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CHOOSING OPTIMAL VARIANTS OF PHOTOGRAM-
METRIC SURVEY WHEN SOLVING THE PROBLEMS
OF ROCK MECHANICS AND MINE SURVEYING

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ABSTRACT: In this article the authors set forth the technique of choosing optimal variants of photogrammetric survey by analytical modelling by means of computers. For choosing an optimal variant of photogrammetric survey when solving a concrete problem, the influence of the various factors (base length, angles of convergence, control points number) upon the accuracy of space coordinates determination is investigated on these analytical models.

Not long ago the sole objective of photogrammetry in mining was provision of mines with graphic documentation: plans of mine surveying, profiles, photomaps and photographs. Lately though the scope of problems has considerably widened, especially through investigation of deformations at different sorts of objects: pit slopes, buildings, constructions, frameworks of mining machines, geomechanic models, imitating rock pressure manifestations, underground supports etc. This variety of tasks and conditions emphasizes choosing optimal variant of surveying. Here the preliminary evaluation of some factors' (namely: distance, base length, angle of convergence and inclination, siting and number of control points, brand of camera - photogrammetric or amateur, accuracy of photographs measuring) influence upon the accuracy of results is necessary. Immediate analysis of this influence is hardly feasible and practically im-

possible in many cases.

In VNIMI this problem is solved by a method of computer analytical modelling.

Parametric adjustment of measurements by the least square method in calculation of resection in space is taken as a basis of modelling. The resection in space is calculated with the aid of a well known collinearity equation of image rays:

$$\begin{pmatrix} x \\ z \\ f \end{pmatrix} = KM \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

The corrections to preliminary values of data of outer and/or inner orientation are derived from the solution of the set of equations in the form $AX + L = v$ if the condition $pvv = \min$ is met. Along with that it is possible to determine corrections to the ground coordinates (it is done if only their approximate values are available). Spatial coordinates X, Y, Z and their mean square errors are calculated from the adjusted values of the orientation data when any mutual arrangement of camera and object is possible.

Precalculation of possible accuracy of surveying requires an analytical model consisting of the totality of points the coordinates of which on the photographic images and in the system of axes of the objects are calculated according to the central projection laws. This model is built and processed by a computer.

In processing of the model two independent components $m_{1X(Y,Z)}$ and $m_{2X(Y,Z)}$ of the mean square error $M_{X(Y,Z)}$ of coordinates X, Y, Z are calculated. Component $m_{1X(Y,Z)}$ stems from errors of the orientation data and $m_{2X(Y,Z)}$ - from photo-coordinate errors.

Value of $M_{X(Y,Z)}$ is calculated as a root mean square value of the components $m_{1X(Y,Z)}$ and $m_{2X(Y,Z)}$. The component $m_{1X(Y,Z)}$ is derived from the formulae:

$$m_{1X(y,z)} = \mu \sqrt{Q_{X(y,z)}}, \quad (1)$$

$$Q_{X(y,z)} = F_{X(y,z)} B^{-1} F_{X(y,z)}^T, \quad (2)$$

where $F_{X(y,z)}$ and $F_{X(y,z)}^T$ are respectively vector and transpose of the vector the elements of which are partial derivatives of functions X, Y, Z with respect to the orientation data;

B^{-1} - inverse matrix of coefficients of normal equation of corrections;

μ - standard error of unit weight.

The constituent $m_{2X(y,z)}$ is calculated from the formula:

$$m_{2X(y,z)}^2 = \sum_{k=1}^4 f_k^2 m_k^2 \quad (3)$$

where m_k - mean square errors of photographic measurements of coordinates x, y and parallaxes p, q ;

f_k - partial derivative of functions X, Y, Z with respect to coordinates and parallaxes.

Analysis and generalization of the processed data and thus revealed regularities of errors distribution serve as

a basis for the choice of optimal survey variant while solving practical problems.

The investigations results are presented in the form of tables and plots of errors distributions.

The study of some factors' influence upon the accuracy of spatial coordinates determination is analysed below.

THE NUMBER AND SITE OF CONTROL POINTS

The study was accomplished on an analytical model which consists of 50 points evenly distributed (5 rows with 5 points in each) along the near and far (from the centre of projection) facets of rectangular parallelepiped 1000 x 1500 x 1500 mm in size.

Normal perspective of the survey is imitated with the following parameters: basis - 1000 mm, overlap - 50%, distance from basis to the near plane of the parallelepiped - 2500 mm, focal length and size of pictures are 200 mm and 13 x 18 cm respectively. Only the elements of exterior orientation are corrected in the processing of the model.

Fig.1 displays some of the investigated sketches of control points. Values of Q_y (see formula 2), mean square errors of m and its components m_I and m_2 (formulae 1, 3) are presented in table I.

The confined space of this paper does not permit to present in full details the results of the studies, therefore table I exhibits only Q_y , m_{Iy} , m_{2y} , m_y for the six sketches of control points.

Figs.2,3 show the distribution of values Q_y , Q_x , Q_z (decreased for Q_y by a factor of 100) only on sketches 2 and 3 (fig.1) respectively at near and far facets of the model. In calculation of errors it was assumed: $\mu=0,01$, $m_x=m_z=0,01$ mm.

Results of the studies suggest that number of control points influences mainly the values of errors of spatial coordinates whereas their siting sketch affects primarily character of errors distribution on the object of survey. Choosing this or that sketch of control points siting we may enhance (all other things being equal) the accuracy of coordinates determination in prescribed sites of the object or reach uniformity of accuracy throughout the object.

Table I

Name of the Model's facet	Sketch of the points siting					
	I	2	3	5	8	9
Number of control points						
	4	4	4	6	6	8
$Q_y \cdot 10^{-2}$						
Near	14,6	58,6	21,7	51,9	13,0	8,8
Far	950,9	181,2	108,3	160,0	56,5	38,6
Upper	482,8	82,7	86,4	67,8	32,2	23,7
Lower	482,8	157,2	43,6	144,1	37,3	23,7

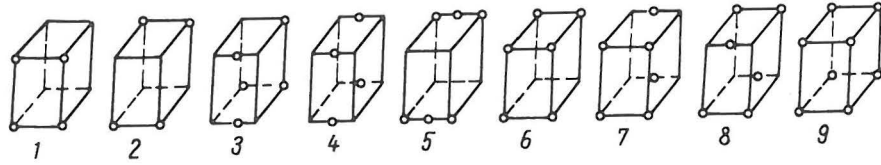


Fig. 1

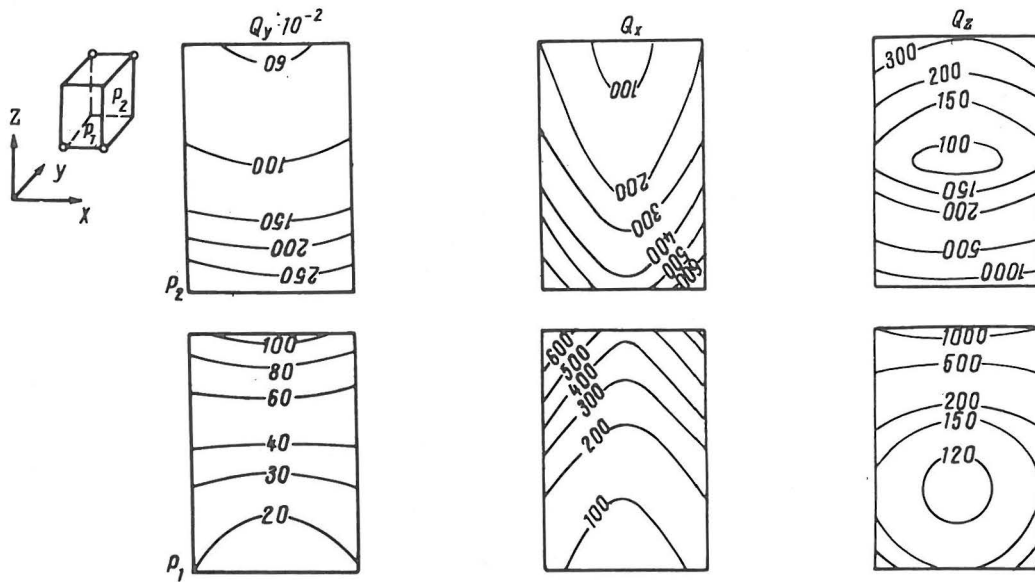


Fig. 2

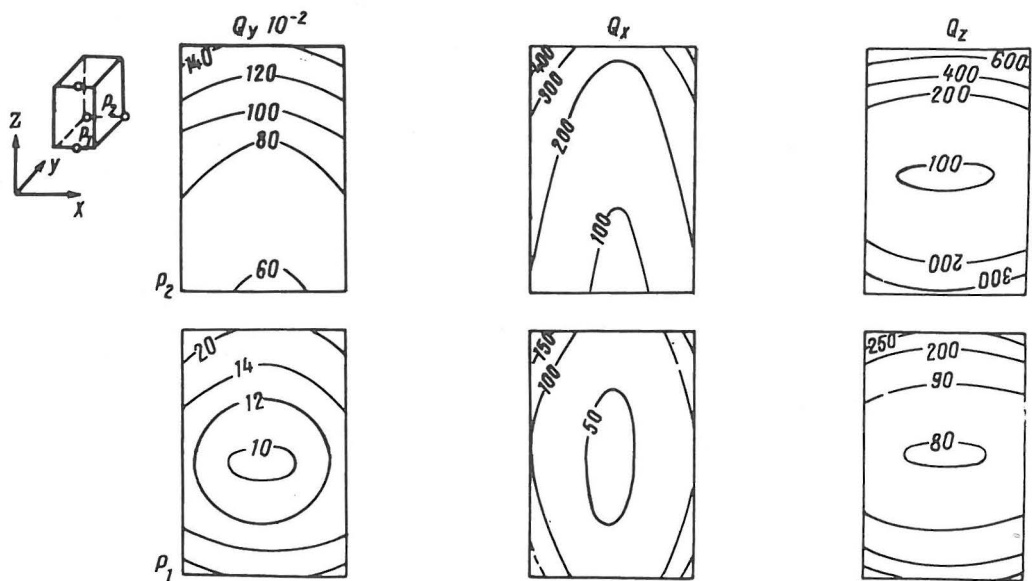


Fig. 3

Table I (contd.)

Left	482,8	119,9	54,5	106,0	39,8	23,7
Right	482,8	119,9	54,5	106,0	39,8	23,7
Mean values of errors in the model						
m_{Iy} , mm	2,14	0,98	0,74	0,91	0,54	0,47
m_{2y} , mm	0,26	0,26	0,26	0,26	0,26	0,26
m_y , mm	2,16	1,02	0,78	0,95	0,60	0,54

CONVERGENCE OF SURVEYING AXES

Analytical models of symmetrical convergent surveying are calculated by a computer. The parameters of these I2 models are displayed in table 2. "Object of surveying" is a vertical plane with 25 points (the 5 rows of 5 points each). Figs 4b and 4c show only 9 points, remaining 16 are symmetrical about AA and BB axes. In calculation the basis was assumed to be horizontal and parallel to the object plane at the distance of 1000 mm from it. Focal length of the pictures was assumed to be equal 1000 x 1000 mm in size (fig.5), whereas No I2 imitates surveying of the plane which expands with increase of the basis and angle of convergence and conforms in size to the area of stereoscopic pairs with 100% overlap (fig.6).

In models processing only elements of exterior orientation were determined with use of four control points. For convenience of the results interpretation the values of were assumed to be equal 0,01 and $m_x = m_z = 0,01$ mm.

Table 3 displays errors of coordinates determination in I2 models.

Table 2

Number of the model	I	2	3	4	5	6	7	8	9	10	11	12
MM	600	600	600	600	600	1000	1400	2000	3300	7200	2000	2000
γ°	0	20	40	50	60	55	55	55	120	150	180	50
$\frac{B}{Y}$	0,6	0,6	0,6	0,6	0,6	1,0	1,4	2,0	3,3	7,2	2,0	2,0

- Notes: 1. B , γ° , Y are the parameters of the survey: B is a base, γ - angle of convergence, Y - distance from basis to survey object along the perpendicular.
2. Model No 11 presents a borderline case of convergent photography (survey with oncoming axes).

Figs 5 and 6 display distribution of errors of space coordinates on the object plane of models No I and No I2. First

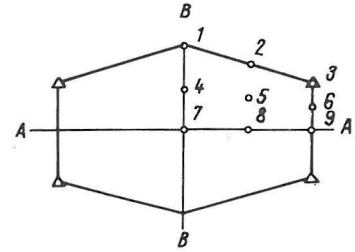
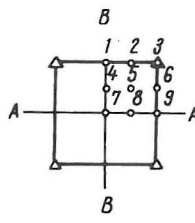
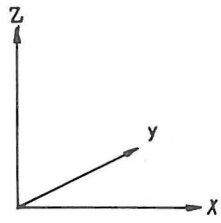


Fig. 4

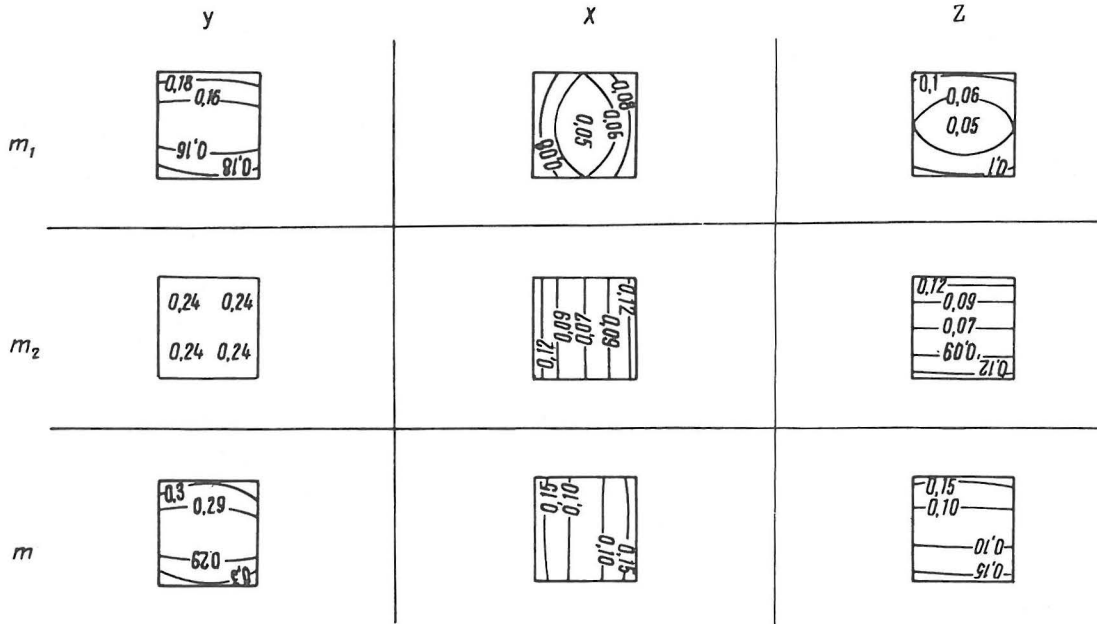


Fig. 5

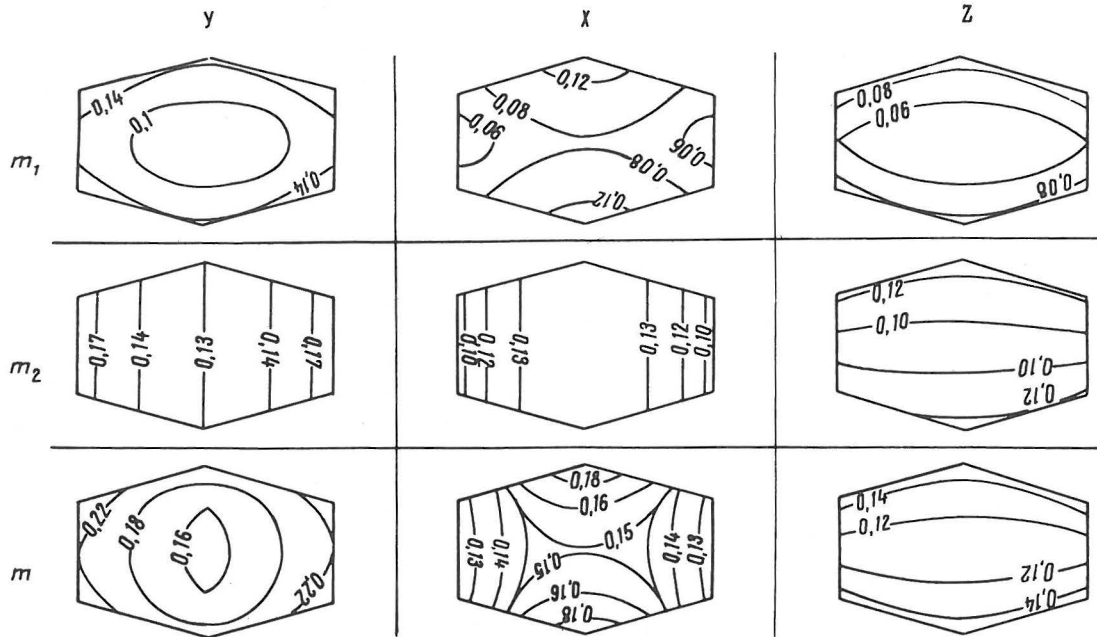


Fig. 6

of them imitates normal survey with relation $B:Y= 0,6$ and the second - symmetrical convergent survey where $B:Y= 2, \quad = 50$.

The accomplished researches enable us to draw following conclusions:

1. The most favourable relation $B:Y$ is in the range from 1,5 to 2,5. In this case accuracy of space coordinates determination is not influenced by variations of angle of convergence from 60° to 120° . With equidistance stereoscopic pair of symmetrical-convergent survey with 100% overlap covers an area twice or thrice as big as that covered by stereoscopic pair of normal survey. Errors of space coordinates determination along the Y axis reduce by the factor of 1,5-2, along other two axes the variations are negligible.

2. Oncoming survey (with angle of convergence equal to 180°) which assures better accuracy of space coordinates determination at the margin of picture may appear useful in surveying the sections of underground workings. Survey with coincident axes has no advantages over other methods of survey whereas the results of position data determination are much more approximate.

As a conclusion we may say that analytical modelling method permits effectively choose optimal variant of photogrammetric survey which secures necessary accuracy in solving mining and other engineering problems with minimal time and labour expenditures.

Conclusions inferred from analytical modelling are in good agreement with the results of processing of real photographs of special stands.

Processing technique serving as a basis of analytical modelling was used in working out recommendations to detecting deformations in buildings, constructions and also in geomechanical models which are being set up to study rock pressure manifestations.

The same technique was unexpectedly applied in restoration of Voskresensk Cathedral in New Jerusalem Abbey, Istra, near Moscow which was destroyed during the World War II.

It was necessary to determine some dimensions of the Cathedral's lost elements. Several amateur archive photographs from which stereoscopic pairs could be compiled were used for the purpose. The points of the Cathedral's remaining part were identified on the photographs and their space coordinates were detected by geodetic methods. Some of the identified points were used as control points, the rest were meant to check up the solution of problem.

Accuracy of the dimensions detected by the comparison of results of different stereoscopic pairs processing and by deviations from geodetic values was of the order of 15-20 cm which appeared quite satisfactory for the demands of the Cathedral's restoration.

Table 3

N. of point	Number of Model											
	1	2	3	4	5	6	7	8	9	10	11	12
	Errors						m_y, mm	m_x, mm	m_z, mm			
1	0,30	0,33	0,30	0,28	0,29	0,23	0,17	0,15	0,14	0,24	-	0,19
	0,09	0,10	0,10	0,10	0,10	0,11	0,13	0,13	0,24	0,42	-	0,20
	0,17	0,18	0,16	0,14	0,15	0,13	0,12	0,13	0,17	0,32	-	0,15
3	0,31	0,34	0,37	0,35	0,36	0,28	0,21	0,16	0,24	0,37	0,18	0,25
	0,17	0,17	0,15	0,12	0,13	0,09	0,14	0,18	0,33	1,40	0,09	0,12
	0,18	0,19	0,18	0,16	0,17	0,14	0,13	0,13	0,18	0,34	0,09	0,15
7	0,29	0,31	0,27	0,25	0,27	0,21	0,15	0,14	0,11	0,20	-	0,15
	0,09	0,09	0,10	0,09	0,09	0,10	0,12	0,15	0,20	1,20	-	0,15
	0,09	0,09	0,09	0,09	0,09	0,09	0,11	0,12	0,14	0,29	-	0,10
9	0,28	0,31	0,32	0,31	0,32	0,24	0,19	0,17	0,22	0,34	0,17	0,22
	0,16	0,15	0,12	0,11	0,12	0,10	0,13	0,16	0,31	1,30	0,08	0,12
	0,09	0,09	0,09	0,09	0,09	0,10	0,11	0,13	0,17	0,18	0,09	0,11
Mean square errors in the whole model (with 25 points)												
	0,29	0,32	0,32	0,30	0,31	0,24	0,18	0,17	0,20	0,32	0,28	0,20
	0,13	0,12	0,12	0,12	0,11	0,11	0,13	0,17	0,29	1,25	0,51	0,14
	0,13	0,11	0,13	0,12	0,12	0,12	0,11	0,13	0,14	0,32	0,09	0,13

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