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ON SOLUTION ALGORITHMS FOR LARGE SPARSE SYSTEMS OF NORMAL EQUATIONS IN PHOTOGRAMMETRY

ABSTRACT

The question of which algorithm is best used for the solution of normal equations with different structures has been investigated. The solution methods are direct solution for banded resp. banded-bordered matrices, a special direct solution technique for arbitrary sparse systems of equations and the method of conjugate gradients. The algorithms have been applied firstly to photogrammetric bundle adjustment with self-calibration, which leads to a banded-bordered matrix of the normal equations, and secondly to calculations of digital height models which are generated by a simple version of the method of finite elements and which lead to band matrices.

INTRODUCTION

In bundle adjustment with self calibration (e.g. GRÜN 1976) and interpolation of digital height models by the method of finite elements (EBNER, REISS 1979) the resulting systems of normal equations are sparse. A sparse system means that a large number of coefficients are zero. The distribution of the non-zeros in a sparse matrix may be arbitrarily.

For such arbitrary sparse matrices, a special direct solution algorithm was developed (GUSTAVSON 1972). In the following the application of this special technique in the field of photogrammetry will be discussed and compared first with ordinary direct solution for banded, respectively banded-bordered matrices and second with the iterative method of conjugate gradients.

A reason for the investigations (namely to find out, which algorithm needs less calculation time) was a former comparison, which was carried out by SCHEK, STEIDLER, SCHAUER (1977) in the field of geodetic networks and of prestressed cable nets. Their results showed surprisingly fast solutions. In the case of geodetic networks the structure of the matrix of the normal equations is arbitrary sparse, in the case of prestressed cable nets it is a band matrix. At that time it was supposed that band algorithm may not be the best solution for banded systems. This assumption was supported by an investigation on the subject of bundle adjustment of SCHENK (1972), who suspected to obtain a faster solution using a so-called "coordinate method", than with convential solution techniques like the direct solution with band algorithm or the iterative method of conjugate gradients (CG). This "coordinate method" is similar to the above mentioned direct algorithm for sparse systems.

STEIDLER (1980) investigated two fields of application, which are summarized in this paper. The first one is the photogrammetric bundle adjustment with self calibration. The matrix of the normal equation in this case is banded bordered and usually a direct solution algorithm for banded bordered systems or the iterative method of CG was applied. These methods are compared here with the direct technique for arbitrary sparse systems. Also the characteristics of the CG method are regarded, when the number of control points is decreasing. To accelerate the convergence "scaling" and "preconditioning" (methods to improve the condition of a matrix) have been applied.

The second field of application is the generation of digital height models by a simple version of the method of finite elements, which leads to a very regular strip matrix. Here also the direct solution algorithm for band matrices, the direct technique for arbitrary sparse matrices and the method of conjugate gradients are compared. The number and distribution of reference points was changed for the CG calculations additionally.

The main goal of the comparison was to find out, which algotithm needed less calculation time. When calculating with direct solutions, the number of multiplications has been counted and compared with the calculation time, and also the number of nonzero-elements was considered. As a secondary problem the

storage capacity was regarded, because if disc memory is avail-
able, storage capacity is not a great factor anymore.

A DIRECT SOLUTION TECHNIQUE FOR LARGE ARBITRARY SPARSE SETS OF LINEAR EQUATIONS.

The first problem when dealing with arbitrary sparse matri-
ces is the storage mode. It would be a poor solution to store
the total matrix with its zero elements. Therefore, a special
storage mode was suggested by GUSTAVSON (1972). The matrix has
to be set up in the form of two vectors which merely indicate
positions and one vector which contains the numerical values of
the matrix. A small example is shown at figure (1).

Having fixed the storage mode one can start the solution of
the system:

$$Ax = b.$$

The solution itself is subdivided into three parts:

(1) symbolic factorization with "reordering",

(2) numeric factorization;

(3) forward and backward substitution.

Symbolic factorization means a decomposition only with the in-
dices of the matrix A into a product $A = L \cdot U$.
L is the lower triangular matrix and U is the upper triangular.
The result of the symbolic factorization gives the information,
where "fill-ins" arise in the factorized matrix. It is well
known that after factorization, the structure of the matrix has
changed in such a sense, that more non-zero elements have to
be considered. To minimize the number of non-zero elements,
and consequently to reduce computing time and storage require-
ments, it is helpful to perform a reordering scheme. Two possi-
bilities have been investigated:

(1) Reordering of the matrix and the right - hand side such,
    that rows are sorted by increasing length. (ROWORD-
    SCHEME).

(2) Do a symbolic elimination and use as i-th pivot row
    that row of the remaining (n-i, n-i) updated matrix
    which leads to the minimal number of multiplications in
    the next factorization step. Because of symmetry this
    means a row with minimal length. (OPTORD-SCHEME).

The ROWORD-scheme requires less calculations, but the OPTORD-
scheme is generally more efficient. However, it also does not
necessarily lead to the factorization with minimum fill-ins.

The following example may clarify the storage mode and the
symbolic factorization:

$$\text{Matrix A:} \begin{bmatrix} 5 & 2 & 3 & 1 & 0 & 0 & 0 \\ & 4 & 0 & 0 & 2 & 0 & 1 \\ & & 6 & 0 & 0 & 0 & 2 \\ & & & 3 & 0 & 0 & 0 \\ \text{symm.} & & & & 5 & 0 & 3 \\ & & & & & 4 & 0 \\ & & & & & & 1 \end{bmatrix} \quad \text{is stored in three vectors:}$$

AN = 5,2,3,1,4,2,1,6,2,3,5,3,4,1

JA = 1,2,3.4,2,5,7,3,7,4,5,7,6,7
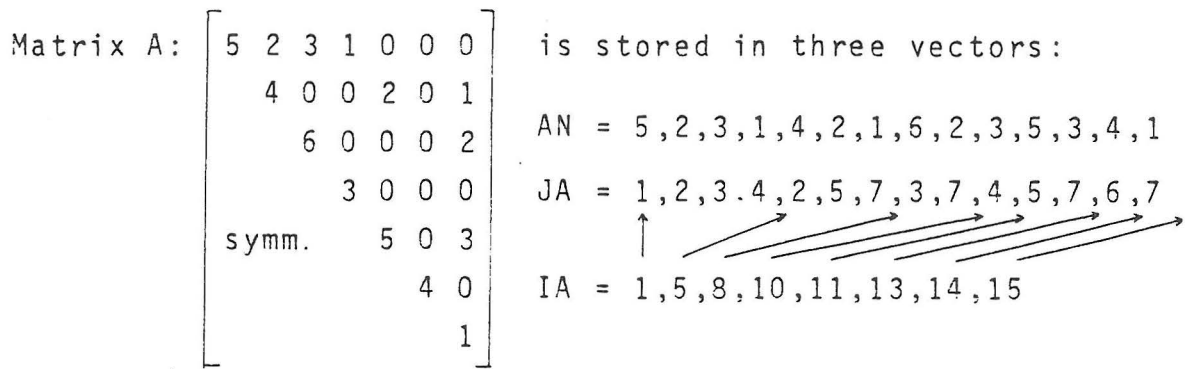
IA = 1,5,8,10,11,13,14,15

FIGURE (1): Storage mode of a sparse matrix.

AN contains the non-zero elements in row-wise order, JA contains the corresponding column subscripts and IA gives the indices of the first elements in each row.

The numeric values, stored in AN are not needed for the symbolic factorization. The first four elements of JA relate to the first row. This row can cause fill-ins in the second or third row: in the second row elements (2,3) and (2,4) may be created, if they do not exist already, likewise, an element (3,4) may be created in the third row. This process will be continued up to the (n-1)-th row. The result for our example is:

JL = 1,2,3,4,2,3,4,5,7,3,4,5,7,4,5,7,5,7,6,7

IL = 1,5,10,14,17,19,20

JL and IL represent the symbolic sparse matrix after factorization. Fill-ins are underlined.

In the next step, the numeric factorization is performed, i.e. the numeric calculation of the triangular decomposition.

The advantage of the separation of symbolic and numeric factorization is the effect, that symbolic factorization is performed only once, even with the calculation of non-linear systems of equations. The numeric factorization is computed in each iteration.

At the third step the forward and backward substitution follows:

$$Ly = b$$
$$Ux = y$$

This can easily be done in a very short time and the separation from the numeric factorization is helpful, when calculating several right hand sides. If reordering was realized in step (1), a back transformation must be executed. It should be pointed out here that the above explained method requires a computer with virtual storage capacity to be really efficient. Other machines would need too long I/O-transport-times, if calculation is not possible directly in core.

The method mentioned here is one of several versions: a very detailed survey with an extensive bibliography is given by DUFF (1976). The other solution algorithm, direct solution for banded resp. banded-bordered matrices as well as the method of conjugate gradients are well known and not explained here.

COMPARISON OF THE SOLUTION ALGORITHMS

SCHEK, STEIDLER, SCHAUER (1977) pointed out, that the solution with a direct algorithm for sparse systems is much faster than the method of conjugate gradients, when calculating geodetic networks. They investigated several geodetic nets with arbitrary sparse matrices of normal equations, and two prestressed cable nets with a band matrix of normal equations. As supposition was assumed that band algorithm may not be the fastest solution for banded systems. This and a similar assumption of SCHENK (1972), who also thought that algorithms for banded matrices could be slower than direct techniques for sparse matrices, have been a cause of investigations of STEIDLER (1980), which shall be reported here.

a) COMPARISON OF SOLUTION ALGORITHMS IN BUNDLE ADJUSTMENT WITH SELF CALIBRATION

Two examples are used for the numerical investigations. The three algorithms mentioned above, have been compared, and for the case of conjugate gradients the convergence rate was considered, when control points have been thinned out. CG includes scaling and for critical cases also pre-conditioning (EVANS, 1967). The examples have been calculated by the method of bundle block adjustment with self calibration. In comparison to the method of independent models it is the more rigorous and more flexible method. The first of the two examples is the well known testblock "OBERSCHWABEN" (OEEPE). It was flown with a ZEISS RMK A 15/23 camera at an image scale 1:28 000. The section used in this test consists of 8 strips with 17 images each, giving 136 images altogether. The forward overlap is 60%, the sidelap 20%. This leads to a square form of the block and an ordering in or lateral to flight direction leads to about the same bandwidth of the matrix of the normal equations. The size of the area is about 40 x 40 km^2. 2000 image points have been measured with a ZEISS PSK, the number of ground points being 926. Two versions with different distributions of control points have been calculated, one with all available 335 points, a second with 46. The following figure shows the distributions:

```
·   control points case 1 = 335
●   control points case 2 =  46
┼   projection centres
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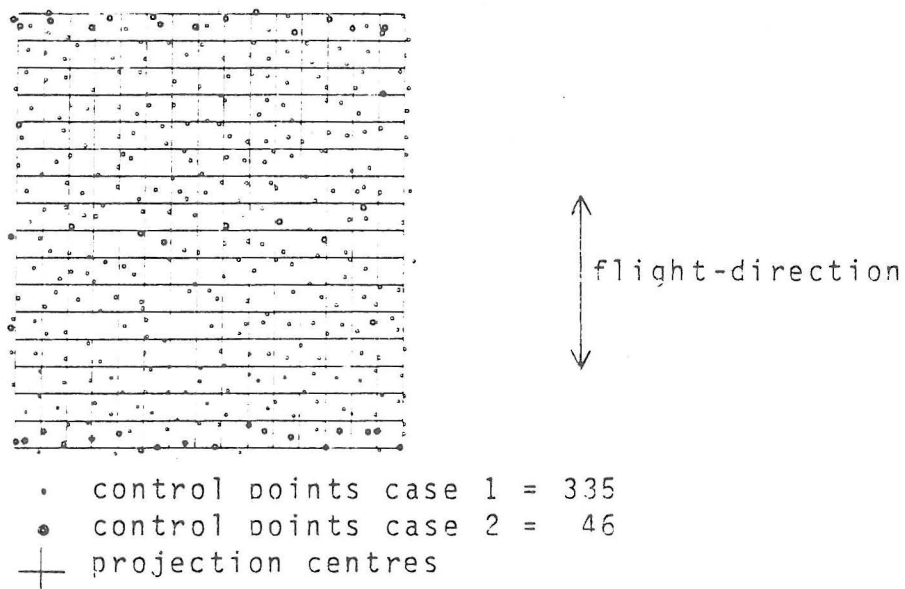
FIGURE (2): Section of testblock OBERSCHWABEN

The second example is the cadastral block "MOOSACH". It was
flown with a normal angle camera, ZEISS RMK A 30/23. The size
of the area is 3.1 x 2.2 km^2 and the image scale is 1:3300.
The block consists of 11 strips, giving 93 images altogether.
2700 image points have been observed with a ZEISS PSK. The
forward and sideward overlap is 60%. The number of ground-
points is about 900, the total number of control points is 25
(15 in x,y, 10 in x,y,z).
The following figure shows the distributions of controls in
the block MOOSACH.

flight-direction

FIGURE (3):

cadastral block "MOOSACH"

+ projection centers

△ control points in x,y,z

□ control points in x,y
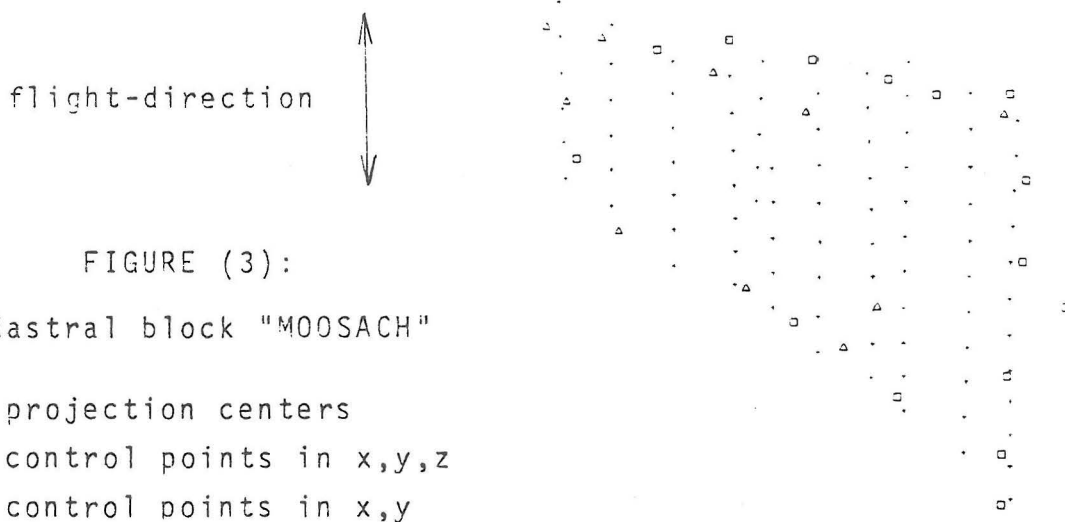
The size of the blocks was chosen in such a way, that it was
possible to calculate them in central memory, because parts
of the investigations had to be calculated at CYBER 175 and
TELEFUNKEN TR 440, which are not equipped with virtual storage.
The unknowns of the ground coordinates have been eliminated.
The resulting reduced matrix of the normal equations is banded

with variing bandwidth; if bundle adjustment with self calibration is performed, the matrix will be banded-bordered, because the additional parameters are additional unknowns and usually ordered at the border of the matrix. For blockinvariant parameters this border will be fully populated.

The program used for the investigations is MBOP (Münchener Bündelorientierung mit zusätzlichen Parametern). It was developed by GRÜN (1976). For self calibration a set of 12 blockinvariant and nearly orthogonal additional parameters was chosen (EBNER 1976).

The number of remaining unknowns in the case OBERSCHWABEN was 828, the bandwidth about 100 plus the border with 12 elements. The number of elements within the band and the border was 94000, of which only 40500 were non-zero.
The initial values of the image coordinates and orientations have been calculated with MBOP.

The following table (1) shows the result of the comparison: (all symbolic calculations were performed with the indices of (6 x 6) submatrices. in order to reduce the index calculations). The calculations have been computed at IBM 370/145 computer.

IBM 370 / 145 unknowns 828 non-zeros before(ne) factorization=40554	(1) band-algorithm	(2)	(3)
		techniques for sparse matrices	
		ROWORD	OPTORD
non-zeros after factorization (nn)	93943	154285	87219
$\frac{nn}{nn_{(1)}}$ 1=(1),(2),(3)	1	1.64	1.33
number of multiplications	5.74 Mio	19.00 Mio	7.92 Mio
$\frac{mult}{mult_{(1)}}$	1	3.3	1.4
time (CPU-sec.)	330	1107	470
$\frac{time}{time_{(1)}}$ 1=(1),(2),(3)	1	3.3	1.4
storage capacity (words)	95 k	300 k	190 k

TABLE (1)

Figure (4) shows the structure of the normal equations after factorization with band algorithm and with the sparse technique.

BAND   OPTORD
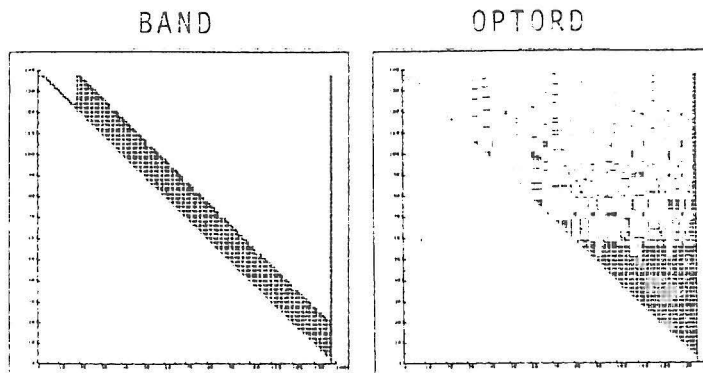

each point represents a 6 x 6 submatrix

FIGURE (4)

It is obvious that the band algorithm needs the least number of multiplications, although the number of non-zero elements is nearly equal to that of the sparse techniques. The calculation time is reduced in the same proportion of the reduction of the number of multiplications. Further the "band" storage mode needs not as much storage as the "sparse" mode, because the latter works with indices.

The second example MOOSACH leads to a system of normal equations with remaining 570 orientation unknowns. The bandwidth is about 100 on average plus the 12 elements of the border. The number of elements within the band and the border is 58100, with only 33300 non-zero elements. Table (2) shows the result of the comparison. The calculations have been computed at IBM 370/145 computer.

IBM 370 / 145 unknowns 570 non-zeros before(ne) factorization=38356	(1) band-algorithm	(2) techniques for sparse matrices ROWORD	(3) OPTORD
non-zeros after factorization (nn)	58100	74500	71100
$\frac{nn_i}{nn_{(1)}}$ i=(1),(2),(3)	1	1.28	1.22
number of multiplications	3.17 Mio	5.19 Mio	5.37 Mio
$\frac{mult_i}{mult_{(1)}}$	1	1.37	1.33
time (CPU-sec.)	138	280	279
$\frac{time_i}{time_{(1)}}$ i=(1),(2),(3)	1	1.32	1.31
storage capacity (words)	60 k	150 k	145 k

TABLE (2)

In this case again band algorithm is superior. Only half of the calculation time and half of the number of multiplications than for the sparse technique is needed. The number of non-zeros after factorization is much less. The storage requirement of the band algorithm is less than the half of the sparse algorithm.
Figure (5) shows the structure of the matrix of the normal equations after factorization with band and sparse method.

BAND   OPTORD
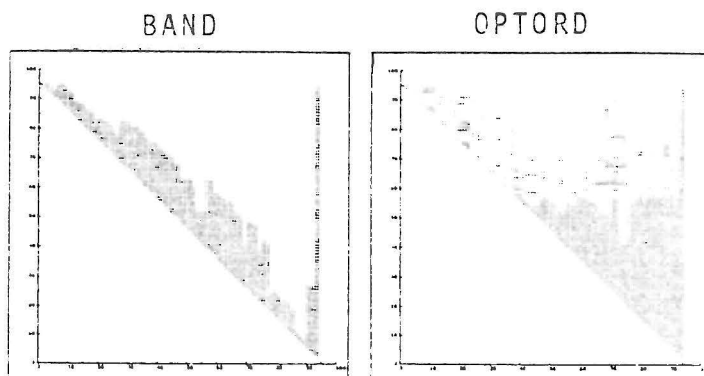

each point
represents
a 6 x 6
submatrix

FIGURE (5)

The next comparison is done between bandalgorithm and the method of conjugate gradients.

The first example (OBERSCHWABEN) is shown in the table (3) with the version of all available 335 control points. The calculations have been performed at TR 440 computer.

TR 440 NU = 328	band-algorithm	method of conjugate gradients (CG)
outer iterations	2	4
CG-iterations	-	14, 45, 52, 46 Σ=157
time (CPU-sec.)	750	1025

TABLE (3)

One can see that the solution with CG is reached after 157 CG-iterations in four outer iterations for the non-linear problem in 1025 CPU-seconds. With the band algorithm the final solution was obtained after two outer iterations in 750 CPU-sec. Outer iterations mean the iterations to solve the non-linear problem.
The same example was also calculated in a second version with 46 control points. Final convergence could not be reached with CG, even though preconditioning was additionally used. This is one of the reasons to reject the CG-method for solving systems with poor control points.

The second example (MOOSACH) is shown in the table (4). The calculations have been performed at CYBER 175 computer.

CYBER 175 NU = 570	band-algorithm	method of conjugate gradients (CG)
outer iterations	2	20
CG-iterations	-	580, 580, . . . 580 Σ = 20·580 = 11600
time (CPU-sec.)	82	1800

TABLE (4)

In this case it was not possible to achieve a solution with CG in a reasonable time; after 20 outer iterations and 1800 CPU-seconds the required accuracy was not attained, in the other case the bandalgorithm needed two outer iterations and a calculation time of 82 CPU-seconds.

The conclusions of the above investigations in a comprised form:
- Direct techniques for sparse matrices are not superior to a band algorithm. The band structure will be destroyed and not even the OPTORD sorting leads to less fill-ins in comparison to the band method. An other point is that the number of multiplications are increasing much more than the number of fill-ins. Here one can see that the relation number of multiplications and number of fill-ins are not proportional.

- The method of conjugate gradients is inferior to the band-algorithm. It needs more calculation time and it is not sure, that convergence can be attained. Especially if the systems

are not equipped with a great number of controls, it is possible that no convergence will be obtained, even if scaling and preconditioning is applied.

b) COMPARISON OF THE SOLUTION ALGORITHMS AT THE GENERATION OF DIGITAL HEIGHT MODELS BY THE METHOD OF FINITE ELEMENTS

One of several possible ways to generate digital height models was suggested by EBNER in 1978. It is based on the finite element method and leads to a problem of least squares adjustment. The occuring system of normal equations has a rigorous regular band structure, but within the band it is very sparse. This fact was the reason to compare the above described solution algorithms in addition to the comparisons of the irregularly shaped band matrices, which result from the bundle adjustment with self calibration.

First applications of the height interpolation by finite elements have been presented by EBNER and REISS in 1978 and a mini-computer program system called "HIFI" was presented by EBNER, HOFMANN-WELLENHOF, REISS and STEIDLER in 1980, where two types of finite elements alternatively have been applied, a bicubic and a bilinear one.

For the following comparisons the bilinear interpolation only was investigated. The formulation of the interpolation is reported in the above mentioned description of "HIFI", and therefore it will be described very briefly here, only in order to explain the resulting structure of the matrix of the normal equations.

Starting from arbitrarily distributed reference points the unknown heights at the nodes in a regular square grid are to be computed, (m = number of grid rows, n = number of grid columns, m·n = number of unknowns, see figure 6a). The terrain surface is approximated by a large number of bilinear finite surface elements, which are linked together at the nodes and are continuous but not differentiable in the first derivation.

In order to receive a surface of minimum curvature through the given reference points (apart from eventual filtering) at every grid point curvature equations are formulated in four discrete directions: the two of the coordinate axes and the two of the diagonals (figure 6a). The curvature equations are linear and contain the difference of two adjacent slopes-related to the distance one - which can be interpreted as a numerical approach of the curvature at the node (i,j). For example the equation of the curvature of the x-direction at node (i,j) is:

$$v_{i,j}^{(x)} = h_{i-1,j} - 2 h_{i,j} + h_{i+1,j} - 0;$$

It forms an observation equation: the observation of the curvature is assumed to be zero with a small deviation. $v_{i,j}$ being the according residual.

The reference points contribute additional equations. They contain the four surrounding unknown grid heights in such a way that the reference point lays on the surface (apart from eventual filtering). They also are linear and do only cause additional contributions to already existing unknowns (EBNER et al.1980). This means the structure of the normal equations is dependent only on the equations of curvature (figure 6). The bandwidth is $(2n + 3)$ and is proportional to the number n of nodes in one row. Every normal equation however contains only up to 9 non-zero elements (figure 6b).
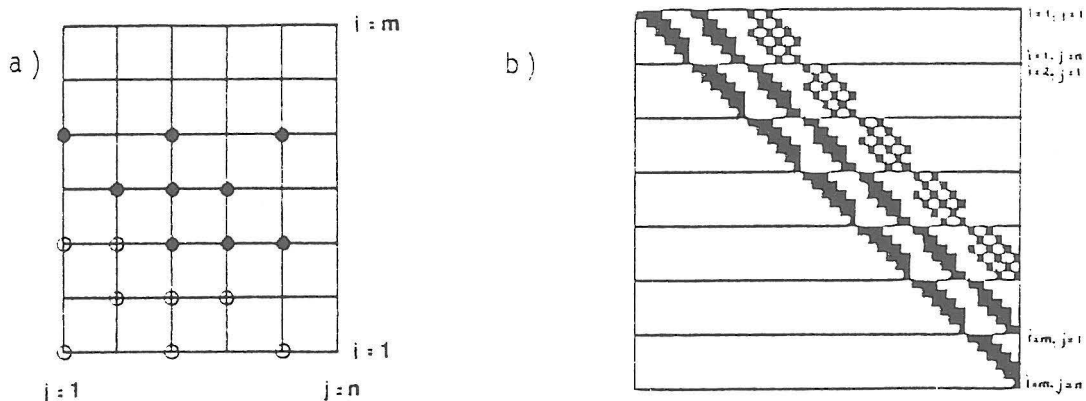

FIGURE 6: a) m by n grid of a DHM with the connections of
            a point (i,j)
         b) structure of the matrix of normal equations
            resulting from a 7 x 6 grid.

This strip matrix stands in contrast to the much less regular banded-bordered matrices of the bundle adjustment with self calibration. Because of this reason the comparison between the solution algorithms seems to be very interesting.

The test material which was used for the comparisons of the algorithms, are three data sets, namely VERNAGT 1, VERNAGT 2 and OBERGURGL. VERNAGT 1 contains 850 reference points, 31 x 11 unknown heights and is a section of VERNAGT 2, where points of intersection between contour lines and grid lines were digitized. In VERNAGT 2, which covers an area of 3.2 x 3.4 km^2 and with a maximum height difference of 700 m, 6200 reference points have been observed. The number of unknowns is 65 x 69 = 4485. The grid spacing is 50 m. Figure 7 shows the distribution of the reference points, which have been used, and the contour lines which were derived from the interpolated DHM. They have been drawn automatically using a plotting routine of the Leibniz-Rechenzentrum Munich at a drum plotter "CALCOMP 936 S".
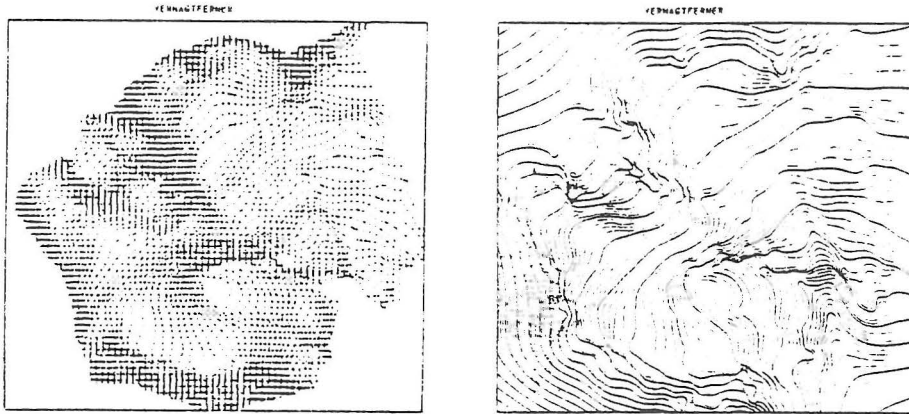
FIGURE (7)

The digital height model OBERGURGL covers an area of 750 x 500 m^2. The grid spacing is 12.5 m. The reference points have been observed along contour lines and the number of unknowns is 61x41 = 2501. The comparison between "sparse techniques" and "band algorithms" is illustrated by the example VERNAGT 1 They have been calculated at an IBM 370/145 computer. Table (5) shows the results.

VERNAGT 1 m = 31 n = 11 nu = 341 nv = 2700	band- algorithm	tecnniques for sparse matrices sorted by OPTORD
non-zeros after factorization (nn)	8032	12 239
number of multi- plications	101264	1 511 151
calculation time (CPU-sec.)	7	50
storage cap. (words)	9 k	25 k

nv = number of non-zeros before factorization
nu = number of unknowns

TABLE (5)

Figure (8) shows the structures of the matrix of the normal equations with band algorithm and with direct techniques for sparse matrices after factorization.
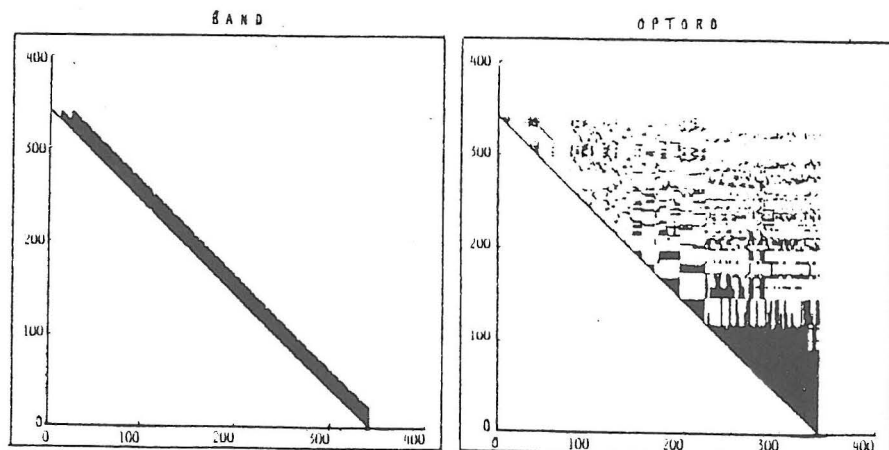
FIGURE (8)

One can see clearly that using the bandalgorithm the band becomes full. In the sparse case the bandstructure was completely destroyed.

The comparison between the bandalgorithm and the method of conjugate gradients is illustrated using all three sets of data in table (6) (calculated at a CDC CYBER 175):

data set	unknowns	nv	band-algorithm			CG with scaling		$\frac{time_{CG}}{time_{band}}$
			time	mult	nn	time	CG-iterat.	
VERNAGT 1	31 x 11=341	2700	0.13	0.1 Mio	9032	0.19	66	1.46
VERNAGT 2	65 x 59=4485	39165	52.27	44.5 Mio	618930	168	955	3.21
OBERGURGL	61 x 41=2501	21599	11.51	9 Mio	207261	20.4	226	1.77

nv = non-zeros before factorization
nn = non-zeros after factorization

TABLE (6)

In all three cases the band algorithm was the fastest. The method of CG included scaling. It should be pointed out here, that with CG an accuracy of about 3 mm was attained, compared with the direct solution. This accuracy may be very high for digital height models. The calculation time in the poorest case is three times longer than with band algorithm. It may be reduced if lower accuracy would be sufficient. In the case of saving storage capacity it can be stated, that the method of conjugate gradients needs much less storage, namely twice the number of non-zeros before factorization, the band algorithm needs space for the number of non-zeros after factorization; but when using discs the storage may not be very important.

To show the convergence of the method of conjugate gradients several versions of VERNAGT 2 have been computed. The reference point situation was changed, this means the number of points has been successively reduced until only one remained at last.

Table (7) shows the results:

number of reference points	number of CG-iterations	calculation time (CPU-sec.)
6247	355	168
3124	386	173
1562	912	179
1249	923	180
1041	938	182
781	959	186
624	971	188
416	1059	202
249	1167	227
124	1309	254
61	1646	321
30	2122	412
11	2754	537
3	2961	561
1	2959	561

A solution was reached in each example, even in the singular case with only one reference point (CG-method gives the minimum norm solution in singular cases). In this case the solution was reached after 2960 iterations, when theoretically 4500 (= number of unknowns) should be expected.

Finally it can be stated that for the generation of digital height models the application of "sparse" techniques seems to be rather inefficient. This statement may be restricted because of the comparison of only one example, but due to the same structures of any matrix of normal equations the conclusion may be advocated.

The band algorithm leads to the fastest solution, even faster than CG, especially in the poorer cases with only few reference points. But using CG there are no difficulties with convergence in any case. Regarding the storage capacity, CG needs the least, because only the non-zero elements before factorization have to be stored, contrary to band and sparse technique, where the non-zeros after factorization are needed. But this fact does not seem very important, when enough space on disc is available.

CONCLUSION

It was pointed out that direct techniques for arbitrary sparse matrices are inferior to the direct solution for band matrices in the case of regular structures, how they appear at bundle block adjustment with self calibration and at generation of digital height models by the method of finite elements. The sparse techniques needed more time and more storage capacity for the investigated examples. Besides they demand a computer with virtual storage capacity.

The method of conjugate gradients has to be rejected in the case of bundle adjustment with self calibration, because convergence cannot be reached in every case, even not if scaling and preconditioning is applied.

Better results are obtained with CG at calculation of digital height models. Indeed band algorithm was faster, especially in the cases with only few reference points, but convergence could be attained in all cases.

More storage capacity is needed if calculating with band algorithm, because in case of CG only the non-zeros before factorization are needed. But this point should not be overestimated because today nearly all computers are equipped with disc storage capacities.

The results obtained in this investigation confirm the conventional usage of band algorithm in bundle adjustment and seem to be very important because this fact could not be anticipated.

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