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COMPUTING AND CHECKING THE PROJECTION CENTERS IN ANALOGICAL PLOTTERS

Abstract:

A simple procedure is proposed, in which the coordinates of the projection cen-ters of each independent model are computed by space resection on the photocarrier's fiducial marks. These coordinates are then compared with those obtained by space intersection from the same marks, thus checking their stability along the observation cycle.

An electronic computation program, and some results are finally reported.

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Computing and checking the projection centers in analogical plotters

1. - In aerial triangulation with analogical plotters the coordinates of the projection centers (PCs) are generally determined at the beginning, halfway and at the end of a group of observations (a strip, or even a block); a precision grid, the coordinates of which are known with a standard error of about $\pm 1\,\mu m$, is observed to this purpose in a perfect horizontal attitude of the projection cameras. By this way it is obviously impossible to determine these coordinates in each model, as effected by the analytical plotters.

Now the PCs may be largely displaced at every variation of the camera's attitude; a determination of the PCs in each model would be therefore highly desirable. This seems possible also with analogical plotters, provided that their photocarriers have engraved proper fiducial marks; if we use a suitable differential procedure, there is no need that these marks be strictly calibrated and referred to the principal point.

This is what we want to investigate. The research is lead through the following stages: a) - mathematical setup of the problem; b) - compilation of an electronic program; c) - experimentation, carried out with the Santoni Simplex II C plotter of the University of Ancona.

Mathematical setup

2. - The $X_0Y_0Z_0$ coordinates of the PC of a photogram, properly placed in an analogical plotter, may be determined by various procedures, among which we shall take into consideration the space intersection and the space resection.

2.1 - Let us consider a general point A in the plate's plane, and let us read its horizontal coordinates XY in two different planes $Z = Z_1$ and $Z = Z_2$. We have (fig. 1):

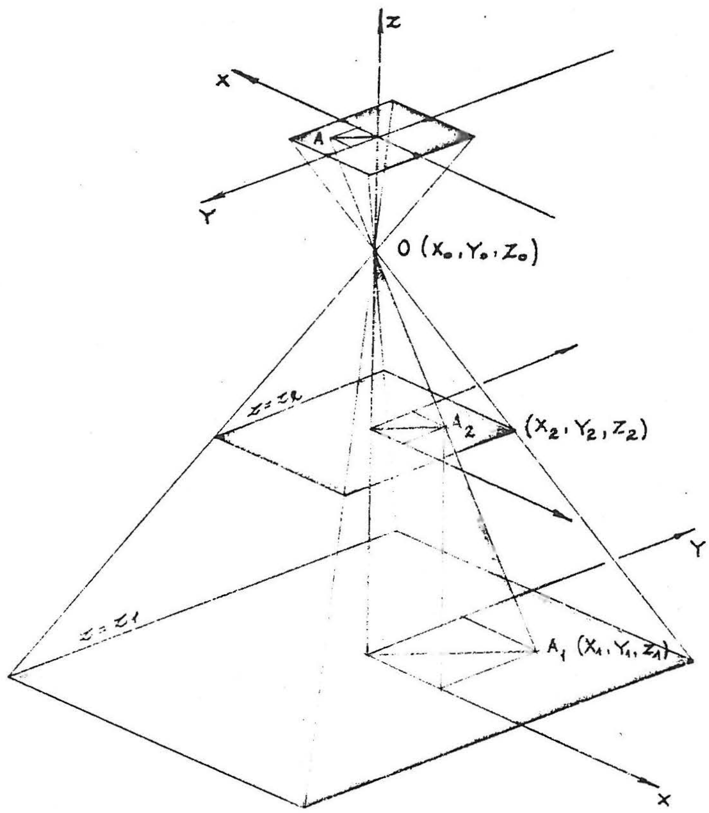

Fig. 1

$$1)\qquad \frac{X_1 - X_2}{Z_1 - Z_2} = \frac{X_1 - X_0}{Z_1 - Z_0} \quad ; \quad \frac{Y_1 - Y_2}{Z_1 - Z_0} = \frac{Y_1 - Y_0}{Z_1 - Z_0}$$

Each A point gives therefore 2 equations, respect. in X and in Y. As the problem's unknowns are 3, $X_0 Y_0 Z_0$, we must have at least two A points. We generally have $n > 2$ points, therefore a LS computation is necessary.

With the assumptions:

$$2)\qquad \begin{cases} a = -(X_1 - X_2) \ ; \ b = -(Y_1 - Y_2) \ ; \ c = (Z_1 - Z_2) \\ T_x = -(cX_1 + aZ_1) \ ; \ T_y = -(cY_1 + bZ_1) \end{cases}$$

we obtain from 2):

$$\begin{cases} c X_0 \qquad + a Z_0 + T_x = 0 \\ \qquad c Y_0 + b Z_0 + T_y = 0 \end{cases}$$

and hence, with further assumptions:

$$\begin{cases} X_0 = x_0 + dx \; ; \; Y_0 = y_0 + dy \; ; \; Z_0 = z_0 + dz \\ t_x = c(x_0 - X_1) + a(z_0 - Z_1) \; ; \; t_y = c(y_0 - Y_1) + b(z_0 - Z_1) \end{cases}$$

we obtain the transformed system:

$$3) \quad \begin{cases} c\,dx \qquad + a\,dz + t_x = v_x \\ \qquad c\,dy + b\,dz + t_y = v_y \end{cases}$$

which shall be solved by a LS computation.

It is possible to compute the focal length from the same observations. In fact, each observed point A_i gives a value of the focal length:

$$4) \quad f_i = -c \cdot \frac{\sqrt{x_i^2 + y_i^2}}{\sqrt{a_i^2 + b_i^2}}$$

where $x_i y_i$ are the image coordinates of A_i, and $a_i b_i c$ are defined by 3). In a quick approach, we may attribute to each value f_i the weight:

$$5) \quad p_i = \left(a_i^2 + b_i^2\right)^2 \; ;$$

indeed, as the image $x_i y_i$ coordinates may be considered as error free (they are precise grid coordinates), the standard error of f_i is inversely proportional to $(a_i^2 + b_i^2)$. Hence we obtain a mean weighted value:

$$6) \quad f = \frac{[p_i f_i]}{[p_i]}$$

Remarks:

i) - precision grid plates are employed, whose image coordinates are exactly known; but they remain unutilized, except in the computation of f. The values which we obtain for $X_0 Y_0 Z_0$ and for f are very strong and well checked;

ii) - the knowledge of the camera's inner orientation, and particularly of the focal length, is not necessary. On the other hand, every misplacement of the grid plate on the carrier is fully repeated in the PC's horizontal coordinates;

iii) – the plate must be strictly horizontal; it is obviously impossible to use this method to determine the PCs of each oriented model along the operation's course.

2.2 – The method of <u>space resection</u> is based on the measure of the instrumental coordinates XYZ of at least 3 points ABC. The image coordinates of these points, and the camera's inner orientation elements must be now exactly known.

The method utilizes the relations which connect the horizontal coordinates $X_i Y_i$ of a point P_i, with its image coordinates $x_i y_i$, the focal length F, the PC's coordinates $X_o Y_o$, and the spatial attitude $\varphi \omega \kappa$ of the camera.

In shorthand notation we may write:

$$7)\quad \begin{cases} X_i = X_o + f(h, x_i, y_i, F, \varphi, \omega, \kappa) \\ Y_i = Y_o + g(h, x_i, y_i, F, \varphi, \omega, \kappa) \end{cases} ;$$

hence, being $dh = dZ_o$, and being $x_i y_i F$ constant at the variation of the camera's height and attitude:

$$8)\quad \begin{cases} dX_i = dX_o + \frac{\partial f}{\partial Z_o} dZ_o + \frac{\partial f}{\partial \omega} d\omega + \frac{\partial f}{\partial \varphi} d\varphi + \frac{\partial f}{\partial \kappa} d\kappa \\ dY_i = dY_o - \frac{\partial g}{\partial Z_o} dZ_o + \frac{\partial g}{\partial \omega} d\omega + \frac{\partial g}{\partial \varphi} d\varphi - \frac{\partial g}{\partial \kappa} d\kappa \end{cases}$$

The partial derivatives which appear in these formulas were evaluated by Von Gruber [1] and Hallert [2] in the case of vertical exposures, by Boge [3] in the most general case. Assuming Boge's formulas with some simplification for vertical exposures, the above system can be written:

$$9)\quad \begin{cases} dX_o - \frac{X}{Z} dZ_o + \frac{XY}{Z} d\omega - \frac{X^2+Y^2}{Z} d\varphi + Y d\kappa - dX_i = v_{x_i} \\ dY_o - \frac{Y}{Z} dZ_o + \frac{Y^2+Z^2}{Z} d\omega - \frac{XY}{Z} d\varphi - X d\kappa - dY_i = v_{y_i} \end{cases}$$

where:

$$10)\quad \begin{cases} -dX_i = X_{computed} - X_i = -\frac{Z_i}{D_i}\left(\cos\varphi \cos\kappa \cdot x_i + \cos\omega \operatorname{sen}\kappa \cdot y_i + \cos\omega \operatorname{sen}\varphi \cos\kappa \cdot F\right) - X_i \\ -dY_i = Y_{computed} - Y_i = -\frac{Z_i}{D_i}\left(-\cos\varphi \operatorname{sen}\kappa \cdot x_i + \cos\varphi \cos\kappa \cdot y_i + \operatorname{sen}\omega \cos\kappa \cdot F\right) - Y_i \\ D_i = -\operatorname{sen}\varphi \cdot x_i + \operatorname{sen}\omega \cos\varphi \cdot y_i + \cos\omega \cos\varphi \cdot F \end{cases}$$

The system 9) shall be solved by a LS computation, with an iterative procedure, supposing that the 1st approximation values (X_o', Y_o', Z_o', 0, 0, 0) are known, and having measured in the actual camera's attitude the coordinates X_{1i}, Y_{1i} of n points ($i = 1,n$; $n \geqslant 3$). The iterations are continued until the residuals are no longer significantly improved, both in the linear and angular elements.

Remarks:

i) - the computations are heavy, and need a mean size computer;

ii) - the riguorous knowledge of the camera's inner orientation, and of the image coordinates of at least 3 points is requested;

iii) - the instrumental measurement of the coordinates of these points is done in one only plane, in any camera's attitude. This is a big advantage, which permits the determination of the PCs even in every model, in the observation course itself;

iv) - unless we have a large number of calibrated points, the PC's definition is rather poor. The eccentricity of the plate's setting on the photocarrier is fully repeated in the X_oY_o coordinates of the PC. However, this inconvenience is negligible if we use the fiducial marks engraved on the photocarrier itself, and apply the differential procedure described in the following paragraphs.

2.3 - To obtain the goals stated in para 1. it seems convenable to state the operations' sequence as follows:

a) - at the beginning and the end of each group of observations (a block, or more blocks), we must determine the PCs of both cameras by observation of precision grids in two planes, by space intersection (procedure 2.1). At the same time we shall determine the focal length;

b) contemporaneously we shall determine the PCs utilizing the fiducial marks engraved on the photocarrier. We shall use the observations from the lower plane and the focal length above obtained, with the space resection procedure (see 2.2);

c) - in every model, or each 2-3 models we shall determine the PCs of both cameras. That will be done after the relative orientation, utilizing the fiducial marks engraved on the photocarrier, by space resection;

d) we shall compute the differences ΔX_0, ΔY_0, ΔZ_0 between the PCs' coordinates obtained as in b), and the same obtained as in c). These differences will be applied to the $X_0 Y_0 Z_0$ coordiantes obtained as in a); the result is the corrected coordinates of the PCs in that model.

By this way, the PCs' coordinates introduced in the independent model aerial triangulation are obtained by a differential procedure; the ΔX_0 ΔY_0 ΔZ_0 corrections are actually independent both from the calibration of the photocarrier's marks, and from the knowledge of the inner orientation elements.

The above operations are performed by a computation program, which is organized in three stages:

i) - firstly, it computes the space intersection with observation in two planes, and the focal length;

ii) - then it computes the space resection with observations in one plane;

iii) - the program sets up the transformed system; the solution of the general LS problem is attained by a specific subroutine, which normalizes the system and yelds the normal system of n equations in n unknowns (n = 3 for space intersection; n = 6 for space resection); solves it by the reciprocal matrix procedure; and yelds the residuals of the observation equations and the variance-covariance matrix.

A copy of the program is available on request.

Experimentation

3. - Some experiments were effected at the Galileo Santoni Simplex II C plotter of the Ancona University, in three different days. The observations were recorded using the plotter's REC III device, which types the coordinates with an accuracy of 1-2 μm (last digit is 1 μm). Each utilized coordinate is the mean of 4 successive observations; the actual m.s.e. of this mean, i.e. of the input coordinate, may be evaluated in about ± 1 μm (internal accuracy).

The photocarriers' fiducial marks were calibrated by the Galileo firm a few days before the observations; the discrepancies respect the nominal values (0, $\pm$ 80 mm) do not exceed 10 μm, with a m.s.e. of about ± 1 μm.

An accurate verification and correction of the plotter's general conditions were not performed at the beginning of the operations; in fact, not always such operation is possible in current activity, and some errors may be introduced, as shown below.

3.1 - A summary of the obtained results is reported in the Annex 1, where some tests effected in different conditions and camera's attitudes are reported. From a synoptic glance on it, some conclusions are possible:

a. - heavy discrepancies (up to 0.2 mm) result in the planimetric X_0Y_0 coordinates obtained by space intersection (two planes) and space resection (one plane), carried out on the same observations. This is probably due to the concurrency of many causes, like: i) - imperfect knowledge of the camera's inner orientation; ii) - imperfect knowledge of the image coordinates; iii) - imperfect correction of the plotter. The correct values are those obtained by space intersection, which operates on the differences $(X_2 - X_1)$, $(Y_2 - Y_1)$, and thus eliminates the major part of the above said errors;

b. - the altimetric Z_0 coordinate remains almost unchanged along the whole series of experiments, whatever be the computation procedure and the camera's attitude. Any variation in the camera's attitude, also of only 2^g, produces heavy variations in **the planimetric X_0Y_0 coordinates (real or apparent, see [5]), but does not trouble the Z_0 coordinate and its accuracy;**

c. - a combination of large ($\varphi\ \omega\ \kappa$) variations yelds quite anomalous variations in the $X_0Y_0Z_0$ coordinates and in their accuracy. This is probably due to the program's limitations.

3.2 - We may now draw the following conclusions, whose validity can be probably extended to other analogical devices organized as Simplex II C:

i) - the first quite evident conclusion is that only the altimetric Z_0 coordinate can be defined with a high reliability, and be computed in each independent model at the same time of its observation;

ii) - secondly, it seems evident that to obtain the PC's coordinates it is better to use the photocarrier's fiducial marks than precision grids. The 4 marks which are available (5 in single plates) are fully sufficient to give good PCs;

iii) - as a consequence, we may presume that when using analog plotters the preference should be given to those adjustment and computation procedures which impose the coincidence only of the Z_0 coordinate, and not also of X_0 and Y_0. In a routine block adjustment there will all the same be a sufficient constraint redundancy; in any case a lower redundancy is preferable to using wrong data.

Such a procedure is possible, and we show it in another paper [4] which we present at this Congress.

Rome, April 1980

Aknowledgements

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[3] - BOGE, E. Walter - Resection using iterative least squares Photogrammetric Engineering, XXXI n° 4, July 1965.

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SUMMARY OF THE OBTAINED RESULTS

Test Nº	Camera (a)	Image (b)	Nº Obs. Points	Procedure (c)	Attitude (grad.) ω	φ	κ	P. C. Coordinates (μm) Xo	±	Yo	±	Zo	±
1	L	G	6	I	0	0	0	9 951	11	9 860	11	8 656	19
				R				0 173	65	0 027	48	8 676	23
2	L	P.C	4	I	0	0	0	9 962	14	9 864	13	8 672	30
				R				0 079	16	0 032	13	8 649	4
3	L	P.C	4	R	2	0	0	0 152	20	0 000	17	8 682	26
4	"	"	"	"	5	0	0	0 052	104	0 035	87	8 628	31
5	"	"	"	"	0	2	0	0 041	92	0 011	69	8 657	20
6	"	"	"	"	0	5	0	0 073	145	9 963	91	8 609	26
7	"	"	"	"	0	0	2	9 985	67	9 991	57	8 627	17
8	"	"	"	"	0	0	5	9 996	59	9 985	51	8 670	15
9	"	"	"	"	0	2	5	0 105	70	0 807	53	8 658	16
10	"	"	"	"	5	2	0	9 984	1157	9 162	860	8 727	296
11	"	"	"	"	5	5	5	6 745	2714	0 370	1875	8 801	610
12	R	G	7	I	0	0	0	9 954	7	9 039	7	9 717	12
				R				0 085	83	0 106	62	9 730	30
13	R	P.C	4	I	0	0	0	9 981	9	9 847	9	9 728	20
				R				0 136	176	0 261	150	9 700	46
14	R	P.C	4	R	2	0	0	0 227	133	0 157	113	9 745	36
15	"	"	"	"	5	0	0	0 315	159	0 234	133	9 698	48
16	"	"	"	"	0	2	0	0 222	40	0 257	30	9 762	9
17	"	"	"	"	0	5	0	0 489	36	0 302	23	9 724	6
18	"	"	"	"	0	0	2	0 032	133	0 274	113	9 774	34
19	"	"	"	"	0	0	5	0 281	50	0 273	43	9 706	13
20	"	"	"	"	2	5	0	0 298	1053	9 428	663	9 820	194
21	"	"	"	"	5	5	5	7 950	2809	6 050	1681	0 035	539

(a) - L=left; R=right
(b) - G=precision grid; PC=photocarrier
(c) - I=space intersection (2 planes); R=space resection (1 plane)