# ORIENTATION THEORY FOR SATELLITE CCD LINE-SCANNER IMAGERIES OF HILLY TERRAINS

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COMMISSION II

## ABSTRACT

A new and general orientation theory of satellite line-scanner imageries can be constructed based on affine transformation. First, the basic theory of affine line images is derived and geometrical properties of the model construction and the one-to-one correspondence between the model and object spaces are discussed. Then, the transformation of central-perspective line images into affine ones is investigated for the case where photographed terrains are hilly. The proposed orientation method is tested with simulated examples so as to clarify the difficulties when applying it to practical cases.

## INTRODUCTION

Satellite CCD line-scanner imageries such as SPOT imageries are usually analyzed using the collinearity equations based on projective transformation. However, this approach may not be effective due to high correlations among the orientation parameters, which arise from the facts that the CCD line-scanner has a very narrow field angle and that height differences in the photographed terrain are small in comparison with the flying height of the satellite. In order to overcome this difficulty, this paper derives a new and general orientation theory of satellite CCD line-scanner imageries based on affine transformation. The proposed method may be effective for the case where height differences in the photographed ground surface are rather small, because parallel projection is independent of the flying height of the platform. The weak point of the method is that central-perspective line-images must be transformed into affine ones and the transformation cannot be carried out without errors due to height differences. However, the transformation errors become negligibly small for the case where the terrain is hilly. This orientation method is tested with simulated line-scanner imageries and the obtained results are discussed.

## BASIC CONSIDERATIONS

### ORIENTATION PROBLEM OF AFFINE LINE-IMAGES IN A PLANE

Let a two-dimensional object be imaged into a line based on affine transformation as is demonstrated in Figure-1. The relationship relating an object point P(Y,Z) and its image point $p_c(y_c)$ can be written in the form

$$y_c = a_1 Y + a_2 Z + a_3 \qquad (1)$$

in which $a_i$ ( i = 1,2,3 ) are independent coefficients. Geometrically, these three orientation parameters are considered to be a rotation parameter $\omega$, a translation parameter $Y_0$ and a scale factor s. It will be noted that another translation parameter $Z_0$ gives no influences on the geometry of the affine image.

In considering the model construction and the one-to-one correspondence between the model and object planes, we must employ three overlapped affine line-images as is shown in Figure-

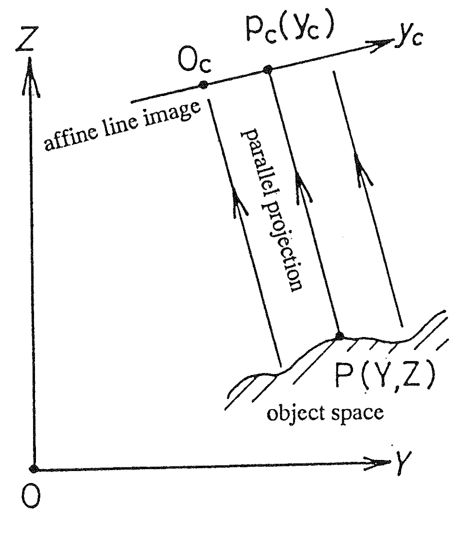


Figure-1 : parallel projection of an object space into a line in a plane

2. The basic equations are

$$y_{c1} = a_{11} Y + a_{12} Z + a_{13} \quad (2)$$

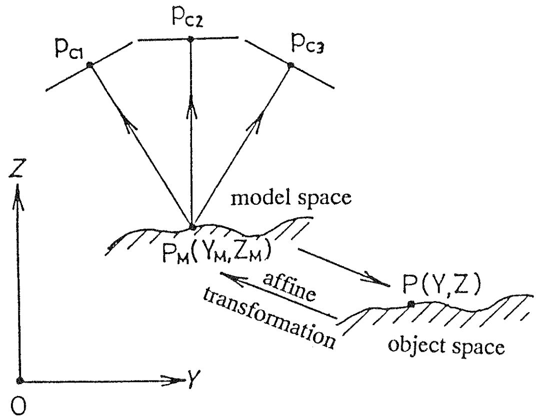


Figure-2 : relative and absolute orientation of three overlapped affine line images

for the first affine line-image,

$$y_{c2} = a_{21} Y + a_{22} Z + a_{23} \quad (3)$$

for the second one, and

$$y_{c3} = a_{31} Y + a_{32} Z + a_{33} \quad (4)$$

for the third one, respectively. Equations 2, 3, and 4 can also be expressed in the following form with respect to the object space coordinates (Y,Z)

$$\begin{aligned} a_{11}Y + a_{12}Z + a_{13} - y_{c1} = 0 \\ a_{21}Y + a_{22}Z + a_{23} - y_{c2} = 0 \\ a_{31}Y + a_{32}Z + a_{33} - y_{c3} = 0 \end{aligned} \quad (5)$$

The condition that Equation 5 is satisfied for an arbitrary object point P(Y,Z) is given in the determinant form as

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} - y_{c1} \\ a_{21} & a_{22} & a_{23} - y_{c2} \\ a_{31} & a_{32} & a_{33} - y_{c3} \end{vmatrix} = 0 \quad (6)$$

Under the condition of Equation 6 we can form a two-dimensional space ($Y_M$,$Z_M$) using the three overlapped affine line-images, which can be transformed into the object space (Y,Z) by the two-dimensional affine transformation having six independent coefficients, i.e.,

$$\begin{aligned} Y_M &= B_1Y + B_2Z + B_3 \\ Z_M &= B_4Y + B_5Z + B_6 \end{aligned} \quad (7)$$

From the fact that an affine line-image has three independent orientation elements, we can accordingly find the following characteristics in the orientation problem of overlapped affine images that:

1) Three orientation parameters among the nine ones of the three overlapped affine images can be provided from the model construction condition(Equation 6), and
2) All the nine orientation parameters can be uniquely determined, if we set up the basic equations for three control points.

## THREE-DIMENSIONAL ANALYSIS OF AFFINE LINE-IMAGES

CCD line-scanner imageries are required to be analyzed three-dimensionally so as to utilize them for mapping the ground surface. In order to relate the orientation theory derived above with the three-dimensional analysis of affine line-images, we will first consider the relationship between an object point P(X,Y,Z) and its image point $p_c(0, y_c)$ on an affine image with respect to the three-dimensional reference coordinate system. This relationship can easily be found as a special case of the analysis of two-dimensional affine imageries (Okamoto,(1989, 1992)) and has the form

$$\begin{aligned} 0 &= A_1X + A_2Y + A_3Z + A_4 \\ y_c &= A_5X + A_6Y + A_7Z + A_8 \end{aligned} \quad (8)$$

Substituting the first equation in Equation 8 into the second one, we obtain

$$\begin{aligned} 0 &= X + D_1Y + D_2Z + D_3 \\ y_c &= D_4Y + D_5Z + D_6 \end{aligned} \quad (9)$$

The first equation of Equation 9 denotes the equation of a plane where the object space is imaged on the line based on affine transformation, and the second equation expresses the relationship between the line image and an image of the object space orthogonally transformed into the

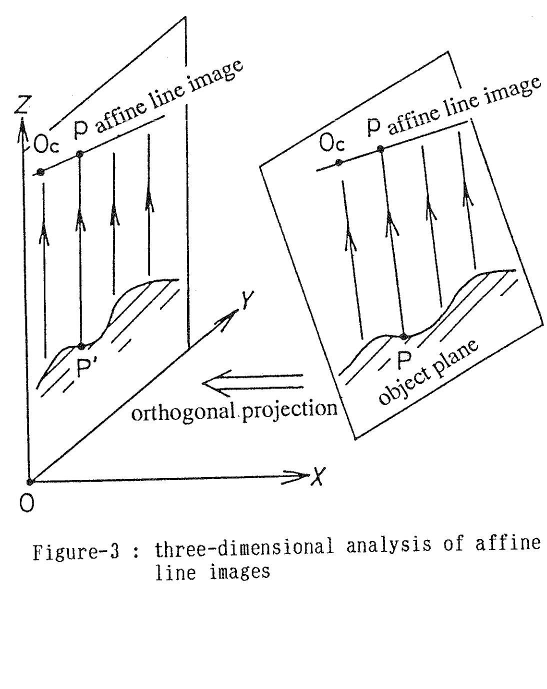


Figure-3 : three-dimensional analysis of affine line images

Y-Z plane of the reference coordinate system (X,Y,Z)(See Figure-3.). Also, we can see from Equation 9 that the three-dimensional analysis of an affine image can be separated into the following two processes: the determination of the plane including the object space and the affine image with respect to the reference coordinate system and the orientation of the image in the Y-Z plane, because the first and second equations in Equation 9 have no common coefficients. The orientation theory derived previously can rigorously be applied to the second phase of the three-dimensional analysis of overlapped affine images, because the second equation is equivalent to the basic equation (Equation 1) for the analysis of affine line-images in a plane.

## TRANSFORMATION OF CENTRAL-PERSPECTIVE LINE-IMAGES INTO AFFINE ONES

In reality, satellite CCD line-scanner imageries are taken central-perspectively. Thus, in a rigorous sense, we must analyze these imageries based on projective transformation. However, such analysis may not be effective, because the satellite CCD line-scanner conventionally has an extremely narrow field angle and height differences in the terrain are very small for the flying height of the platform. This may especially be true when the photographed terrain is hilly. In order to overcome this difficulty, we will employ the orientation theory derived in the previous section for the analysis of satellite CCD line-scanner imageries by transforming them into affine ones. This transformation will be explained as follows.

Let the ground surface be flat and a central-perspective line image be taken at convergent angle $\omega$ (See Figure-4.). Further, the image is assumed to intersect the terrain at its principal point H. p(y) denotes a real image point and $p_g$ is the intersecting point of the ray $\overrightarrow{O_A p}$ and the ground surface. The corresponding affine image point $p_a(y_a)$ can be found by drawing the normal to the central-perspective line image from $p_g$. The relationship between the central-perspective image point p(y) and the corresponding affine one $p_a(y_a)$ is given in the form

$$y_a = y(1-(\tan\omega)\,y/c) \qquad (10)$$

in which c denotes the principal distance of the scanner. The rotation angle $\omega$ and the interior orientation parameters ($y_H$, c) of the scanner are approximately known in the conventional analysis of satellite CCD line-scanner imageries. Thus, the image transformation errors due to the errors of the orientation elements are considered to be small, if the ground surface is flat. In addition, such errors can be corrected in the orientation calculation using Equation 9. However, we must consider the image transformation errors due to height differences in the terrain(See Figure-5). Let $\Delta z$ indicate height difference of a ground point from the average height and $\alpha$ denote the half of the field angle of the scanner. The image transformation error $\Delta y$ due to neglecting the height difference $\Delta z$ is shown as $\overline{p_a p_a'}$ in Figure-5 and is given in the form

$$\Delta y = \Delta z(\tan(\omega+\alpha) - \tan\omega)\cos\omega \qquad (11)$$

In a case where the field angle is 4deg. ,the rotation angle is 30deg., and the maximum height difference in the terrain is 500m, the maximum image transformation error amounts to 10.3m at the ground scale, and the average error may be 5m. Considering that the pixel size of SPOT imagery is $13\times13$, $\mu m$ the theoretical error at the ground scale may be 2.6m. Accordingly, we can conclude that the orientation theory derived in the previous section is effectively applied for the analysis of satellite CCD line-scanner imageries, if the maximum height difference is smaller than 300m.

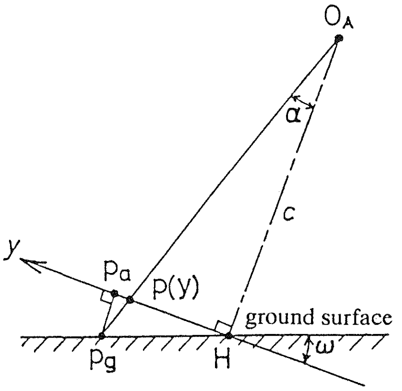


Figure-4 : transformation of a central-perspective line image into an affine one

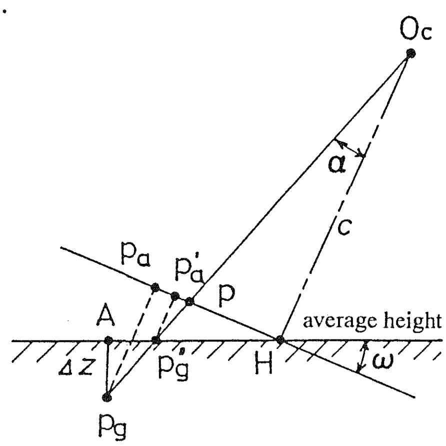


Figure-5 : image transformation error due to height difference in the terrain

## TESTS WITH SIMULATED EXAMPLES

The proposed orientation theory was tested with simulated satellite line-scanner imageries. In the construction of the simulation models overlapped line-scanner imageries were assumed to be employed, which were taken in a convergent manner from the three different flight paths of the satellite as is shown in Figure-6. The image point coordinates of 65 object points were calculated by means of the collinearity equations under the following conditions:

flying height : H = 800km
field angle of the line-scanner : $\alpha$ = 4 deg.
focal length of the scanner : c = 1000 mm

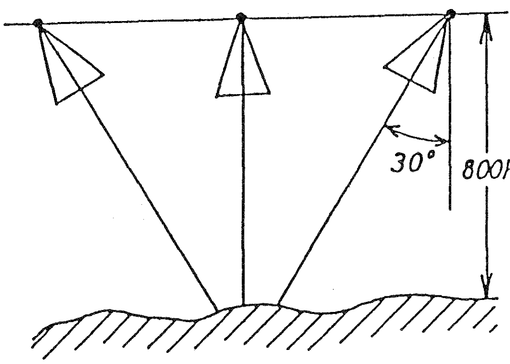


Figure-6 : three overlapped CCD line-scanner imageries taken from the three different flight paths of the satellite

convergent angle : $\omega$ = ± 30 deg.

Further,two kinds of simulation models were constructed : Simulation Model I in which all three overlapped imageries are taken normal to the flight paths and the maximum height difference in the terrain is 500m, and Simulation Model II where the line-scanner imageries are taken oblique to the flight paths as is demonstrated in Figure 7 and the maximum height differences are 100m, 250m, and 500m. The rotation angles $\kappa$ are assumed to be 45deg. Then, the perturbed image coordinates were provided in which the perturbation consisted of random normal deviates having a standard deviation of 3.3 micrometers. In addition, maximum errors of the orientation parameters of the scanner along the flight path were assumed to be as follows: ±15 minutes regarding the rotation parameters ($\omega$, $\varphi$, $\kappa$) and ± 1.0 km regarding the translation parameters ($X_0$, $Y_0$, $Z_0$). The flying course (60 km) of the platform is divided into three sections and the exterior orientation parameters are assumed to vary linearly in each section(See Figure-8). Errors of the interior orientation elements are 1.0mm for the principal distance of the scanner and 0.5mm for the principal point coordinate.

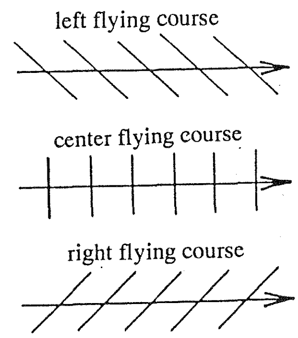


Figure-7 : three overlapped CCD line-scanner imageries taken oblique to the flight paths

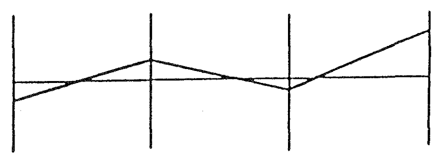

Figure-8 : changes of the orientation parameters along the flight path

The two simulation models were analyzed using the proposed orientation theory for different configurations of ground control points(See Figures 9a and 9b). The obtained results regarding the standard error of unit weight, the average internal error at the check points, and the average external error were given in Tables-1 and 2. We can find in Tables-1 and 2 the following characteristics of the orientation problem of satellite CCD line-scanner imageries using affine transformation:

1) When the overlapped imageries are taken normal to the flight paths(Simulation Model I), the obtained accuracies are not so high. In particular, if the number of ground control points is diminished, the constructed model is deformed which causes large external errors.
2) In order to increase the connecting ability of adjacent models(Equation 6) , the line-scanner imageries should be taken oblique to the flight path (Hofmann(1986), Ebner and Mueller (1986)). In Simulation Model II the obtained

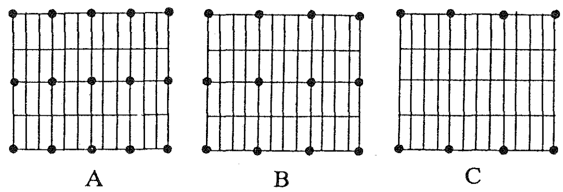


Figure-9a : congigurations of control and check points in the analysis of satellite CCD line-scannerimageries taken normal to the flight paths

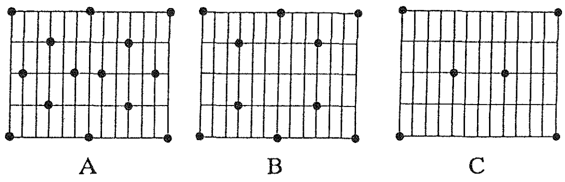

Figure-9b : congigurations of control and check points in the analysis of satellite CCD line-scanner imageries taken oblique to the flight paths

| | A | B | C |
|---|---|---|---|
| standard error of unit weight | 3. 4 μm | 3. 3 μm | 3. 4 μm |
| average internal error | 5. 1 m | 5. 3 m | 11. 1 m |
| average external error | 7. 8 m | 6. 1 m | 15. 8 m |

Table-1 : the obtained results for the analysis of satellite CCD line-scanner imageries taken normal to the flight paths of the satellite (the maximum height differnce in the terrain : 500 m)

external errors are small and stable regardless of the number of ground control points.

3) Height differences give almost no influences on the external errors, if the photographed terrains are hilly.

## CONCLUDING REMARKS

In this paper a new and general orientation theory using affine transformation has been derived for the analysis of satellite CCD line-scanner imageries. This is because in the geometry of an affine image the flying height of the platform plays no role unlike in that of a central-perspective image and thus high accuracies can be expected even when the terrain has very small height differences in comparison with the flying height of the satellite. However, in order to employ this theory for the analysis of the present satellite CCD line-scanner imageries, we must transform the central-perspective line images into affine ones and the image transformation errors due to height differences in the terrain are inevitable. Therefore, the proposed orientation theory is applicable only in the range where the image transformation errors are negligibly small.

| | A | B | C |
|---|---|---|---|
| standard error of unit weight | 3. 4 μm | 3. 3 μm | 3. 3 μm |
| average internal error | 3. 6 m | 3. 9 m | 4. 8 m |
| average external error | 3. 8 m | 4. 0 m | 6. 0 m |

Table-2a : the obtained results for the analysis of satellite CCD line-scanner imageries taken oblique to the flight paths of the satellite (the maximum height difference in the terrain : 100 m)

| | A | B | C |
|---|---|---|---|
| standard error of unit weight | 3. 3 μm | 3. 3 μm | 3. 4 μm |
| average internal error | 3. 9 m | 4. 4 m | 5. 2 m |
| average external error | 4. 1 m | 4. 5 m | 6. 9 m |

Table-2b : the obtained results for the analysis of satellite CCD line-scanner imageries taken oblique to the flight paths of the satellite (the maximum height difference in the terrain : 250 m)

| | A | B | C |
|---|---|---|---|
| standard error of unit weight | 3. 4 μm | 3. 3 μm | 3. 4 μm |
| average internal error | 4. 3 m | 5. 1 m | 5. 9 m |
| average external error | 4. 6 m | 5. 0 m | 8. 3 m |

Table-2c : the obtained results for the analysis of satellite CCD line-scanner imageries taken oblique to the flight paths of the satellite (the maximum height difference in the terrain : 500 m)

The proposed orientation theory has been tested with simulated examples and has proved to be very effective for the analysis of satellite CCD line scanner imageries taken of hilly terrains.

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