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TERRAIN ANALYSIS AND ACCURACY PREDICTION BY MEANS OF THE FOURIER TRANSFORMATION

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Abstract

Profiles are measured in different geological terrain types. The profiles are analyzed by the power spectra which are used for a statistical description of the different landscapes. By means of the spectrum, the standard deviation between the terrain surface and a digital terrain model is estimated.

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Introduction

The power spectrum is a well-known quantity to electrical engineering. It is used to examine the power content at different frequencies in signals and can be applied to deterministic as well as random signals. Some of the fluctuations of the terrain surface are most similar to random signals, so it is reasonable to apply the power spectrum technique to terrain surfaces. By means of the power spectrum the different types of landscape can be distinguished (see Ayeni, 1976 and 1978), and the standard deviation between a digital terrain model and the terrain can be estimated. This standard deviation is valid in the entire model.

The choice of interpolation method to make the digital terrain model is of less importance since the accuracy depends primarily on the nature of the terrain surface and the spacing of the control points (Kubik and Botman, 1976).

In the following, data sampling and the procedure of power spectrum estimation are briefly discussed, together with the connection between the nature of the terrain surface and the spectral estimate. Furthermore, some estimates of the spectra of different geology are shown, and finally an example illustrating the prediction of the standard deviation by means of the power spectrum is given. The latter is verified by a test area.

Data Sampling

We assume that the terrain surface is a stochastic process; however, raw data samples rarely appear in this form. Therefore the first operations on the samples are to estimate the systematic trends and the mean to subtract these values from the data points to get zero mean. A straight line or other polynomial trend can be calculated by the least-squares method and removed.

We now assume that this pre-processing has been carried out to remove the mean value and non-stationary trends from the data. For the sake of convenience we also confine ourselves to use equi-spaced data samples. If the sample interval is Δx meters and the profile length is L meters, the data will contain wavelengths from L to $2 \cdot \Delta x$, or frequencies from $1/L$ to $1/(2 \cdot \Delta x)$. Frequencies in the data which are higher than $1/(2 \cdot \Delta x)$ will be folded into the lower range from $1/L$ to $1/(2 \cdot \Delta x)$ and confused with these. This concept is known as aliasing. To avoid aliasing we must choose Δx so small that no wavelength smaller than $2 \cdot \Delta x$ exists in the terrain. In practice this is impossible because we are measuring terrain surfaces, and we have no reason to expect that the terrain contains any smallest wavelength. Consequently, we shall always expect an aliased spectral estimate. However, it is possible, to some extent, to remove the aliasing error. The shortest wavelength is $2 \cdot \Delta x$, and the corresponding frequency is called the Nyquist frequency. Because of the aliasing, the estimated value at the Nyquist frequency is approximately twice the corresponding value of the spectrum.

Estimation Procedure

It is easy to show (Schwartz and Shaw, 1975) that a raw spectral estimate can be calculated from the data

$$S_N(f) = L \cdot C_f^2 = L(A_f^2 + B_f^2)$$

where

$$A_f = \frac{2}{N} \sum z_i \cos\left(\frac{2\pi}{L} \cdot \Delta x \cdot i \cdot f\right)$$
$$B_f = \frac{2}{N} \sum z_i \sin\left(\frac{2\pi}{L} \cdot \Delta x \cdot i \cdot f\right) \qquad (1)$$

L is the length of the profile and z_i is the sample value at the position $\Delta x \cdot i$. This has been examined empirically by Frederiksen, Jacobi, and Justesen (1978). The estimation of the power spectrum is based on random profile sampling in the area. For our purposes, the best estimate is calculated from profiles of a reasonable length and a sample spacing as small as possible, say a few centimeters. The small sample spacing will secure information to estimate the high frequency part of the spectrum. However, the decrease of spacing will increase the number of samples unacceptably. The effect of a realistic sample spacing is aliasing and the fact that we fail to obtain the high frequencies. Similarly, we cannot increase the profile length to infinity, and the result is a frequency separation of 1/L. This is due to the harmonic functions which produce wavelengths of L, L/2, L/3... (2 · Δx). At a fixed frequency, f, the estimated value is representing an average of the spectrum in the frequency range from f - 1/2L to f + 1/2L. So, to estimate all the "energy" in the range, we must multiply by L the square sum of the Fourier coefficients.

Direct use of (1) to estimate the power spectrum causes a rather fluctuating estimate which is difficult to interpret. To obtain the main shape of the spectrum, a smoothing procedure is applied. Tukey (Blackman and Tukey, 1966) has suggested a procedure which causes the following averaging in the frequency domain

$$S_N(f) = \frac{1}{4} \cdot S_N(f-1) + \frac{1}{2} \cdot S_N(f) + \frac{1}{4} \cdot S_N(f+1)$$

The estimation of the power spectrum may be described briefly as follows:

1. Profiles are measured with equi-spaced samples. The profile length must be reasonably long to reduce bias problems.
2. Each profile is Fourier transformed and smoothed.
3. The smoothed spectral estimates are averaged to get the main shape of the spectrum. This also reduces the variance of the spectrum.

The Spectral Estimate

A spectral estimate may appear as shown in Figure 1. Notice the log-log coordinates. Even if the smoothing procedure has been applied, the estimate is still irregular, and it is an aliased spectrum. The estimate is plotted versus the wavelengths instead of frequencies because this is more reasonable when dealing with terrain surfaces. The spectral estimate is approximated by straight lines for two reasons: the lines make it easier to compare different spectral estimates, and they also make it possible to obtain a handy expression for the standard deviation between a digital terrain model and

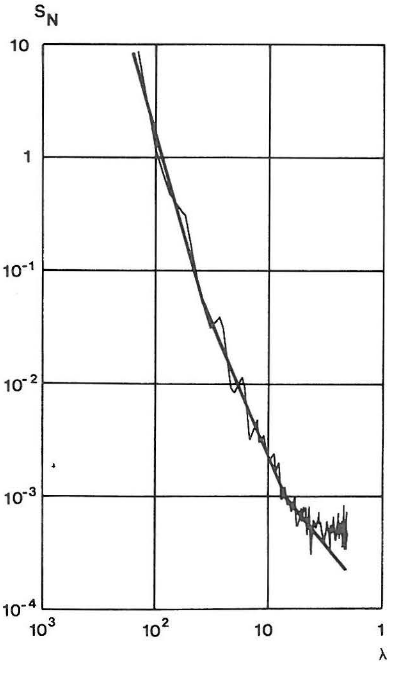

Figure 1

Straight line approximation of spectral estimate the terrain represented by the spectral estimate. The expression for the standard deviation will be discussed later.

We shall now characterize the spectral estimate by the slope and absolute placing of the lines which approximate the spectral estimate. In Figure 2 a spectrum is shown as a straight line representing a hypothetic terrain surface. Due to the line approximation the estimate can be expressed as:

$$\log S_1 = \log E_1 + \alpha \log \lambda \quad \text{or} \quad S_1 = E_1 \lambda^{\alpha}$$

where α is the slope, λ the wavelength, and E_1 the "energy" at $\lambda = 1$ meter.

Another terrain is represented by $S_2 = E_2 \cdot \lambda^{\alpha}$. Both spectral estimates have the slope α and contain the same wavelengths. From

$$\frac{S_2}{S_1} = \frac{E_2}{E_1} = \text{constant}$$

we conclude that the spectra are proportional. This means that the two terrain surfaces contain the same pattern of fluctuations and look alike except for a scale factor in the z-coordinate.

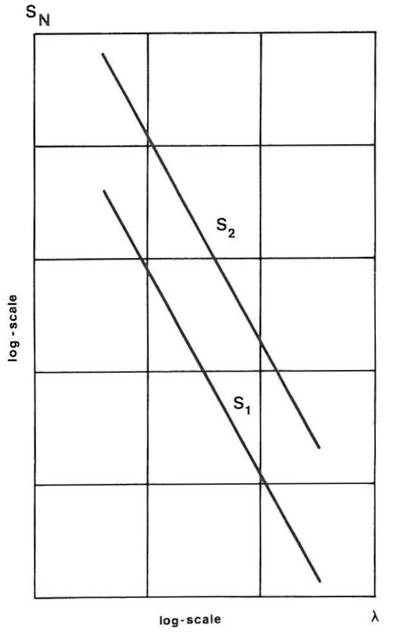

Figure 2

For a spectrum with the slope $\alpha = 2$ for all wavelengths, we get

$$S = E \lambda^2$$

or

$$C = \frac{\sqrt{S}}{\sqrt{L}} = \frac{\sqrt{E}}{\sqrt{L}} \cdot \lambda = \text{constant} \cdot \lambda$$

which indicates proportionality between the amplitudes and the wavelengths. In this terrain the surface fluctuations are independent of scale, which means that the landscape looks the same independently of the altitude from which is it observed. This phenomenon appears in certain mountain areas.

If the slope of the spectrum is greater than 2, the landscape is hilly. Most of the "energy" is found in the amplitudes corresponding to the long wavelengths. The height of the hills defines the placing of the spectrum, and the surface is smooth since only small amplitudes are present in the range of short wavelengths. A spectrum with a slope less than 2 is representing a flat landscape with a rough surface since the surface contains relatively large variations with short wavelengths.

Spectra of Different Geology

Following the estimation procedure, the spectra of some different geological terrain types have been estimated. Heights have been measured in profiles photogrammetrically or by levelling. The geology and the corresponding profile data are listed in Table 1.

Table 1

Region/Geology	Number of profiles	Number of samples per profile	Sample distance (meters)	Measuring method
Denmark near Viborg out washed plain	10	300	1	Levelling
Denmark near Horsens terminal moraine	10	300	1	Levelling
Denmark near Nakskov lodgement till	10	300	1	Levelling
Washington Land sedimentary rocks	6	400	50	Photo-grammetry
Disko Island plateau basalts	8	300	30	Photo-grammetry
Norway, south-east of Oslo / precambrian basement	7	500	15	Photo-grammetry

The spectra of three terrain types in South Denmark are shown as straight line approximations in Figure 3. The geology in the areas are out washed plain, terminal moraine, and lodgement till located near the cities of Viborg and Horsens in Jutland and Nakskov in Lolland, respectively. The relative shapes of the spectra confirm the intuitive impression of the variations in the three types of landscape. The terminal moraine is a hilly

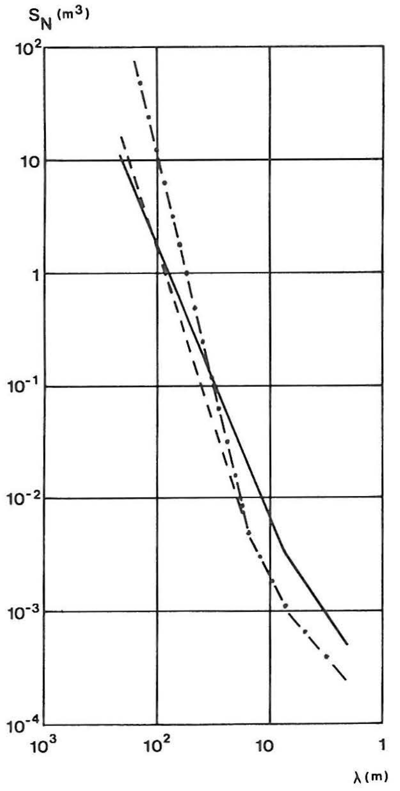

Figure 3

Spectral estimates of

——— out washed plain
—•—• terminal moraine
— — — lodgement till

landscape, while the lodgement till and the out washed plain are rather flat. In the range of wavelengths from about 11 m to 2 m, we observe that the spectra of the moraine landscapes coincide. Both areas are farm lands today, while the measured part of the out washed plain has not been cultivated. Though it has not been examined in detail, one might draw the conclusion that the cultivation has caused a smoothing of the surface for short wavelengths.

In order to evaluate how this technique works in landscapes which are very different from those of South Denmark, three mountain areas have been examined. The profile measurements have been carried out photogrammetrically in a Zeiss Planitop F2 with automatic registration equipment. The photo scale in the Norwegian area located south-east of Oslo is 1:15,000, while the sedimentary rocks in Washington Land, North Greenland, and the Disco Island with its plateau-basalts have been measured on a scale of 1:56,000.

The spectra are shown in Figure 4. The spectra are estimated for rather long wavelengths

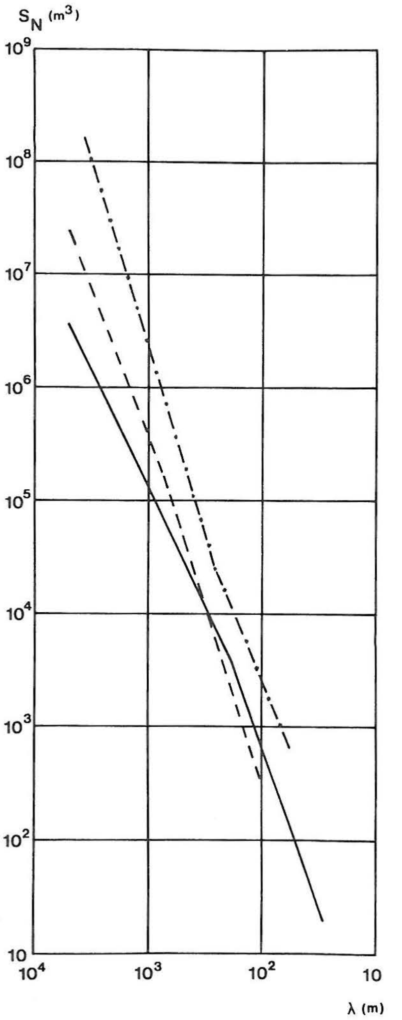

Figure 4

Spectral estimates of

—— precambrian basement
—•— plateau-basalts
— — sedimentary rocks

due to the photo scales. But also in this case the relative positions and slopes of the spectra are in accordance with the visual impressions of the landscapes. Further interpretation as to the geology should be carried out by geologists, but for technical surveying purposes we can use the range of short wavelengths of the spectrum for accuracy estimation. This is demonstrated in the following section.

The Spectrum and the Standard Deviation of a Digital Terrain Model (DTM)

If we know the spectrum of a terrain surface and in particular the high frequencies, we can estimate the standard deviation between a digital terrain model and the terrain surface

$$s_o^2 = m_z^2 + \sum_{f=1/(2\cdot\Delta x)}^{\infty} C_f^2 \qquad (2)$$

The expression is discussed by P. Frederiksen, O. Jacobi, and J. Justesen (1978). The term m_z^2 is the variance of the data points on which the DTM calculations are based. The sum is calculated from the frequency $1/(2\cdot\Delta x)$ to infinity. Δx is the separation of the data points, so this simple expression is valid only in the case of equally-spaced samples in the x and y directions.

It may seem inconvenient that we have to sum over a frequency range which is not recorded in connection with the DTM measurements. But if the power spectrum is known and if the spectrum can be approximated by a simple function or neglected for frequencies higher than a fixed f, the sum can be calculated easily.

Jacobi (1980) has given a spectral estimate for a moraine landscape. The spectrum is approximated by a straight line, so the formula becomes

$$s_o^2 = \frac{E\cdot(2\cdot\Delta x)^{\alpha-1}}{\alpha-1} + m_z^2$$

The data of the line approximation are

$$E = 10^{-4} \quad \text{and} \quad \alpha = 2.5$$

giving

$$s_o = 1.89\cdot 10^{-4}\cdot \Delta x^{1.5} + m_z^2 \qquad (3)$$

A modest test area of 4225 (65 x 65) points arranged in a square grid with the sample spacing of 10 meters has been measured photogrammetrically. The photo scale is 1:4,000, and the m_z is approximately 0.10 m. Every fourth point in both directions of the test area was used to construct the DTM. This is a regular data grid containing 17 x 17 points separated by 40 meters. From (3) we can calculate an estimate of the standard deviation with Δx = 40 m.

$$\tilde{s}_o = \sqrt{1.89 \cdot 10^{-4} \cdot 40^{1.5} + 0.10^2} = 0.24 \text{ m}$$

By means of a linear least-squares prediction every data point in the test area was estimated and compared with the measured value. At last the standard deviation was calculated from the 4225 residuals

$$s_o = 0.26 \text{ m}$$

If the test area is representative of the continuous terrain surface, we must conclude that the calculated value based on the spectrum is a rather good estimate of the standard deviation.

Comments

The spectral estimates have been approximated by straight lines, but for geological interpretation purposes local peaks in the estimate may be of particular interest. So, other smoothing procedures or filters may be applied to bring out terrain characteristics. It is obvious that not only geology defines the spectra, but also erosion, growth, and cultivation influence the shape of the estimate. However, the effects may appear in different parts of the spectrum.

For technical surveying the range of short wavelengths is of special interest. From Formula (2) we see that the interpolation accuracy is determined by the high frequencies which are not recorded in the data points of the DTM. Further experiments must be carried out to interpret the connection between the terrain variations and the spectrum to obtain handy expressions for the estimation of the standard deviation.

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