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HIFI - A MINICOMPUTER PROGRAM PACKAGE FOR HEIGHT INTERPOLATION BY FINITE ELEMENTS *)

ABSTRACT

A general minicomputer program package for height interpolation is presented. It is written in FORTRAN and interpolates a digital height model (DHM) from arbitrarily distributed reference points and breaklines, using the Finite Element Method. From the DHM digital profiles can be derived for orthophoto production and digital contour lines can be interpolated and plotted. The paper describes the principles of interpolation and the structure of the programs. Finally the efficiency of the package is demonstrated by the results of processed practical examples.

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S 51 - 644

1. INTRODUCTION AND REVIEW

The paper presents a general FORTRAN program package for height interpolation. It was developed for use with the mini-computer Hewlett-Packard HP 1000, but it can also be transferred to other minicomputers and to large computers.

The program package interpolates the heights of grid points from arbitrarily distributed reference points and points along break lines. The meshes of the grid are squares, the size of which can be chosen by the user. From the digital height model (DHM), formed by these grid points, digital contour lines and digital profiles can be derived optionally.

The Finite Element Method is used for interpolation of the DHM. This general method has been successfully applied in numerical mathematics and several fields of engineering over more than two decades /1/, /2/. During the last few years the method has also been applied in geodesy /3/, /4/. Like these applications the DHM interpolation is based on relatively simple and regular structures. Local surfaces are defined for all meshes of the grid of interpolation points. These FINITE ELEMENTS are tied together accordingly and form the total interpolation surface. HEIGHT INTERPOLATION BY FINITE ELEMENTS - which leads to the program package name of HIFI - results in an interpolation surface of minimum curvature, which approximates the given reference points with optional filtering and represents the break lines adequately.

Two types of finite surface elements are used for the individual grid meshes, either bilinear or bicubic. The bilinear variant had already been presented in 1978 at the International Geodetic Week in Obergurgl /5/ and at the ISP Commission III Symposium in Moscow /6/. If the bilinear elements are replaced by bicubic elements the mesh width of the grid can be extended and the computational effort decreases. On the other hand, consideration of break lines is much more complicated with bicubic elements. The HIFI program package therefore applies the bilinear variant for terrain with break lines and the bicubic variant if no break lines are to be considered.

The development of the program package is supported by Carl Zeiss. HIFI can be used in combination with the analytical plotter PLANICOMP C 100 and the new analytical orthoprojector ORTHOCOMP Z2 /7/.

Two program versions are available. The smaller HIFI-P, interpolates a DHM from terrain data without break lines and uses bicubic finite elements. Reference points may be arranged along contour lines, along profiles, in a grid or may even be arbitrarily distributed. From the DHM, interpolated by HIFI-P, digital PROFILES can be derived and used for orthophoto production with the ORTHOCOMP Z2.

The extended program version HIFI-PC interpolates a DHM from arbitrarily distributed reference points and break lines, using bilinear finite elements. If no break lines are given, the bicubic variant is automatically applied. Further on

HIFI-PC offers the options of deriving digital PROFILES (as above) and digital CONTOUR LINES from the DHM. The contour interval can be chosen by the user and the computed contour lines can be plotted by a digital tracing table from Zeiss.

Both program versions supply absolute orientation of the model data onto given control points. Apart from profiles and contour lines further informations can be principally derived from the DHM. An appropriate example for the production of slope maps is given in /8/.

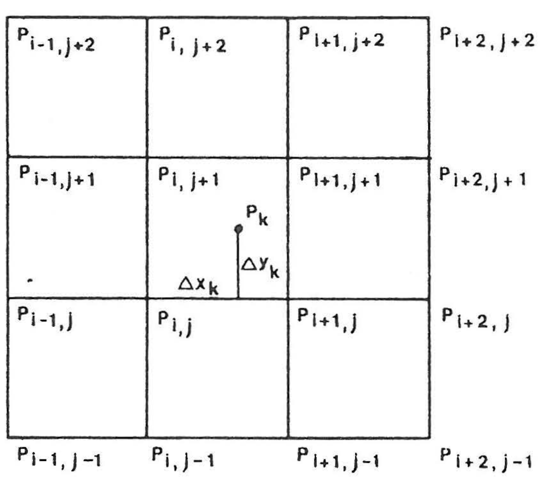
A more detailed description of the principles of interpolation of DHM, profiles and contour lines is given in the following chapter. HIFI-P and HIFI-PC are then presented in the form of block diagrams. Finally the efficiency of the program package is demonstrated by the results of processed practical examples.

2. PRINCIPLES OF DHM INTERPOLATION AND DERIVATION OF DIGITAL PROFILES AND DIGITAL CONTOUR LINES

2.1 DHM interpolation by bilinear finite elements

In this case a separate bilinear polynomial is used for each grid mesh and the interpolation surface, formed by these finite elements is assumed to be continuous. The mathematical formulation is founded on the use of bilinear base splines S_{ij} which are defined for all grid points P_{ij}. The base of these splines is locally bounded to the four surrounding grid meshes with a maximum value of 1 at the grid point $P_{i,j}$. For a more detailed representation see for instance /3/. The interpolation surface then is the result of a superposition of local surfaces $\alpha_{i,j} \cdot S_{i,j}$. This means, that each base spline $S_{i,j}$ makes a contribution to the interpolation surface. In the case of bilinear finite elements the values $\alpha_{i,j}$ are identical with the functional values or heights of the interpolation surface at the grid points $P_{i,j}$.

If a point P_k of coordinates x_k, y_k is located inside a mesh, as shown in figure 1, the corresponding height h_k of the interpolation surface follows as a linear function of the heights of the four surrounding grid points $P_{i,j}$, $P_{i+1,j}$, $P_{i,j+1}$, $P_{i+1,j+1}$ and the respective coefficients are bilinear functions of the local coordinates Δx_k, Δy_k.



$$\Delta x_k = (x_k - x_i)/d\,,\quad 0 \le \Delta x_k < 1$$

$$\Delta y_k = (y_k - y_j)/d\,,\quad 0 \le \Delta y_k < 1$$

$$d = x_{i+1} - x_i = y_{j+1} - y_j$$

Fig. 1

The requested grid heights of the DHM are defined as unknowns and estimated from the heights h_k of the available reference points and a general curvature minimization of the interpolation surface with filtering at the reference points. This problem can be interpreted as an adjustment of indirect observations and be solved using the least squares method.

The height h_k of the respective reference point P_k (x_k, y_k, h_k) is treated as an observation of the unknown height of the interpolation surface at position x_k, y_k. This unknown height however, is a linear function of the four surrounding grid heights, as shown above. So, reference point P_k leads to an observation equation, containing four unknowns, the coefficients of which are linear functions of Δx_k and Δy_k. Equation (1) represents this relation in detail, v_k is the least squares correction of h_k.

$$\begin{aligned} v_k = {} & (1-\Delta x_k)(1-\Delta y_k) h_{i,j} + \Delta x_k (1-\Delta y_k) h_{i+1,j} \\ & + (1-\Delta x_k)\Delta y_k \, h_{i,j+1} + \Delta x_k \, \Delta y_k \, h_{i+1,j+1} - h_k \end{aligned} \qquad (1)$$

All equations of type (1) can be weighted individually, according to the accuracy of the observations h_k and to the amount of filtering desired.
With the bilinear variant curvature minimization can not be realized rigorously, because all second derivatives of the interpolation surface are zero. Therefore second differences of the heights are used instead. For each grid point such second differences are formulated in x- and y-direction and are interpreted as fictitious observations of amount zero and certain stochastic properties. The corresponding observation equations for grid point $P_{i,j}$, shown in figure 1, then read:

$$\begin{aligned} v_{1,i,j} &= h_{i-1,j} - 2\,h_{i,j} + h_{i+1,j} - 0 \\ v_{2,i,j} &= h_{i,j-1} - 2\,h_{i,j} + h_{i,j+1} - 0 \end{aligned} \qquad (2)$$

The most simple way is, to treat all fictitious observations as uncorrelated and of equal accuracy, which is adequate for homogeneous and isotrop terrain. The suggested concept however, is not restricted to this assumption, but allows for more general stochastic properties of the fictitious observations, by which the local shape of the terrain surface can be modelled too. A relevant example for one dimensional interpolation is given in /6/. The general realization of the concept is still being worked on.

Break lines, given as a sequence of single points in space, are considered in a direct and simple way, as shown in fig. 2. For this purpose the respective break line is initially represented as a polygon, eventually as a filtered one. Then the intersections of the break line with the vertical planes, passing through the grid lines of the DHM, are computed and the observation equations (2) are altered in such a way, that connections across the break line are excluded and sharp bends may appear at the intersection points.

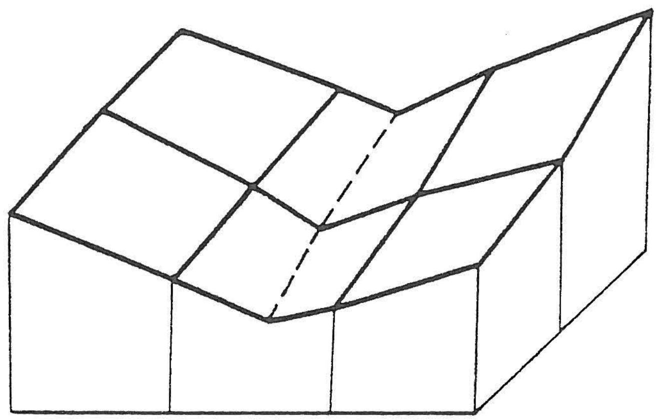


Fig. 2

The equations of type (1), put up for all reference points and the total of equations (2), altered accordingly in the neighbourhood of break lines, form a system of observation equations, from which, in case of a diagonal weight matrix, a banded structured system of normal equations is obtained. Assuming a rectangular area of n grid lines and m grid points per line, the number of unknowns is $m \cdot n$ and the band width is proportional to m. A direct solution of the normal equations, as used in the HIFI program package, then leads to a total computing time of $\mathrm{const} \cdot m^3 n$, whilst the corresponding computing time per grid point is proportional to m^2. From there it follows that the computational effort can be reduced considerably by a limitation of the number m of grid points per line. This is possible by subdividing the total area into several strips with the full number n of lines, but with less grid points per line. These strips have to overlap adequately and are treated successively.

Two related publications should be mentioned /9/, /10/ in connection with the presented DHM interpolation by bilinear finite elements. The concepts of interpolation suggested in these articles, show close similarities to the method used here. Both publications and the present one were all independently produced and confirm the topicality of the suggested method and its efficiency.

2.2 DHM interpolation by bicubic finite elements

In this case a bicubic polynomial is used for each grid mesh and the interpolation surface, formed by these more general finite elements, is assumed to be continuous. Continuity however, is not restricted here to the functional values, but extends also to the first and second derivatives. The resulting smoothness of the interpolation surface allows for an extension of the mesh width of the grid in comparison to the bilinear variant. This is shown in figure 3.

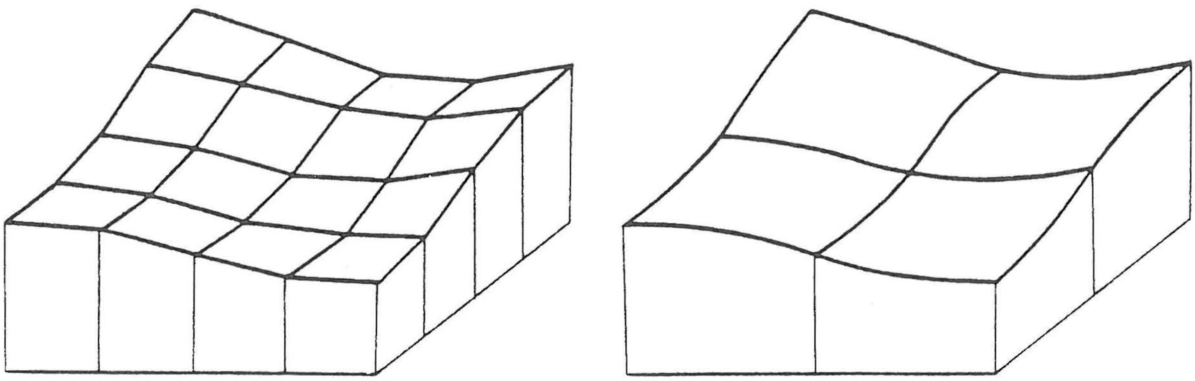


Fig. 3 Interpolation surfaces built respectively of bilinear (left) and bicubic (right) finite elements

As in 2.1 the mathematical formultation is founded on the use of base splines $S_{i,j}$, which are defined for all grid points $P_{i,j}$. In the present case however, bicubic base splines are used. The base of these splines is extended to the 16 surrounding grid meshes, with a maximum value at the grid point $P_{i,j}$. The interpolation surface again is the result of a superposition of local surfaces $\alpha_{i,j} \cdot S_{i,j}$, but the individual values $\alpha_{i,j}$ specifying the contributions of the base splines $S_{i,j}$ to the interpolation surface, are not identical with the heights $h_{i,j}$ at the grid points.

With the assumptions of figure 1 the height h_k of the interpolation surface at position x_k, y_k then follows as a linear function of the parameters α of the 16 surrounding grid points. The respective coefficients are bicubic functions of the local coordinates Δx_k, Δy_k.

In the adjustment the parameters α are defined as unknowns and are estimated from observation equations for the reference points and for the curvatures at the grid points. These equations are much more complicated than equations (1) and (2) and can not be represented here in detail. It shall be stated however, that the observation equations for the reference points typically contain 16 unknown parameters α, whereas the curvature equations contain only 9. The curvatures themselves result from the second derivatives of the interpolation surface. As mentioned previously no consideration of break lines takes place with the bicubic variant.

The comments in 2.1 concerning the structure of the system of normal equations, the band width and the subdivision of the total area into overlapping strips are also valid here. The results of the adjustment are the parameters α, by which the interpolation surface is completely determined. The heights at grid points or at arbitrary positions x_k, y_k can be directly computed from the parameters α of the surrounding 9 or 16 grid points. This allows for a rigorous densification of the DHM in a simple way.

A DHM interpolation, based on piecewise bicubic polynomials had already been suggested by Kubik in 1971 /11/. The bicubic variant of the HIFI program package is similar to Kubik's concept. The chosen formulation by base splines however, leads to a considerable reduction of the computational effort.

2.3 Derivation of digital profiles from the DHM

From the DHM, interpolated according to 2.2, digital profiles can easily be derived and used for controlling an orthoprojector. To exhaust the height information of the bicubic DHM, it is recommended that the distance between adjacent profiles and between points along the profiles should be chosen smaller than the mesh width of the DHM. The heights of the individual profile points are computed from the parameters α, as described in 2.2.

A derivation of digital profiles is also possible from a DHM, interpolated according to 2.1. In this case, choosing the distance between profiles equal to the mesh width of the bilinear DHM is recommended.

2.4 Derivation of digital contour lines from the DHM

Starting with a DHM, interpolated by bilinear or bicubic finite elements, digital contour lines of desired interval can be computed and prepared for plotting with a digital tracing table. This is done in three successive steps.

In step one the intersections of the contour lines with the vertical planes, passing through the grid lines of the DHM, are determined by nonlinear interpolation in the respective vertical planes. In case of a bilinear DHM the grid points and break line points, located in this vertical plane, are used as reference points. The chosen interpolation is similar to the method of Akima /12/ and works without filtering. The resulting interpolation curve has continuous first derivatives in all points except at break line points, where sharp bends are allowed. If the DHM is the result of an interpolation by bicubic elements, the interpolation surface is continuous up to the second derivatives and is completely described by the parameters α, as shown in 2.2. Consequently, the intersections of the contour lines with the vertical grid planes can also be computed directly from the parameters α.

All points of the contour lines, determined in step one, are located at grid lines of the DHM. In step two the course of the contour lines is interpolated from these points. If the DHM consists of bilinear elements and break lines, the intersections of these lines with the contour lines are used as additional reference points. The interpolation is again similar to Akima's method /12/. The resulting interpolation curve passes through the reference points and has continuous first derivatives. Sharp bends are only allowed at the intersections with break lines. If the DHM is interpolated according to 2.2 the course of the contour lines can be directly computed from the parameters α and the points obtained in step one.

Finally in step three the nonlinear functions, representing the course of the contour lines, are broken up into polygons and prepared for plotting. The length of the pieces of these polygons varies and is chosen in such a way, that the plotted contour lines appear smooth. The only exceptions are the intersections with break lines, where sharp bends may appear.

3. INFORMATION ON THE PROGRAM PACKAGE

3.1 Program version HIFI-P

This smaller version of the program package interpolates a DHM by bicubic finite elements. From the DHM digital profiles can be derived and used for orthophoto production with the Zeiss ORTHOCOMP Z2. HIFI-P is written in FORTRAN and runs on a minicomputer Hewlett-Packard HP 1000. In connection with the operating system RTE IV and the ORTHOCOMP Z2 or the PLANICOMP C100 a minimum core capacity of 96 K words is required. As already mentioned in 2.1 the total area can be subdivided into several strips of limited width, to save computing time. This subdivision is done automatically. The strips overlap adequately and are treated successively. With this provision even large areas can be processed in reasonable computing time. A block diagram of the program version HIFI-P is shown in figure 4.

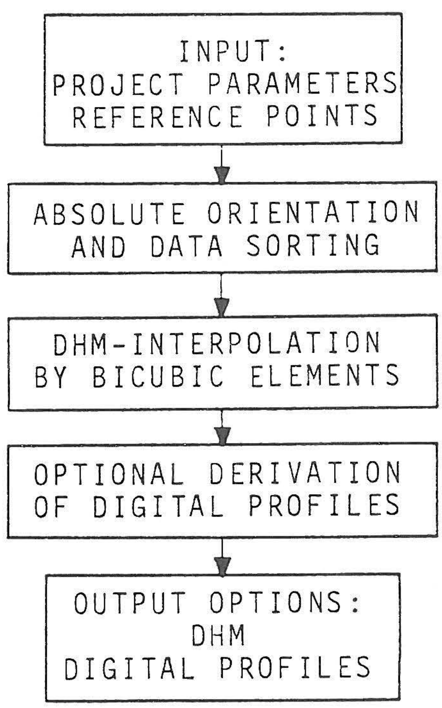


Fig. 4 Block diagram of the program version HIFI-P

3.2 Program version HIFI-PC

This extended version of the program package interpolates a DHM of either bilinear elements, if break lines are to be considered, or of bicubic elements, if no break lines are given. From the DHM digital profiles and digital contour lines of desired interval can be derived. HIFI-PC is also written in FORTRAN and runs on a minicomputer Hewlett-Packard HP 1000. In connection with the operating system RTE IV and the ORTHOCOMP Z2 or the PLANICOMP C 100 from Zeiss, a minimum core capacity of 128 K words is required. A subdivision of the total area into overlapping strips, as described in 3.1, is also possible. The computed contour lines can be plotted by a digital tracing table from Zeiss. A block diagram of the program version HIFI-PC is shown in figure 5.

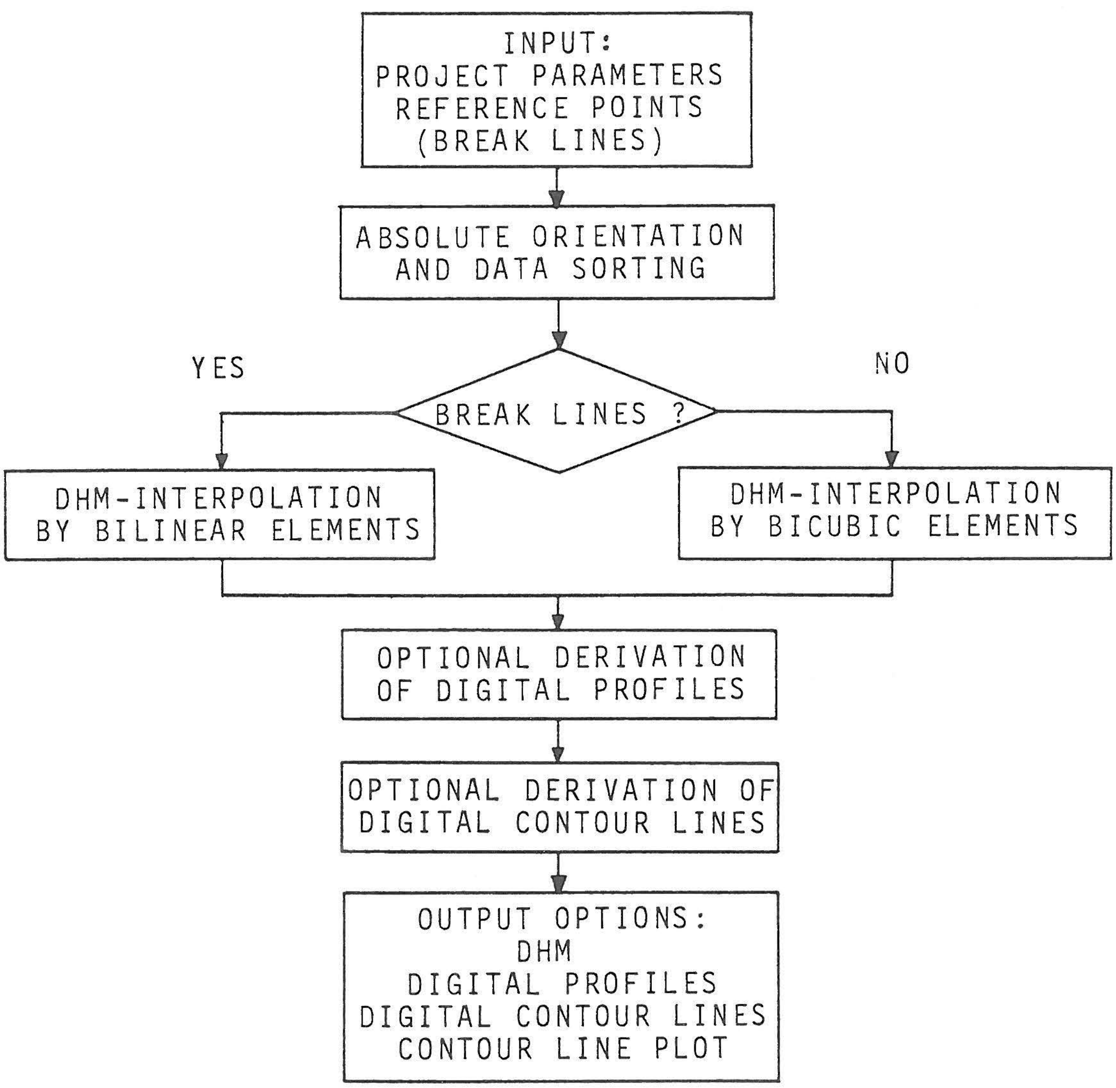


Fig. 5 Block diagram of the program version HIFI-PC

4. PRACTICAL APPLICATIONS AND RESULTS

First results of practical applications of height interpolation by finite elements have been presented in /6/, /8/ and /13/. In this paper two more, recently processed examples shall be treated.

The data acquisition for the projects described was undertaken with the analytical stereo plotting system Zeiss PLANICOMP C100 at the Chair for Photogrammetry of Munich Technical University. To compute the digital height models, a preliminary program version was used which runs at the CDC Cyber 175 of the Leibniz Computing Centre Munich (this program uses bilinear surface elements and considers breaklines if necessary).

4.1 Test "Hoellentalferner 1977"

Input:

1 stereo model (image scale 1 : 11000, camera Zeiss RMK 30/23), covering the glacier "Hoellentalferner" in the Bavarian Alps.

Data acquisition:

- recording of contour lines (10 m interval)
- recording of break lines (including the glacier boundary)
- recording of planimetric lines (boundaries of old snow and firn)
- on-line-plotting of the recorded lines on the digital tracing table, Zeiss DZ 6 (scale 1 : 2 500)

operator: H. Rentsch

Data processing

- interpolation of a DHM (10 m grid) for a 570 x 560 m^2 subarea (computing time on the Cyber 175: 40 CPU sec)
- transfer of the DHM-data to the minicomputer Hewlett Packard HP 1000 using CCT
- derivation of contour lines from the DHM (computing time on the HP 1000: about 2 min)

Output:

Map of the computed contour lines, plotted on the digital tracing table, DZ 6 (scale 1 : 2 500), plotting time about 22 min.

Figure 6 shows the part of the on-line-drawing, which covers the DHM-area (included are 10 m - contours, the boundary of the glacier, the line of old snow and the firn boundary).

In figure 7 a perspective view of the interpolated DHM is shown (the plotting program used is based on the subroutine FXY3D of the Leibniz Computing Centre Munich and the image was plotted at a drum plotter CALCOMP 936 S). Apart from the glacier boundary figure 7 also shows the recorded break lines, which are not included in figure 6.

Figure 8 shows the contour lines, derived from the DHM and automatically drawn by the DZ 6.

Remarks:
For interpolation of the DHM 40 CPU sec were needed on the Cyber 175 (no subdivision of the area). By dividing the whole area into 4 strips the same interpolation will require about 25 min on the HP 1000 minicomputer.

4.2 Test "Donauwoerth"

Input:

1 stereomodel (image scale 1 : 14 400, camera Zeiss RMK 15/23). The model covers the left half of a 1 : 5 000 map sheet, the area is 2 350 x 1 200 m^2.

Data acquisition:

- recording of the heights of a 10 m square grid
- recording of profiles (comb - shaped scanning, distance between profiles 20 m)
- recording of contour lines (interval 2.5 m, additional contours if necessary)
- recording of break lines and spot heights
- recording of roads and the boundaries of woods

operator: M. Spindler

Data processing:

Up till now two sub areas of different terrain character have been investigated (size 600 x 600 m^2). Sub sets of the recorded data were used as reference points and the data density of the sub sets was varied accordingly. The mesh width of the interpolated DHM was always 10 meters and break lines were considered. The resulting interpolated grid heights of the DHM's were compared with the directly measured and recorded heights. Table 1 shows the RMS values of the corresponding height differences.

The standard deviation of the height measurements at grid points was estimated from repetitive measurements and amounted to about 0.25 m. This value corresponds to 0.1‰ of the flying height. Consequently the values in table 1 are given only in tenths of meters.

Remarks:
Table 1 demonstrates that the measurement of a regular grid, where the floating mark stops at the respective grid point until the operator restarts the program, leads to obviously better results than the dynamic procedure of profiling. Moreover grid measurement takes less time. The same results were also obtained in a previous study /14/.

A more elaborate investigation for the test area "Donauwoerth" is in preparation at the Chair for Photogrammetry of Munich Technical University and will be published at a later date.

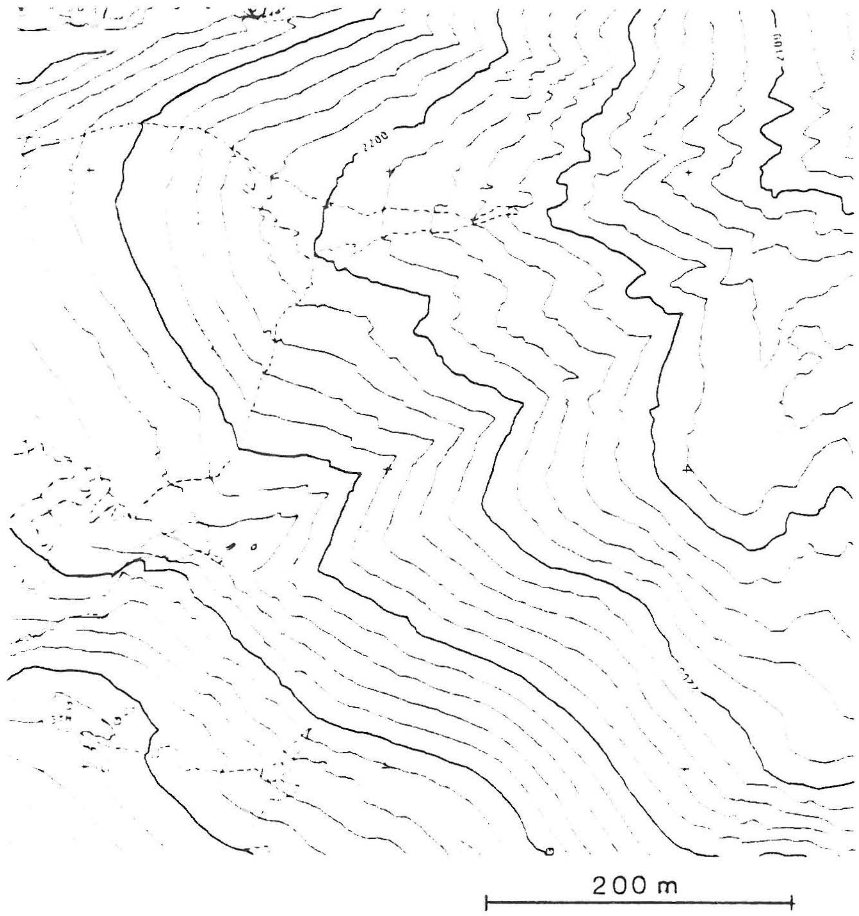


Fig. 6 Part of the on-line-drawing "Hoellentalferner" (contour interval 10 m, index contours 50 m)

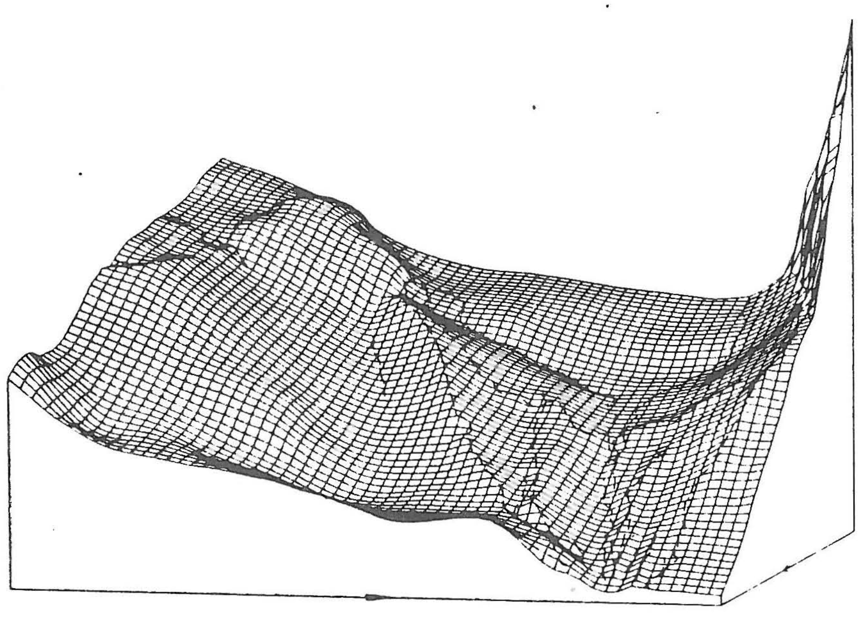


Fig. 7 Perspective view of the DHM "Hoellentalferner"

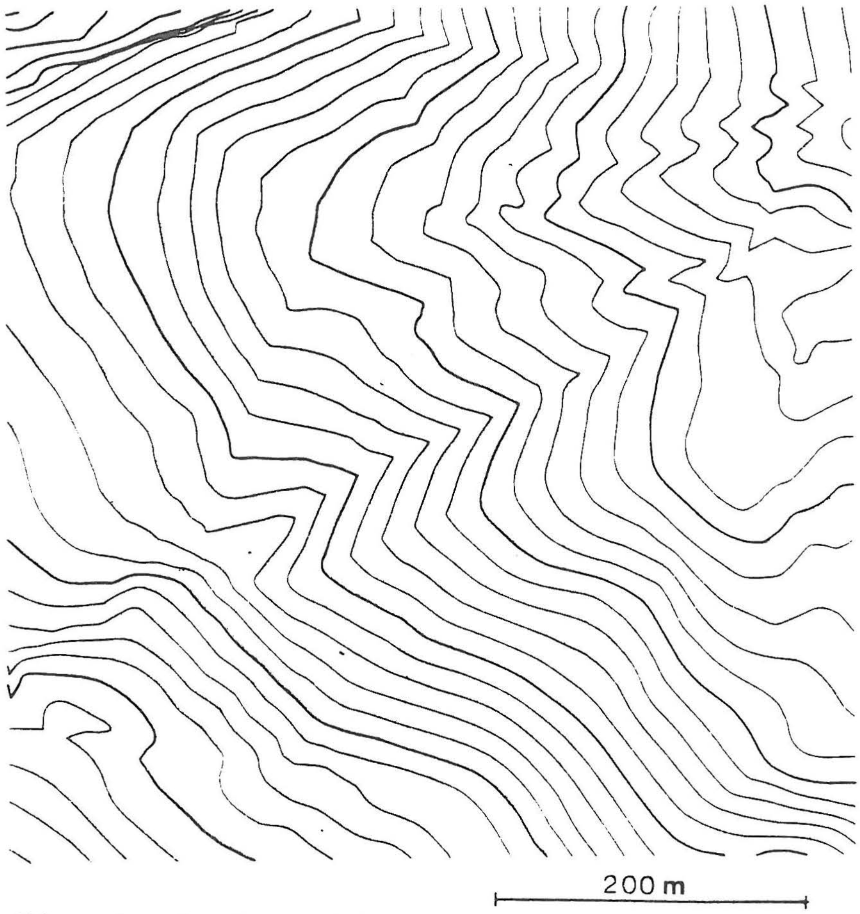


Fig. 8 Contour lines derived from the DHM "Hoellentalferner" and drawn automatically

reference points used for interpolation	terrain shape	
	steep; heavily changing slopes; distinct breaklines	smooth; slightly distinct breaklines
20 m - grid	0.4 m	0.3 m
30 m - grid	0.5 m	0.4 m
40 m - grid	0.6 m	0.4 m
20 m - profiles	0.6 m	0.5 m
40 m - profiles	0.7 m	0.6 m
2.5 m - contours	0.5 m	0.5 m

Table 1 RMS values of the height differences between interpolated and measured 10 m - grid points

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REFERENCES

/1/ Strang, G. and Fix, G.J.: An Analysis of the Finite Element Method. Prentice-Hall, Englewood Cliffs, N.J., 1973.

/2/ Zienkiewicz, O.C.: The Finite Element Method. Third Edition. Mc Graw-Hill Book Company, London, 1977.

/3/ Sünkel, H.: Lokale Interpolation von Schweredaten durch Spline-Funktionen. Vorgelegt beim Alpengravimetrie - Kolloquium, Wien, 1977.

/4/ Moritz, H.: Introduction to Interpolation and Approximation. Sammlung Wichmann. Neue Folge, Band 10, 1-45, 1978.

/5/ Ebner, H.: Zwei neue Interpolationsverfahren und Beispiele für ihre Anwendung. Bildmessung und Luftbildwesen, 15-27, 1979.

/6/ Ebner, H. and Reiss, P.: Height Interpolation by the Method of Finite Elements. Presented Paper, ISP-Commission III-Symposium, Moscow, 1978. Nachrichten aus dem Karten- und Vermessungswesen, Reihe II, Heft 36, 79-94, 1978.

/7/ Faust, H.W.: ORTHOCOMP Z2, der analytische Orthoprojektor von Carl Zeiss. Presented Paper, Commission II, ISP-Congress, Hamburg, 1980.

/8/ Ebner, H. und Reiß, P.: Photogrammetrische Herstellung von Gefällstufenkarten. Schriftenreihe der Arge Flurb, Heft 3, 17-27, 1980.

/9/ Zumofen, G. und Leoni, M.: Neue Programmsysteme zur Berechnung und Darstellung von Isolinien mit Hilfe von Kleincomputern. Vermessung, Photogrammetrie, Kulturtechnik, 189-194, 1977.

/10/ De Masson d'Autume, G.: Surface Modelling by Means of an Elastic Grid. Presented Paper, ISP-Commission III-Symposium, Moscow, 1978. Photogrammetria, 65-74, 1979.

/11/ Kubik, K.: The Application of Piecewise Polynomials to Problems of Curve and Surface Approximation. Rijkswaterstaat Communications 12, The Hague, 1971.

/12/ Akima, H.: A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures. Journal of the Association for Computing Machinery, 589-602, 1970.

/13/ Peipe, J., Reiß, P. und Rentsch, H.: Zur Anwendung des digitalen Geländemodells in der Gletscherforschung. Zeitschrift für Gletscherkunde und Glazialgeologie, 161-172, 1978.

/14/ Rüdenauer, H.: Problemanalyse und Untersuchungen zur zweckmäßigsten photogrammetrischen Datenerfassung für die digitale Verarbeitung zu straßenbaulichen Zwecken. Forschungsauftrag 2.012 G 74 C des Bundesministers für Verkehr. Institut für Photogrammetrie und Ingenieurvermessungen der Technischen Universität Hannover, 1978.