evaluated as a result of the least squares adjustment except the tolerance t_p, which has been defined as the upper bound for the norm of a vector containing p uncorrelated random variables (observational errors $\bar{\varepsilon}$). The practical problem, which has now to be solved is the estimation of the maximum t_p of the random function $\bar{t}_p$ (see (4.8)), each term of which is bounded. The procedure which is usually recommended for establishment of t_p is the following (Baarda (4), Grün (7), Molenaar (12)): It is in the first place assumed that $\bar{\varepsilon}$ are normally distributed with zero as the expectation for the mean and σ^2 as the apriori known reference variance. In such a case the statistic

$$(5.1) \qquad \chi_p^2 = \bar{t}_p^2 / \sigma^2$$

is distributed as χ^2 with p degrees of freedom. It is, now, only necessary to select the so-called confidence level α, find in the χ^2 - tables corresponding critical value $\chi^2_{\alpha,p}$ and compute the tolerance

$$(5.2) \qquad t_p^2 = \chi^2_{\alpha,p}\, \sigma^2$$

Naturally, in place of the χ^2-distribution, we may use the F-distribution which, if the reference variance is apriori known, leads to exactly the same results. We may summarize the basis for application of the above-mentioned method :

(1) True errors ($\bar{\varepsilon}$) are assumed to be normally distributed.
(2) The expectations of true errors are zero and their variance is apriori known.
(3) Confidence level α has to be selected.

Each of the above-mentioned points is liable to criticism. In the following we shall discuss each of them in more detail in order to find out, what are the dangers and how the remedy can, eventually, be found.

5.1. Normal distribution

The experiments show that the observational errors with which we have to deal in photogrammetry, surveying, astronomy, etc., are often not normally distributed, see Romanowski (15). The shapes of distribution curves obtained experimentally differ somewhat from normal ones and they lack the tail areas. This lack of tail areas complies with the definition of blunder given earlier : in both cases the absolute value of an observational error is limited. On the other hand the application of the normal distribution has great practical advantages : it is theoretically very well defined and easily available in tabular form. The differences between the shapes of the actual and the normal density curves are usually not very large if the tails are cut off, and the same is valid for the χ^2-distribution (see fig.1.) The mentioned shortcomings must, however, be kept in mind in order to avoid misinterpretations. In the first place, the probabilities derived with the help of the normal (or χ^2) distribution function have only a symbolic value. This is especially critical in the tail areas, which do not exist in the actual distribution. In order to illustrate that, we shall give a simple example : assume that true errors are approximately normally distributed with known variance and W=I. We may imagine an observation i for which $q_{ii}=1$. In such a case all off diagonal elements in row i are equal to zero (because matrix Q is idempotent). From (4.1) we obtain $v_i = \varepsilon_i$ which, if (4.5) is satisfied, gives $|\varepsilon_i| \leqslant t_1$. Obviously, every true error larger than t_1 shall be discarded by the test and there will never be a wrong decision whatever α we chose (no observational error larger than the tolerance can stay undetected).

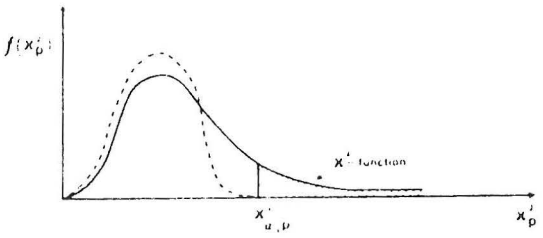

Figure 1

5.2. Mean value and variance of true observational errors

We shall not discuss here the cases, when expectations of true errors are not zero, which means, in other words, that so-called systematic errors exist. Obviously, there is a strong influence of systematic errors on blunder detection. But we intend to treat the blunder detection in strictly productional environment. Then, we assume that the mathematical models and the measuring instruments are tested in advance sufficiently well in order to guarantee that systematic errors are at least practically negligible. The same reasoning justifies then the assumption that the variance is apriori known. No serious production can be started without knowing the accuracy attainable with the available measuring instruments and methods. Compared to the systematic error problem, the problem of the variance is somewhat easier to solve, because, if the apriori value is not known, we have the aposteriori estimation of the variance always at our disposal. Some authors (Pope (14)) plead for the usage of aposteriori estimated variance instead of apriori value because "significant differences between the two are evident in the practice". This is, of course, always possible by inserting the estimated variance instead of apriori value in expression (5.1) and by using the F-distribution instead of the χ^2-distribution. Pope himself uses somewhat different approach and so-called τ-distribution. The usage of aposteriori variance has, however, the following serious disadvantage : if there is a

blunder among the observations, the estimated variance becomes large, and additionally, the critical value of the F-distribution becomes larger, if the degrees of freedom of the estimated variance becomes smaller. This means that the tolerance t becomes much too large and the test (4.6) too intensitive.

5.3 Selection of confidence level α or tolerance t

After selecting α, and assuming σ^2 is known, the tolerance (5.2) may be established. Practically it means that, by doing so, we define the maximum observational errors, which shall be tolerated (or in other words, which are normally not exceeded by standard observational procedures). Some authors who use the formalism of statistical hypothesis testing for blunder detection (Baarda (4), Molenaar (12)), consider the selection of α as being a matter of personal taste. Such a procedure is natural by usual statistical testing, where more objective criteria are not available, and where the selected α reflects only the degree of cautiousness the statistician shows against or in favour of acceptance of the hypothesis. In our case, however, by selecting α we shall establish the maximum magnitude of an error, which can happen if the observations are executed with due care. Such a value is, now, not anymore a matter of taste, but exists objectively. It seems, then, more appropriate not to express the tolerance indirectly by α-value, but to give it directly. There are authors who support such a presentation (Molnar (13), Förstner (5)). If only test (4.6) is planned nothing else but t_1 must be determined empirically. Otherwise we have to dispose with empirical values for all required t_p. Such a set of values would be the most valuable one, because it would adapt our test to the actual error distribution. If it is not available, the apriori knowledge of σ^2 is required. With help of σ, t_1 and χ^2-tables the required quantities t_p may be established

6. Remarks on known blunder detection methods

6.1. Classical tests on reference variance and residuals

The usually applied F-test on the aposteriori reference variance is equal to the test on sum of squared residuals. The merits of the test are : it gives global check for the whole system and requires little computation. The serious disadvantage is : no blunder location is possible and the sensitivity is low by large redundancy. Obviously, the test on the reference variance is useful only as a rough indication. With the notation used in (4.5) the classical test on residuals may be written as

$$|v_i| \leqslant t_1 \tag{6.1}$$

Here we may also give the blank range

$$\bar{\bar{e}}_{ii} = (1 + \sqrt{q_{ii}\,(1 - \bar{w}_{ii})}\,) \cdot t_1/\bar{w}_{ii} \tag{6.2}$$

which compared to indicator (4.14) always gives a larger value, if applied to the same observation. The value $\bar{\bar{e}}_{ii}$ approaches the value $\bar{e}_{ii}$ as the $\bar{w}_{ii}$ approaches one. The defficiency of classical tests on residuals has been extensively discussed in various papers. Sometimes, anyhow, the merits of the method (e.g. small computational effort) may be exploited. This can be done, as Förstner (5) indicates, when all $\bar{w}_{ii}$ are equal, and even better when they are close to one. Typically, the arithmetic mean possesses such characteristics.

6.2. Data snooping

Baarda was the initiator of modern numerical treatment of blunders. His various publications are invaluable contributions to the discussion of statistical aspects of the problem. He also designed a blunder detection procedure (Baarda (4)), which is well known under the name Data Snooping. It was used later as a basis for various invariants. Therefore, it requires special attention. Baarda uses the following test :

$$(6.3) \qquad W_i \underline{v} / \sqrt{(WQW)_{ii}} \leqslant t_1$$

where W_i is row i of matrix W and $(WQW)_{ii}$ is the diagonal element i of the matrix in brackets. For the case of a diagonal W the above test assumes the form (4.5). There is no attempt for numerical location procedure. Baarda (4) concludes : "Data-snooping will therefore always be a risky activity. Remeasurement of all x^1 will always be the safest way, although this will usually be avoided for reasons of economy, and because conceivably remeasurements will include new gross errors". Obviously Data Snooping is designed only as a blunder detection and not as location procedure. Baarda (4) actually requires that test (6.3) is executed only if a global test on the reference variance indicates blunder, which is clearly a conceptual mistake. Some users try to extend Data Snooping to enable the blunder location (Förstner (5)), while others apply it strictly, even considering every residual not satisfying inequality (7.3) to be a blunder (Grün (8)). The suggestions to compute the coefficients q_{ii} only approximately in order to reduce computational effort (Grün (8)) lead to the situation which is somewhere between the classical test on residuals and Data Snooping with all its consequences.
Although it was developed completely independently from Baarda, we would like to mention here the procedure developed by Pope (14). He gives a very valuable analysis of expression (4.1). The detection procedure is based on an inequality similar to (4.5) and shows some agreement with Data Snooping. It differs from Data Snooping in the following : no test on reference variance is required and for the determination of critical value the so-called tau-distribution is used instead of the F-distribution. The question of blunder location, Pope also leaves open, and requires further diagnosis for all observations whose residuals do not satisfy the inequality.

6.3. Maximum standardized residual

Kraus (9) was probably first to suggest blunder location with the help of the maximum standardized residual d_1. The procedure is very simple : the largest standardized residual, if it exceeds the tolerance (4.5) is considered to be a blunder. Förstner (5) delivers the proof that, if only one blunder is present, the maximum standardized residual always indicates the blunder. Unfortunately he neglects the other random errors as well as the influence of iterations, so that the proof has very limited value. The experiments show clearly that, even if one blunder is present, the maximum standardized residual does not necessarily indicate the blunder (see Molnar (13)). The procedure with maximum standardized residuals, therefore, involves a certain risk. Concluding, we may say, that the procedure with maximum standardized residual does not require greater computational effort then Data Snooping and, for the case of dominating diagonal elements of $\overline{W}$ and one blunder, delivers reliable blunder location. Here should be mentioned also experiments conducted by Förstner (5) to enable treatment of more than one blunder. But because it requires the knowledge of mutual relationship of those blunders, the practical value of such approach is not great.

6.4. Systematic elimination

In order to avoid the difficulties encountered when using maximum standardized residual the inequality (4.17) may be additionally utilized. The blunder is, obviously, located if for a certain residual $d_1 > t_1$ and $\dot{s} \leqslant t_{(r-1)}$ is valid. If the condition for $\dot{s}$ is not satisfied for any residual we conclude that more than one blunder is present and proceed further by computing all possible d_2 and after that d_3 etc. If both conditions are satisfied for more than one d_1 (or d_2, d_3 etc) the blunder cannot be precisely located (any of the observations indicated may be blunder). The objection against the above method is, as mentioned by Molnar (13), that the test on $\dot{s}$ is relatively intensitive. But the test on $\dot{s}$ is not used for detection of blunders, but only to distinguish among the candiates for rejection. The detection is actually completed with help of (4.6), which is the most sensitive test.

6.5. Pre-adjustment detection of blunders

The blunders may be detected even when no adjustment is completed, if conditions equations may be established including observations and constants only (e.g. the sum of angles measured in triangle must be equal to 200^g). The blunder location in such cases is difficult to solve because nothing similar to the maximum standardized residual or relation (4.17) may be found. The only remedy is reobservation or analysis of the coefficient matrix, which, because of its arbitrary structure, is very unreliable in general.
However, the preadjustment tests may be very useful in special cases, particularly when individual condition equations include only a very small number of observations. An important role is played here by a special group of tests, which may be called quick checks. Such checks are characterized by low sensitivity (aimed at detection of large blunders) high location power (each condition equation includes only a few observations) and negligible computational effort. The checks of that type are inevitable by iterative procedures, because large blunders may jeopardize the convergency of iterations. The problem of more sensitive preadjustment tests is very seldom discussed in publications on blunder detection. Especially in photogrammetric applications, it is difficult to establish an appropriate set of condition equations. One of the few contributions to this subject (Molenaar and Bouloucos (12)) shows the deficiencies of such an approach (restriction to the assumption of one blunder, complicated logic for location of that blunder and a set of condition equations which is complete for levelled photogrammetric models only).

7. Sensitivity of blunder detection

In order to obtain practically useable information about blunders the following indicators may be proposed : after completion of adjustment the indicators, e, give information about blunders which may still be present in the system. The indicator e_{ii} (4.11) is of central importance. The formula is valid if no more than one blunder is expected in the system; in particular if residual i is involved the blunder is expected only in observation i. A blunder of magnitude between t_1 and e_{ii} may stay undetected. If two or more blunders are expected, indicator (4.14) for groups of observations may be used. For planning or research purposes the indicators of the type $\bar{e}$ (4.15) give information about maximum magnitude of blunders which may stay undetected when test (4.6) is used for blunder detection. For other types of tests such indicators (blank range) may also be easily computed (e.g. (6.2)).

7.1. Reliability

Baarda (3) defines sensitivity indicator for Data Snooping, which in connection with so-called reliability gained wide popularity. This indicator belongs to the category of apriori indicators and is directly comparable with (4.14). Because it differs from (4.14) we shall at fist try to analyse the differences. Baarda introduces for the mentioned indicator the symbol ∇. Applying the denotation used earlier (6.3) we may write

$$(7.1) \qquad \nabla_i = 6\sqrt{\gamma} / \sqrt{(WQW)_{ii}}$$

where γ is non-centrality parameter of F-distribution - function of α and β. Baarda (4) defined : "Internal reliability is described by the lower bounds $\nabla_0 l_i$ for gross error ∇l_i which can just be detected by the test with given probability β". There are two objections for using ∇_i : (a) theoretical one and (b) practical one.

(a) The value ∇ is derived assuming that a blunder is stochastical variable normally distributed with variance equal to reference variance but non-zero mean. Such a concept of blunder does not agree with reality, except for a few categories of blunders (e.g. observation of wrong target). Malfunctioning of recording device, for example, produces blunders which are clearly uncompatible with above concept.

(b) In order to illustrate the actual information obtainable by ∇ we shall assume that ∇_i is computed for an observation i, which participated in an adjustment. Conceive again n repetitions of all observations, where each realisation of observation i is biased by ∇_i and additionally affected by random errors in the same way as other observation. If the Data Snooping is applied on all n adjustments, and n is large enough, then we expect that in 100p % of all cases the blunder shall be detected in observation i and in 100 (1-p)% of cases not. Simulation with approximately normally distributed errors may easily prove that this will actually be so. But is this the information we need for blunder detection sensitivity? Some authors even erroneously interpret ∇_i in the sense as given for e_{ii}.

8. Economy

The first question, which has to be answered is whether the application of sophisticated numerical blunder location methods is economically justified. If the conditions mentioned in chapters 2 and 3 are satisfied, then the question may be answered by yes for off-line procedures. In particular such an answer may be given, without any doubt, in cases where a located blunder may be easily discarded without any serious consequences for the total adjustment system. When the blunder has to be replaced by an acceptable observation, the precise location means reduction of remeasurements to absolute minimum. The savings achieved in such a way are expected to compete with increased computational effort. In photogrammetry the remeasurements mean usually that the whole model or photo, where blunder was found, has to be reobserved. So, if we have to deal only with the last mentioned category of observations, every location more precise than the identification of model or photo to be measured, has little economical effect. If it is required that the system should be fool proof, because any blunder which escapes direct scrutiny when discovered too late may have severe financial consequences (e.g. penalties to be payed to the contractor), then we encounter different situations. Such a system may be realized in one of the two ways : (a) by planning the configuration of observations in such a way that even simple tests guarantee high sensitivity, or (b) by application of the most sensitive detection methods. The research (see Molnar (13),

Amer (2)) makes it clear that the first solution would often require substantial increase in the number of observations. The second solution requires, naturally, also proper planning. Additionally, at least, diagonal elements of matrix $\bar{W}$ (practically only of Q) have to be computed. Application of the test (4.5) or Data Snooping gives then optimal sensitivity for detection (if t_1 is properly established), but requires reobservation in order to locate blunders. With practically the same computational effort and sensitivity the maximum standardized residual gives additional location possibility and elimiates unnecessary reobservations. The maximum standardization residual is especially risky to use in case of more than one blunder. In order to avoid difficulties, which application of maximum standardized residual may cause, the inequality (4.17) may be used, and this requires no additional computation. Such a procedure may become uneconomical, if more than one blunder is present, because of rapid increase in computational effort, as Mikhail remarks in (10). But the alternative solutions, in case of several blunders, are also not particularly inexpensive. The procedure with maximum standardized residual requires, then, several repetitions of adjustment, not to mention that possible wrong decisions have no positive economical effect also. Taking this all into account, the proposed procedure might turn, anyhow, economically acceptable. The above conclusions are, naturally, valid only if the structure of matrix $\bar{W}$ is unfavorable. In special cases simplified procedures are more economical. E.g. in case of arithmetic mean the maximum absolute residual is reliable and obviously the most economical blunder indicator. There are also other aspects of economy of blunder detection, which have not so much to do with the particular procedure applied. Without any doubt, the economy of the whole procedure is better if the blunders may be detected and located as soon as possible, at best during the completion of observations (on line). In such a case the sophisticated blunder detection procedures are not so obvious. Anyhow, even then, they guarantee discovery of small blunders and rapid remeasurement. Also, the large adjustment systems which include numerous observations are obviously more difficult to handle than smaller systems. The danger that indefinite situations occur is much greater in large systems, because the test on $\dot{s}^2$ becomes less sensitive due to larger redundancy (see (4.16)). There is also risk for larger number of blunders, what makes the detection procedure in any case more expensive. Therefore, decomposition or large adjustment systems into smaller units might show economical advantages, if sensitivity is not reduced too much. Such decomposition is possible for aerial triangulation (relative orientation, model formation, triplet adjustment, strip formation, strip connection, subblock adjustment, etc.). Some of the units may be adjusted simultaneously with completion of observations. Many authors support such a procedure (Mikhail (11), Molnar (13) etc.). Testing of the total system should, anyhow, not be neglected as a final test of highest sensitivity. But if partial tests are properly executed we do expect that only occasionally some blunders will not be detected by them.

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