IMPROVEMENT OF DIGITAL IMAGE RESOLUTION BY OVERSAMPLING

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ABSTRACT:

The resolution of digital images is one important factor that determines the quality of any processing on the images. High resolution scanners are still too expensive to be an alternative for many users. This paper discusses a method to obtain one single high resolution image by multiple scanning, here called oversampling. Each of the scanned images has a lower resolution than the resulting high resolution (HR) image. Between each scan a subpixel movement along the scanning axises is made. Observational equations are formed under the assumption that the gray value of each scanned pixel is an average of the intensities in the corresponding area of the scanned photograph. The gray values of the HR image are computed simultaneously by solving an underdetermined equation system. The system is solved by the pseudo inverse, which minimizes the sum of the squared unknowns. The results are not very promising and the aim is to convince the reader that this approach either needs more investigations or that it is useless.

1. INTRODUCTION

The quality of the input is essential for the quality of the result. This is true for any process and digital image processing is certainly no exception. Resolution is one important factor for the quality of digital images. As an example, let us study the effect of different resolutions in the case of multigrid matching. Here the resolution is deliberately decreased to make the matching faster. Then the resolution is increased step wise back to the original resolution. The accuracy of the matching improves as the pixel size gets smaller. In figure 1.1 we see that the root mean square (RMS) of five different test cases depends almost exponentially on pixel size in multi grid matching (Li, 1989). Though the accuracy increases with smaller pixel sizes, we will gain less the smaller the pixel size is. Also, computations will be more time consuming and the images will take more disk space. However, in special high precision applications of photogrammetry, the small gain in accuray can make the difference between success and failure.

High resolution scanners are quite expensive, while low resolution scanners, except for insufficient resolution, also are poorly calibrated. In the beginning of this project the hope was that, using a cheap table scanner with a resolution of 40 μm or more, one could by oversampling produce images of perhaps 10 to 20 μm pixel size. However, our model has not proved to be very encouraging. The main problems are the instability and size of the equation system.

2. THE METHOD OF OVERSAMPLING

This paper presents an attempt to produce single high resolution images by multiple scanning with a device of lower resolution capacity. The positional relations between the images must be well known, thus this method can not be applied for scanning a photograph randomly. The hope was that by matching images, oversampling could be used for randomly scanned images, e g from multiple scanning in a table scanner. In the test we have
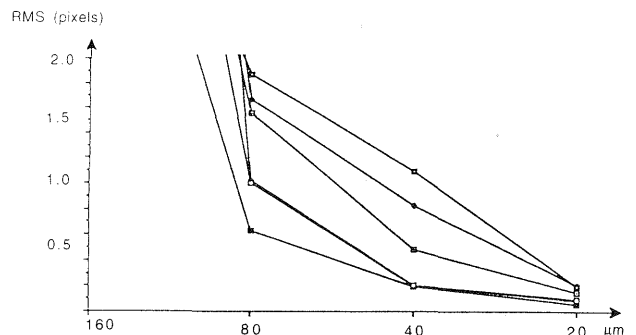
1.1 Relation between root mean square (RMS) and pixel size in five test cases of multigrid matching.(Li,1989)

used a CCD camera mounted inside an analytical plotter. The image stages can be moved with an accuracy of 1-2 μm (Almroth, 1990).

The hypothesis is that each intensity value in the scanned images is an average intensity of the corresponding area of the image. If there is an object, or a part of it, smaller than the pixel size, that object is smoothed when the area is scanned. In a way, our problem is to take the inverse of averaging. By scanning one and the same area several times with successive subpixel movements of the image stage between the 'exposures' we get our observations. For example, if we want an image with twice the original resolution along both axis we scan the area four times with a movement of half a pixel size along the scanning axises. Figure 2.1 shows the scanning procedure in such a case. The shaded area is the position of the resulting high resolution (HR) image related to the scanned images.

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image [1,1]   image [1,2]

image [2,1]   image [2,2]
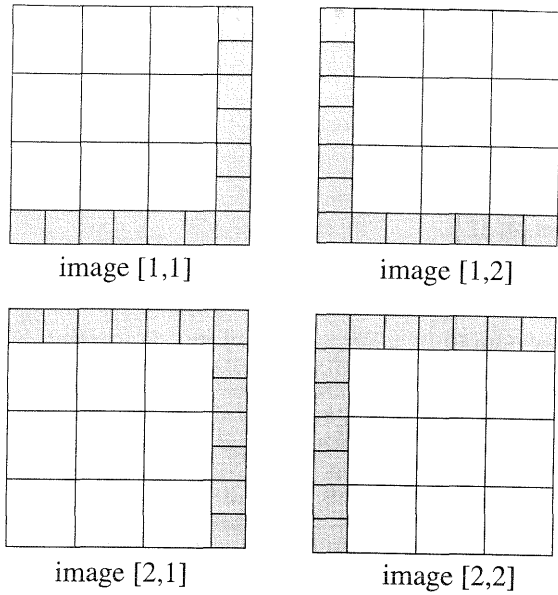
2.1 The scanning procedure. *Four images, here 3 by 3 pixels in each, are scanned. Between the "exposures" the image stage is moved half a pixel size according to the figure. The shaded area shows the extent and location of the resulting high resolution image related to the scanned images.*

If x_{ij} are unknown gray values and l_1 is one observed gray value, then

$$\frac{x_{11}+x_{12}+x_{21}+x_{22}}{4} = l_1 \qquad (1.1)$$

The observation equations are written

$$x_{11}+x_{12}+x_{21}+x_{22} = 4*l_1 \qquad (1.2)$$

The equation system is

$$\mathbf{AX = L} \qquad (1.3)$$

where

$$\mathbf{A} = \begin{bmatrix} a & a & 0 & 0 & . & . & 0 \\ 0 & a & a & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & 0 & 0 & a & a \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} 1 & 1 & 0 & 0 & . & . & 0 \\ 0 & 1 & 1 & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & 0 & 0 & 1 & 1 \end{bmatrix} \quad \mathbf{0} = \begin{bmatrix} 0 & 0 & 0 & 0 & . & . & 0 \\ 0 & 0 & 0 & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & . & . & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ . \\ x_{1N} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{24} \\ . \\ x_{2N} \\ . \\ . \\ . \\ x_{M1} \\ x_{M2} \\ x_{M3} \\ x_{M4} \\ . \\ x_{MN} \end{bmatrix} \qquad L = p*q* \begin{bmatrix} l_{1111} \\ l_{1211} \\ l_{1112} \\ l_{1212} \\ . \\ l_{121n} \\ l_{2111} \\ l_{2211} \\ l_{2112} \\ l_{2212} \\ . \\ l_{221n} \\ . \\ . \\ l_{21m1} \\ l_{22m1} \\ . \\ l_{pqmn} \end{bmatrix} \qquad (1.4)$$

x_{ab} : unknown intensity value at position a,b
l_{cdef} : intensity value at position e,f in image c,d
M,N : size of the resulting image
m,n : size of each observed image
p,q : degree of oversampling (here p = q = 2)

$\dim(\mathbf{A}) = m*p*n*q,(m*p+p-1)*(n*q+q-1)$

$\dim(\mathbf{a}) = \dim(\mathbf{0}) = m*p,m*p+p-1$

The unknowns are the gray values of all pixels in the HR image. Each matrix **a** has as many columns as there are unknown pixels in one row of the HR image. There are as many matrices **a** or **0** in each row of matrix **A** as there are pixels in one column of the HR image.

In case of another degree of oversampling the number of 1's on each line in matrix **a** and the number of **a** matrices on each line in matrix **A** will change. It is possible to have different degrees of oversampling along the two axises. For example, the CCD camera used has a pixel size of 12 by 16 μm. Instead of resampling the image to square pixel sizes the degree of oversampling can be 3 along one axis and 4 along the other. Then the output image would have a pixel size of 4 by 4 μm.

2.1 Pseudo Inverse

There are more unknowns than observations, so the system is underdetermined. This means that there is an infinite number of solutions to the problem. While we in the case of a redundant system usually minimize the sum of the squared residuals, one approach for undetermined systems is to choose the solution that minimizes the sum of the squared unknowns:

$$\min \left(\sum_i x_i {}^t x_i \right) \qquad (2.1)$$

The inverse of the design matrix A that performs this minimisation is called the pseudo inverse, A^+.

It is defined by

$$AA^+A = A \qquad (2.2a)$$
$$A^+AA^+ = A^+ \qquad (2.2b)$$
$$(A^+A)^t = A^+A \qquad (2.2c)$$
$$(AA^+)^t = AA^+ \qquad (2.2d)$$

where t is the transpose of a matrix (Bouillion/Odell 1971). Using the pseudo inverse the solution to a linear system

$$AX = L \qquad (2.3)$$

is

$$X = A^+L \qquad (2.4)$$

For a given matrix A the pseudo inverse A^+ is unique. In 1960 Greville presented an iterative method for computing the pseudo inverse (Boullion/Odell, 1971). The algorithm does not need any inversions of matrices and is easily implemented in a computer.

2.1.1 Example of the use of the pseudo inverse

Suppose that the average of two variables is 50. Then we know that the sum of them is 100. We write the equation

$$x_1 + x_2 = 100 \qquad (2.5a)$$

or written in matrix representation

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 100 \end{bmatrix} \qquad (2.5b)$$
$$\quad A \qquad X \quad = \quad L$$

The system has an infinite number of solutions. The pseudo inverse of matrix A is

$$A^+ = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \qquad (2.6)$$

The solution that minimizes X^tX is

$$X = A^+L = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 100 \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \end{bmatrix} \qquad (2.7)$$

The observation is equally distributed to the unknowns. However, if we know that the average of x_2 and a third variable, x_3, also is 50 then we have

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \qquad (2.8)$$

The pseudo inverse solution will **not** be

$$X = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix} \qquad X^tX = 7\,500 \qquad (2.9)$$

as one intuitively might expect, but

$$X = \begin{bmatrix} 33 \\ 67 \\ 33 \end{bmatrix} \qquad X^tX = 6\,667 \qquad (2.10)$$

This indicates that we can expect a periodic distortion in the results when using the pseudo inverse. The equation systems used, see (1.4) have the same structure as in (2.8) and consequently corresponding periodic distortion was found in tests.

2.2 Computational shortcuts

One major problem is the size of the pseudo inverse. For instance, if the original images are 200 by 200 pixels and the degree of oversampling is three by three, the pseudo inverse will have $600*600$ rows and $602*602$ columns which equals $130*10^9$ elements. If each element is represented by one byte this means that the pseudo inverse needs 130 Gbyte! Of course this is not realistic. Instead we use the fact that for square images and with equal degree of oversampling the matrix **A** and the submatrix **a** will have the same structure (see 1.4). We can calculate the coefficients in matrix A^+ from the pseudo inverse of matrix **a**, called a^+. The dimension of a^+ is 600 by 602 or 361 kbytes.

3. RESULTS
3.1 Test images

Test images have been created to examine the method under controlled forms. Three simple figures were created (3.1). Figure 3.1a, c and e show a point, a line and a step table of gray values. The images are 21 by 21 pixels. The average of a 3 by 3 neighbourhood is taken over the image. These are considered to be our observed intensity values, which could be thought of as coming from nine images, where each image is 7 by 7 pixels. After computing the pseudo inverse and multiply it with our observation vector we get the unknowns. The results, which were hoped to be identical to the original images, are shown in figure 3.1b, d and f respectively. Obviously we have introduced periodic patterns on the images. Though the shapes of the patterns differ they all have a periodicity of three, which is equal to the degree of oversampling. In none of the examples the observation equations seem are satisfied since we have introduced gray tones in completely white and in completely black areas. This is an effect of that we can not limit the solution to be in the interval 0 to 255 and consequently the unknowns with negative values have to be set to 0 afterwards. In the same way unknowns with values greater than 255 have to be set to 255. For example the first 3 by 3 pixels in the original and resulting image look like

$$\begin{pmatrix} 255 & 255 & 255 \\ 255 & 255 & 255 \\ 255 & 255 & 255 \end{pmatrix} \qquad \begin{pmatrix} 279 & 279 & 279 \\ 242 & 242 & 242 \\ 242 & 242 & 242 \end{pmatrix}$$

original image $\qquad$ resulting image

In both cases the average is 255, which is exactly what the observation equation reads, but as 255 is maximum, what will be presented is

$$\begin{pmatrix} 255 & 255 & 255 \\ 242 & 242 & 242 \\ 242 & 242 & 242 \end{pmatrix}$$
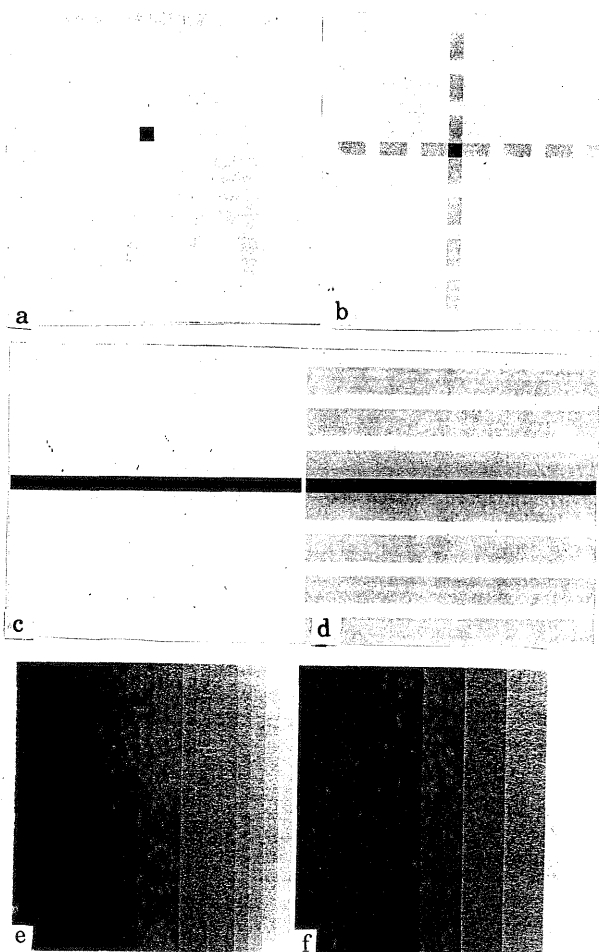
3.1 Test images. a, c and e *show a point, a line and a step table of gray values. The size of each image is 21 by 21 pixels. A 3 by 3 averaging filter was applied to the images. The output of the filters are considered to be our observations. The pseudo inverse premultiplied with the observational vector gives the solution shown in b, d and f.*

Residuals were computed by subtracting the resulting image from the original. Table 1 shows the RMS of the residuals and X^tX for the original and computed image.

	RMS	ΣX^tX original (10^6)	ΣX^tX result (10^6)
point, 3a-3b	5.14	28.61	28.52
line, 3c-3d	17.45	27.31	27.04
step table, 3e-3f	9.80	8.68	8.64

Table 1. *Root mean square of the residuals between original and computed image and X^tX for original and computed image.*

3.2 Real data

A program was written for a Kern DSR-11 analytical plotter. The program uses to a large extent the package of program procedures included with the plotter. The operator decides the window size in pixels and the degree of oversampling. After the operator manually has moved the plates to the area of interest, the program is run and images are scanned and stored according to the procedure in figure 2.1. The image files are transferred to work stations, where computations are done.

Each test has been made with a degree of oversampling of three. Each of the nine images has a size 100 by 100 pixels. The resulting image has a size of 302 by 302 pixels. In figure 3.2 we see one of the tests; a part of a fiducial mark. Figure 3.2a is one of the nine original scanned images. The image in figure 3.2b contains all the observed intensity values from all nine images. The pixels are sorted in the same way as in our observation vector. This image has three times the resolution of the scanned images and each pixel derives from either different pixels in the same scanned image or from different images. If the intensity that the sensor registers corresponds most to the intensity at the centre of the pixel area this would be a useful result. However, the pixel size is still that of the scanner, though we only 'use' 1/9 of the pixel area. Figure 3.2c, finally, shows the image as computed with the pseudo inverse. Because of the size of the equation system, more than 90 000 unknowns, a theoretically incorrect partitioning of matrices was done. In this way the computations could be done iteratively, which was considerably faster. Though this is not exactly what would be the result of a direct solution it is probably not far from it. The image is severely distorted. The pattern and the periodicity are similar to those of the test images, discussed in 3.1.

4. DISCUSSION

Obviously, the pseudo inverse implemented right away does not give a satisfying result. One possibility is to add additional constraints. This would increase the stability of the equation system. To add additional constraints we must have a priori knowledge of the images, like for instance that neighbouring pixels have approximately the same gray value. To find suitable constraints is not an easy task. No matter what constraints we introduce it is most likely that, even though we can produce less distorted images, we have still reduced the quality so that not much is won compared to using the original resolution.

Since the periodicity of the distortion is known, perhaps it can be removed by a cut off filter. This has not been tested yet.

The computational cost for matrix operations of this dimension is quite high. Local calculations should be preferred from this point of view. One example is a method called data cumulation (Albertz/Zelianeos 1988). Different satellite scenes covering the same area were used. Polynominals are made, using the gray values in local areas. From the polynominal the resolution can be increased by resampling. Also this method requires that the relations between the observed images is very well known.
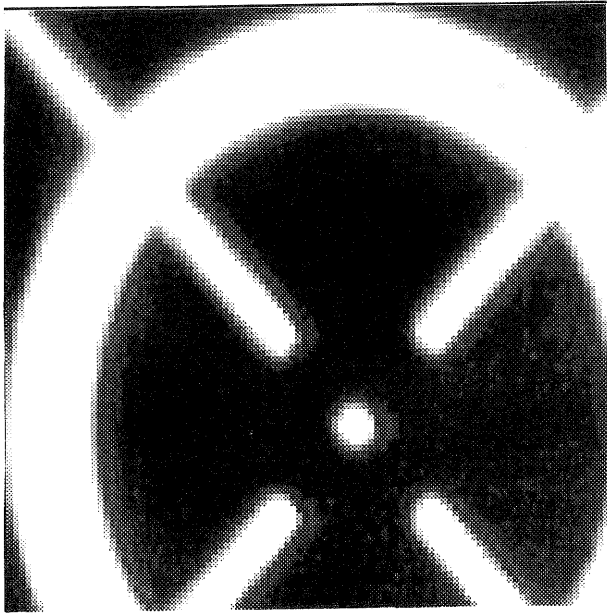
a
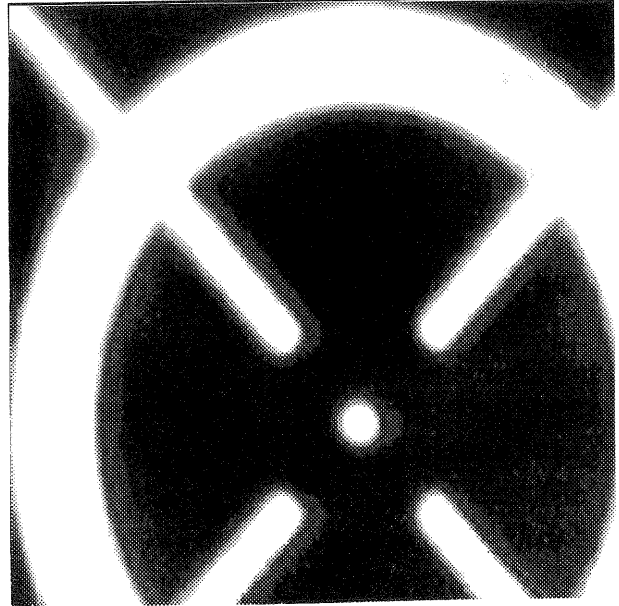
b

c

3.2 Real data. *A fiducial mark was scanned nine times with successive subpixel movements of the image stages between the 'exposures'. a shows one of the scanned images with a resolution of 100 by 100 pixels. b shows the pixels from all nine images sorted in order of geometric appearance. The resolution is 300 by 300 pixels. c is the result of oversampling with the pseudo inverse solution. A periodic distortion is introduced.*

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