Commission V/6

B. Drerup

Sonderforschungsbereich 88/C1
Hüfferstr. 27
D-4400 Münster
W. Germany

A Procedure for the Numerical Analysis of Moire´Topograms

Abstract

The contour lines of a moire´topogram represent a description of a surface that is dependent on the position of the object. In medical applications the positioning of the patient cannot be guaranteed, so that the mere examination of the moire´fringe patterns may lead to wrong interpretations. The moire´topogram must therefore be assessed.

For numerical reconstruction the topogram is digitized by hand along a rectangular grid (raster). The data, that is collected along each grid line allows the reconstruction of profiles, which can be combined to give a spatial representation of the surface. The grid generates a parameter representation of the surface, which facilitates other subsequent mathematical procedures similar to that of rasterstereography described in the paper of Frobin and Hierholzer.

Introduction

Moire´topography is a stereometric method for the measurement of three dimensional shapes. The total spatial information is contained in one two-dimensional image. In the "Classical Case" /Takasaki/ a line-grating positioned in front of the object modulates: 1) the light emerging from the light source S (fig.1), thus generating a pattern of illuminated lines on the object; 2) the light reflected from the object and collected by the lens L. The superposition of the two line patterns produces moire´fringes in the focal plane behind the lens. The fringes may be interpreted as the image of intersection lines ("contour lines") on the objects surface, produced by the intersection of the object with a system of thought contour layers (e.g. E_N in fig. 1) in space. To each contour layer an ordinal number N is allocated. N equals the number of grating lines between the object ray SP and the image ray PQ through L.

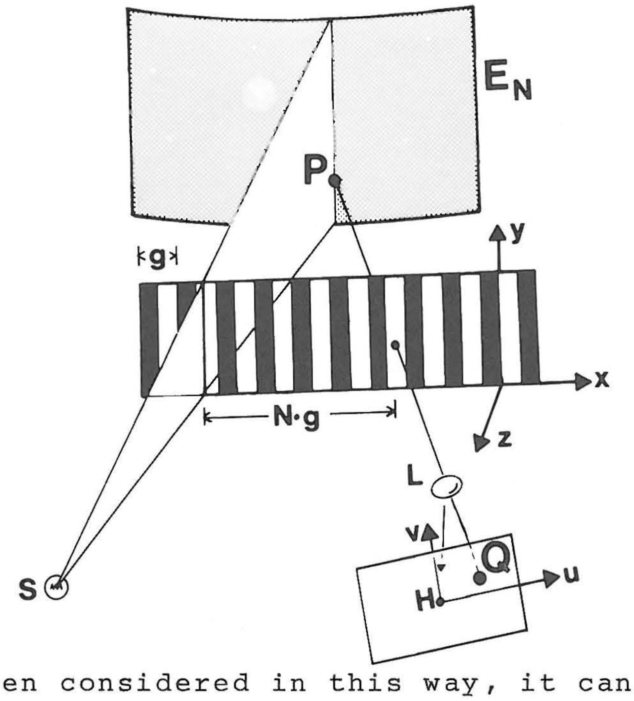

Fig. 1 Schematic diagram of the principle of moire´ topography

When considered in this way, it can be seen that the fringe pattern in a moire´topogram, which is the registration of the moire´fringes, is dependent of the position of the object behind the grating. If conclusions about the shape of the object are to be drawn from the fringe patterns,it is necessary to localize the object in a standardized position. However, this might lead to difficulties in medical applications, especially those where patients with trunk deformities are being analysed. If on the other hand a coordinate free representation of the surface is given, these difficulties will not arise. A procedure, starting from a topogram and resulting in a parameter representation of the registered surface is given in the following. The surface is reconstructed point by point along profiles, from which the parameter representation is calculated. The subsequent procedure starting from a parameter representation of a surface to its coordinate-free representation is given elsewhere/Lipschutz; Frobin,Hierholzer/.

Point Reconstruction

An x,y,z coordinate system is attached to the grating with the y-axis parallel to the grating lines and the x-axis in the grating plane. The film plane is spanned by a u,v coordinate system with the origin at the principal point H. We assume P to be a point on a contour line contained in the contour layer E_N. If the geometry of the optical arrangement is known, as well as the coordinates (u,v) of the image point Q and its ordinal number N, then the point P in space can be reconstructed by the intersection of the image ray with the plane passing through the light source and the N^{th} grating clearance. N is counted from that clearance at which the image ray passes through the grating. By this reconstruction the (x,y,z) and (u,v,N) representation of a point P are equivalent.

In the particular case where the light source and the nodal point of the lens are in the same distance l from the grating, and the film plane is parallel to the grating, the coordinates of P are computed:

$$\begin{aligned} z &= l \cdot N/(N-g/d) \\ x &= u \cdot (l-z)/f \qquad (1) \\ y &= v \cdot (l-z)/f \end{aligned}$$

with the u,v coordinate axes parallel to the x,y coordinate axes respectively and $H=(H_x, H_y, H_z)=(0,0,-l/f)$, f being the camera constant, g/d the ratio of the grating constant to the distance between lens and light source.

Parameter Representation of a Surface

The mapping of the points Q with the plane coordinates (u,v) into the surface points P with the spatial coordinates (x,y,z):

$$\begin{aligned} x &= x(u,v) \\ y &= y(u,v) \\ z &= z(u,v) \end{aligned}$$

is a parameter representation of a surface. As a special mapping we can reverse the central projection through L, and superpose to the topogram a regular raster of coordinate lines with constant spacing between successive lines. The image of the coordinate lines $u=u_i$ and $v=v_j$ on the surface are the v- and u- parameter curves u_i and v_j respectively, which in the following will be loosely called horizontal- and vertical profiles. Coordinate lines and respective parameter curves lie on particular planes, (fig. 2). To all planes the projection centre L is common.

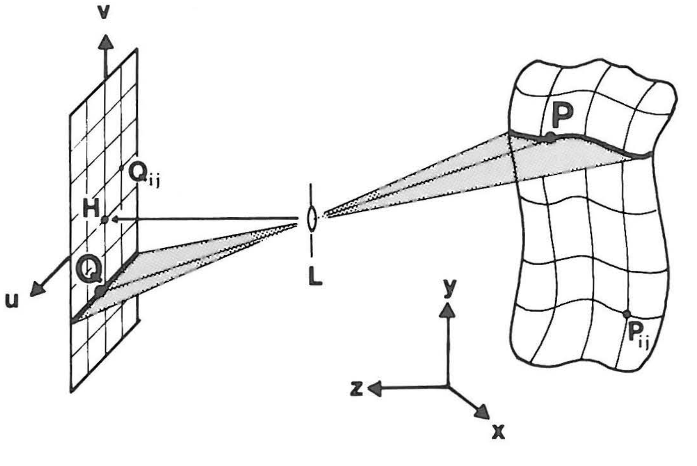

Fig. 2 Coordinate lines in the film plane are mapped into parameter curves on the surface

There is a one-to-one correspondence between each intersection point of two coordinate lines and the intersection point of the two respective parameter curves (overlapping structures are ignored). In the following the parameter description of a surface means such a mapping of the intersection points $Q_{ij}=(u_i,v_j)$ of the u_i and v_j coordinate lines into the image points P_{ij} with the coordinates (x_{ij},y_{ij},z_{ij}). In general Q_{ij} cannot be expected to lie on a contour line, so P_{ij} must be determined by interpolation between the two intersecting profiles. In the following the steps leading to this interpolation are described.

<u>Moire Pattern Analysis</u>

The determination of the spatial coordinates P_{ij} from its image coordinates (u_i,v_j) requires the knowledge of the ordinal number $N=N(u_i,v_j)$, which is computed by interpolation along the u_i and v_j parameter curves. In particular the interpolation involves the following steps.

1) By visual inspection of the topogram the sign of the slope of the profiles is determined: The difference ΔN of the ordinal numbers of two contour lines, which are intersected in succession by a coordinate line may be +1 or -1; $\Delta N=0$ at the peak values of the profile. Having prescribed the direction of running through the coordinate lines, the sections with $\Delta N=+1$ and $\Delta N=-1$ are marked by a "+" and a "-" respectively (fig. 3). The peak values ($\Delta N=0$) are recognized by a change of the sign.

If by using a special grating an asymmterical but still periodical structure of the contour layers along the z-direction is produced/Basse; Kugel,Lanzl; Windischbauer/, this visual inspection can be replaced by an automatic inspection.

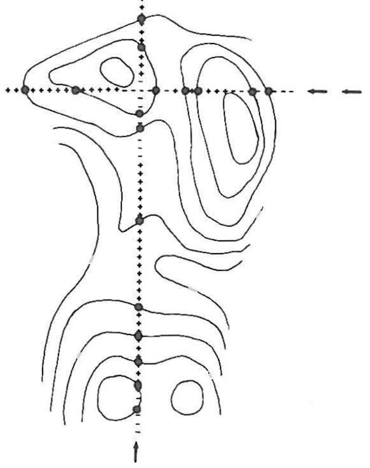

Fig. 3 Segments of the coordinate lines with positive ("+") and negative ("-") slope of the corresponding parameter curves. Note the prescribed direction. The digitized points are marked by a dot

2) The profiles are digitized by measuring the (u,v) coordinates of the points of intersection between the coordinate line and the contour lines. A relative ordinal number is calculated for each point by adding $\Delta M=\Delta N$ to the ordinal of the preceding point, starting with an arbitrary (integer) M_o for the first point of the particular profile.

3) The absolute ordinal number is calculated for each point. This is done by adding a correction constant K to M:

$$N=M+K$$

Note, that N,M and K are integers. To determine K it is nece-

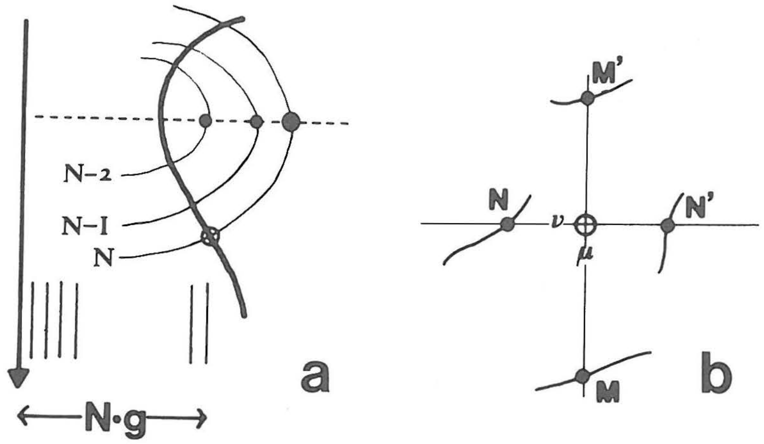

Fig. 4 Determination of the ordinal number of a fringe by:
a) counting the grating lines
b) comparison of the interpolated values ν and μ

ssary to know N in one point of the profile. According to its definition N is the number of grating lines between the object ray and the image ray. For our purpose one grating line is specially marked, so that its shadow on the surface can easily be recognized. The number of grating lines between the marked line and the intersection point of its shadow with a contour line (fig. 4a) is then the ordinal number of that contour line.

The described method fails, if the coordinate line is not intersected by the shadow of the marked line. In this case K is found by intersection with another coordinate line, whose ordinals are already known (fig.4b). Let M,M′,N,N′ be the 4 ordinal numbers neighboring the intersection point of two coordinate lines. From the already absolutely known N and N′the ordinal number ν in the intersection point can be interpolated, which therefore no longer needs to be an integer. Similarly μ can be interpolated from the relative M and M′. If the interpolation is exact, the equation

$$K=\nu-\mu$$

holds. With the constraint that K is an integer, K can also be determined when small interpolation errors occur. For curved objects a linear interpolation is not sufficient in general, but with a cubic spline K could be determined correctly.

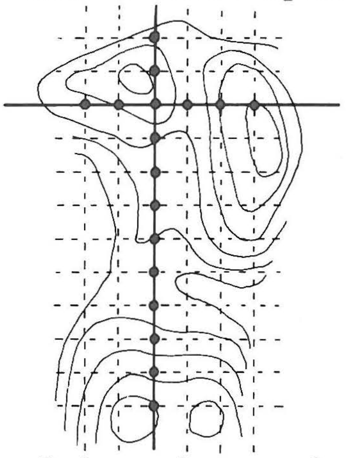

Fig. 5 A surface point is reconstructed for every point of the raster superposed to the topogram

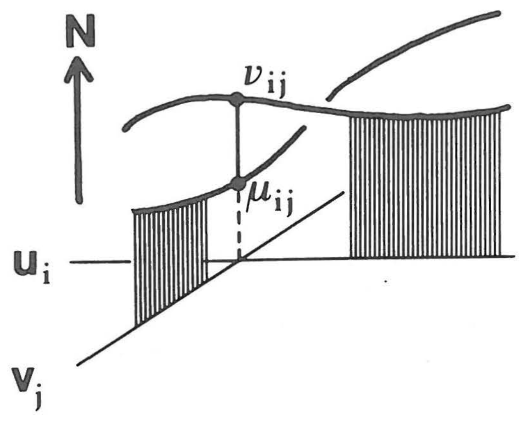

Fig. 6 The values of the interpolated ordinals in general differ. Their differnce is a measure for the digitization error

4) At the intersection points of each two coordinate lines (fig. 5) the ordinal number is interpolated from the parameter curves. In general the reconstructed profiles do not intersect in space due to digitization errors. Instead, different ordinal numbers result for an object point, depending on an u- or v- parameter curve was used for reconstruction (fig.6) Therefore for the ordinal number of P_{ij} a mean value is taken:

$$N(u_i, v_j) = (\nu_{ij} - \mu_{ij})/2$$

ν_{ij} and μ_{ij} being spline interpolated values.

Results

The topogram of a plaster cast of a patient suffering from scoliosis (fig. 7a) was evaluated in the above described manner. Profiles were reconstructed along a cross grid of 40 coordinate lines (fig. 7b). A perspective view of the parameter representation of the surface, reconstructed from these profiles, is shown in fig. 7c. The distance between two succesive profiles on the surface amounts to about 1.75 cm.

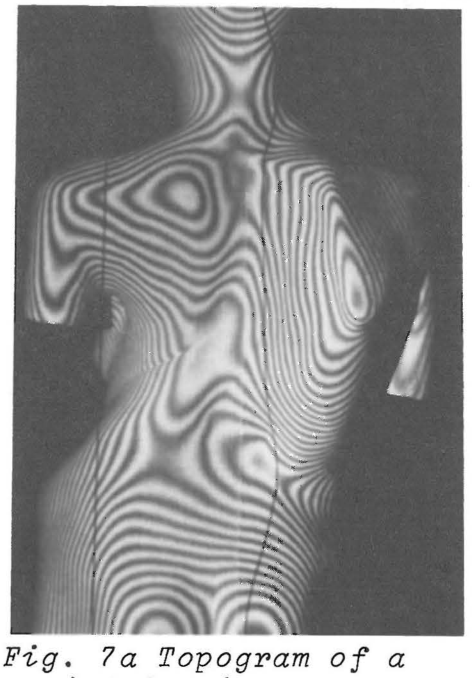

Fig. 7a Topogram of a patient's plaster cast (scoliosis)

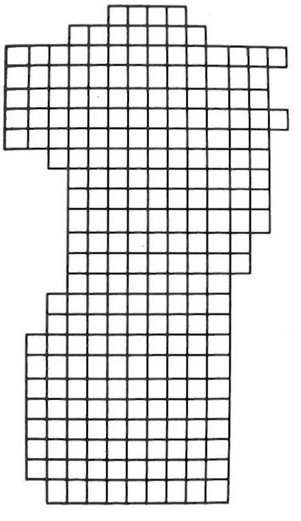

Fig. 7b Raster of coordinate lines

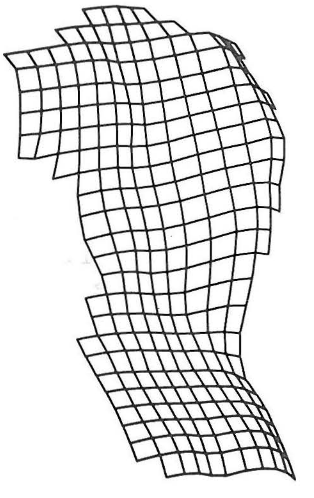

Fig.7c Perspective view of the combined profiles

The difference of the ordinal numbers in P_{ij}

$$\tau_{ij} = |\nu_{ij} - \mu_{ij}|$$

can be taken as a measure for the digitization error. In the above shown case the mean digitization error, taken over all reconstructed points was $\bar{\tau}=0.15$, roughly equivalent to a difference along the z coordinate of 0.5 mm. The variance was determined to be 0.22, equivalent to 0.7mm.

Discussion

The described procedure for the evaluation of topograms mainly consists of two steps: 1) measurement and reconstruction of profiles from the moire´fringe pattern, 2) computation of a parameter representation of the surface from the profiles. This partition is an advantage in those cases, where important information can already be drawn from the profiles /Drerup/. On the other hand it seems desirable that the digitization error of 0.7 mm caused partly by the interpolation is improved.

If a rigorous shape analysis based on its parameter representation is required, a more direct determination of the parameter representation might give better results. This could be done by a method /Yamato/, where discrete segments of the surface are reconstructed by polynomials fitted to the contour lines.

References

G. Basse: Thesis Univ. Hamburg, 1976

B. Drerup: Interner Bericht SFB 88/C1 Nr. 20, Münster 1979

W. Frobin, E.Hierholzer: Interner Bericht SFB 88/C1 Nr. 21 Münster 1979 and: This issue

H. Kugel, F.Lanzl: *Holography in Medicine and Biology*,ed. by G.v. Bally, Springer, Berlin...,1979

M.M. Lipschutz: *Theory and Problems of Differential Geometry*, Mc. Graw-Hill, New York, 1969

H. Takasaki: *Holography in medicine and biology*,ed. by G.v. Bally, Springer, Berlin...,1979

G. Windischbauer: *Holography in Medicine and Bilogy*, ed. by G.v. Bally, Springer, Berlin..., 1979

J. Yamato: Medinfo, Tokyo 1980