THE "FLOWING" ACCUMULATION METHOD AND ITS APPLICATION FOR EARTH SURFACE ANALYSIS

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ABSTRACT:

Method for the description of functions of two variables is proposed. Applications and interpretations are considered on the example of the analysis of the earth surface segment. Input data for computer processing is digital elevation model /DEM/ which is set at regular grid and obtained with a help of an automatized photogrammetrical device.

The DEM is described as a graph. Every point of the DEM univocally responds to the only node of the graph and vice verse. An arc indicates the direction of a maximum decrease in the corresponding point of the DEM. The graph constructed in such a way consists of the directed trees. The quantitative functions which serve as integral characteristics of the surface are built on the nodes of the graph.

Algorithms worked out on the basis of this model provide for the following:
- to carry out preliminary processing of the DEM with the purpose of recovering of the surface structure adequate to the real one;
- to describe a network of valleys thalvegs and borders of their drainage basins, to define their structure;
- to calculate the areas of the drainage basins, the length of thalvegs sections and basins borders, etc.

The model can be used as a basis for the algorithms construction for automation of morphometrical analysis, the flowing calculations, water erosion processes investigation, topomaps processing.

KEY WORDS: DEM, Algorithm, Accuracy, Mapping, Image Interpretation.

1. INTRODUCTION

The method has been established and used as an attempt for solving a number of tasks ocurred when the processes of aerial photographs interpretation were automatized (Antoshchenko-Olenev and Golda, 1986). Both static relief and image terrain models are analysed while interpreting. Geological objects are manifested in the structure of both the image model and the stereo one. Relief of the earth surface keeps footprints of atmosphere and hydrosphere influence on geological environment, as well as processes which have taken place in it. Those "tracks" are coded in relief morphology characteristics. To calculate the morphology characteristics it's necessary to have a technique of surface description which would allow to derive automatically slopes, sceleton lines /thalvegs and watersheds/, to investigate their topography and topology having only the DEM as input data.

Horton (1945), Strahler (1952), Shreve (1966) showed that development of channels networks and their drainage basins submits definite laws and drainage networks are described with the branched structures which have strictly definite hierarchy and topology. However, usually either simple differentiation or more complicated constructions of differential geometry technique are used at attempts to automate process of sceleton lines derivation from topography surface. The ground of such approach is given in a paper of Antipov and Kireytov (1979).

Analysis of that problem (Golda and others 1986) showed that it's impossible to create robust noise immunity algorithms of sceleton lines recognition using only differential geometry methods. Firstly, it's because of technical reasons. We must use derivatives to second order including in calculations, i.e. quantitative implementation needs both interpolation technique and input data of high quality. But numerical differentiation is less precise operation than interpolation, that's why the result becomes dependent on interpolation technique and it's not noise immunited. Secondly, of intuition consideration. To construct a numerical model of drainage network it isn't sufficient to state the belonging of the points of the surface to thalvegs. It's necessary to connect these points into branched structures that suggest turning back to the input surface analysis. The researcher does not consider the picture of the infinitely small surface segments when quoting thalvegs on the topomap, he observes it in the whole. He can decide if the point belongs to the thalveg only taking into consideration the surface curving "up the flowing" and information about the drainage area, and he can make up watersheds on the surface having a knowledge of the framework of the channel network. Hence, formal procedures of sceleton lines derivation cannot be based only on the surface shape research. It's obligatory to define the surface directly as a structure drawing attention to integral characteristics which allow to judge of the grade of substrat "washing up" by water erosion or of the level of relief developing.

2. MATHEMATICAL MODEL

2.1. The Description of the Surface

For surface description we use mathematical model, which is one of the consequences of "flowing" accumulation method proposed by the author before. We consider a surface $Z(X,Y)$ as a discrete function of two variables $Z(X_i,Y_j)$, i.e. two-dimensional array of heights Zmn corresponding to knots of quadrangle grid $\{i,j\}, i=[1,M]$ $j=[1,N]$. $\{i,j\}$ is a set of knots where the surface is determined. We describe the surface as a graph, nodes of which are put on the same $\{i,j\}$ set. Each node of a graph is put bijectically to the only point of the surface. Arcs of the graph are defined as the mapping $P(\{i,j\})$ that is linear chained list groupped in matrix of links Pmn.

$$p(i,j)=(\tilde{k},\tilde{l}) \quad \text{if function}$$

$$f_{ij}(k,l) = \frac{Z(X_i,Y_j)-Z(X_k,Y_l)}{\sqrt{(X_i-X_k)^2+(Y_j-Y_l)^2}}$$

$$i=[2,M-1]$$
$$j=[2,N-1]$$
$$k=i-1,i,i+1$$
$$l=j-1,j,j+1$$

reaches its minimum at $k=\tilde{k}$, $l=\tilde{l}$. $p(i,j)$ value points direction of maximum decrease of $Z(X,Y)$ in (i,j) knot /neighbouring point on the surface/. Therefore, the graph $G(\{i,j\},Pmn)$ built in such a way is a forest consisting of directed trees with reverse orientation of arcs. According to the construction, no more than one arc comes off from each node. A node that does not receive any arc is a leaf. A node from which none of arcs comes off is a root. We call path from leaf to root a flowingline. A (i,j)node is a root of a tree if $p(i,j)=(i,j)$ i.e. function $Z(X,Y)$ reaches its local minimum at $Z(X_i,Y_j)$ point. As for $i=1,M$; $j=1,N$ $p(i,j)$ values are not determined, the corresponding nodes are roots as well.

We call reflexive closure of any node (k,l), i.e. the set of nodes from which there exist paths to the given node (k,l) as flowing accumulation area of (k,l)node /(k,l) FAA/. The boundary of this set on $\{i,j\}$ may be named as the (k,l)FAA boundary. The segment of $Z(X,Y)$ which is mapped into (k,l)FAA is called the drainage basin of $Z(X_i,Y_j)$ surface point /Z(X_i,Y_j)DB/, and its boundary on the surface is called $Z(X_i,Y_j)$DB boundary.

According to the $G(\{i,j\},Pmn)$ construction nodes of each separated tree make closed one-chained domain on $\{i,j\}$. Domains formed of different trees nodes do not join. Each domain is a projection of DB, and its boundary is a projection of DB boundary of the surface point corresponding to the root.

We construct quantitative functions $S(\{i,j\})$ and $K(\{i,j\})$ on nodes of the graph:
- $S(k,l)$ equals to the number of paths ending in (k,l) node;
- $K(k,l)$ is equal to the number of flowing lines crossed through (k,l) node or number of paths from leaf nodes to the given one.

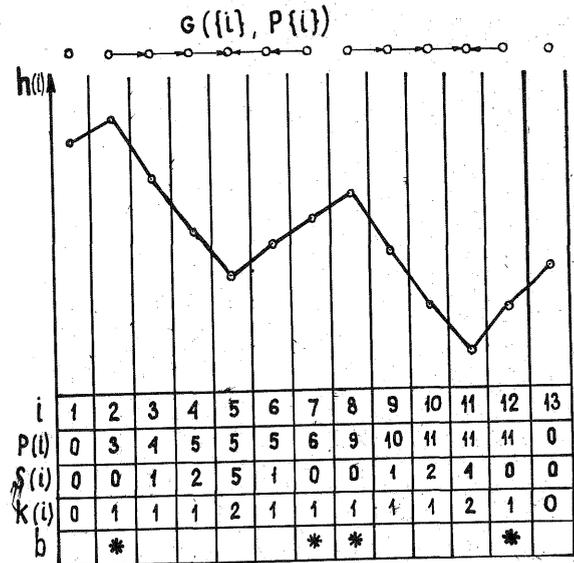Description of one-dimensional function $h(i)$ is shown on fig.1.

i	1	2	3	4	5	6	7	8	9	10	11	12	13
P(i)	0	3	4	5	5	5	6	9	10	11	11	11	0
S(i)	0	0	1	2	5	1	0	0	1	2	4	0	0
K(i)	0	1	1	1	2	1	1	1	1	1	2	1	0
b		*					*	*			*		

fig.1 Description of function $h(i)$.

2.2. Input Data

For computer processing we use the DEM obtained with the help of analog stereodevice stereoproector of Romanovsky and "Format-130" system (Kuznetsov and others 1979). For the DEM building there was used a stereopair of standard aerial photos of 1:25000 scale.

Initial photo is shown on fig.2. The square of the segment is 1 km sq. Initial DEM containing 41 profile lines of length of 401 points was transformed into the DEM of 151×151 size with the help of Fourier-interpolation with 40×18 frequencies renewal. The isolines map of this DEM is shown on fig.3. The range of heights on the segment is 70m. Standard deviation of DEM points altitudes definition is 0,75m, absolute error is 2,5m. The present DEM serves as input data for further processing.

2.3. The Surface Structure Recovery

$S(\{i,j\})$ values are analog of the square of each $Z(X,Y)$DB point, and $K(\{i,j\})$ values describe velocity of concentration of the surface flow into the linear one. Thus, subgraph E of graph G on nodes of which $S(i,j)>S_0$, where S_0 is a square of FAA sufficient for the formation of linear erosion form, reflects the erosion net work according to the level of DEM generalization.
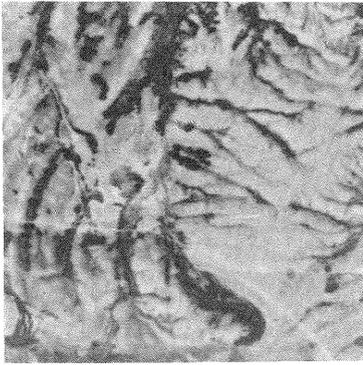
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fig.2. Initial photo of the earth surfa-
ce segment.

In terms of the present paragraph the
thalvegs mapping and FAA boundaries pic-
ture will serve as a criterion of the re-
establishing both shape and structure of
the surface. Using the DEM we construct
the $G(\{i,j\},Pmn)$, $S(\{i,j\})$, $K(\{i,j\})$ and
FAA boundaries for the G roots. Fig.4.
shows the boundaries in white and nodes
of the G where value $S(i,j)>25$, which we
can interpret as thalvegs network mapping.

Having compared fig.2 and fig.4 we can
see that the darinage network is distor-
ted in the DEM. The valleys are broken,
their orders don't correspond to the rea-

lity, a number of closed drainage basins
appeares. Hereby the basins boundaries
are laid down both on the real watersheds
and on the slopes, as well as they cross
the real valleys network. Thus, we can
come to the conclusion that the DEM does
not keep the structure image of the real
earth surface: the relations between thal
vegs, watersheds and slopes are violated,
i.e. the DEM does not satisfy the structu
ral accuracy. Besides the notion "the
structural accuracy" let's introduce the
notions "the structurized DEM" and "the
procedure of DEM structurization".

The notion "structural accuracy" includes
not only accuracy of reflection of a se-
parate point of the real earth surface in
the DEM, i.e. topography accuracy, but al
so accuracy of displaying of "up-and-down"
type of relations between points in the
vicinity of the given one. If the relati-
ons are kept in vicinity of every DEM
point, then they are kept on the whole
surface. So, notion "structured DEM" me-
ans recovering of the real surface shape
using initial DEM processing. True surfa-
ce shape mapping leads to true mapping of
relations between surface structure ele-
ments, such as: connectivity, dendrity,
directing and ordering of valleys network,
conformity and matching of thalvegs and
watersheds networks, as well as slopes
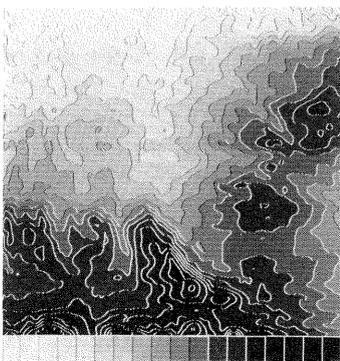sustained by thalveg sections.
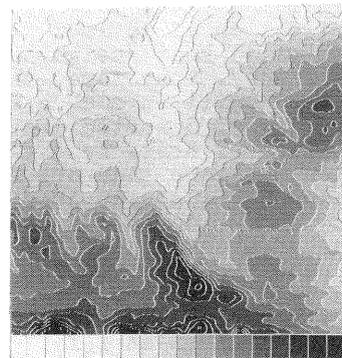

fig.3. Initial DEM isolines map.
Height step is 5m.

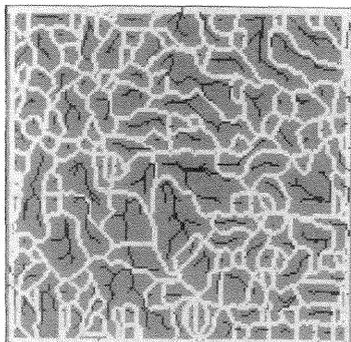fig. 5. "Structurized DEM" isolines map.

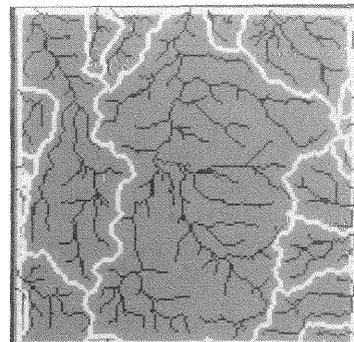fig.4. Valleys network and drainage ba-
sins boundaries on initial DEM.

fig.6.Valleys network and drainage basins
boundaries on "structurized DEM".

Let's observe the shape mapping errors influence on the surface description, taking into accout this point of view. The errors are consequence of transition from continuous relief presentation to discrete one.

As we can see on fig.4 there is a number of closed drainage basins on the surface, which do not exist on the real one. It means G includes far more trees that it needs for the real surface segment description. The root of the tree which is inside of {i,j} but not at its boundary corresponds to a local minimum in the DEM. Thus, the appearance of a local minimum in the DEM which does not exist on the real surface leads to the distirtion of the surface structure description.

As tolerances, errors, confidence bounds are inherent to both numerical modelling of the surface and its instrumental values registration, then the appearance of shape mapping errors is inevitable notwithstanding of DEM construction technique. When we build the DEM having isolines map as input data we realize a transition from continuous line to discrete one and from irregular grid to the regular one, that causes appearance of errors of height measuring in grid knots. The scan by the precise analytical photogrammetrical devices does not guarantee the true surface shape mapping either. Even little amplitude of errors does not change a gist of the matter. Besides, traditional methods of the preliminary processing of digital data such as filtration, interpolation, approximation do not permit to correct errors in mapping of the shape. A local minimum doesn't disappear when we considerably decrease the standard deviation or narrow the interval between confidence bounds. It can only set off or decrease in amplitude depending on the processing parametres. The confidence bounds between which there are true magnitudes of Zmm points allow to change them in any way and consequently relations between the points change as well, but we must keep to the bounds limits. In this case the DEM will always satisfy the topography accuracy. The procedure of the formation of the DEM satisfying the "structural accuracy" using input DEM processing can be reduced to the following: to eliminate the local extremums nonexisting on the real surface, i.e. to fill up unflowing holes and to cut the apexes, the amplitudes of which are less than the tolerance of the initial DEM modelling that, in its turn, causes the minimization of the describing trees number.

The "structurization procedure" may be either two-directed, i.e. both apexes are cut off that is modelling of denudation process and holes are filled up that is modelling of accumulation process, or one-directed, from below. In particular it's enough to fill up the holes which do not exist on the fluvial relief earth surface segment. As noted, a local minimum in the DEM corresponds to a tree root in G. For unflowing process modelling we add a small value Δh to values of heights at

local minimum DEM points and recount arcs of G, in the nodes corresponding to the points vicinities. Both present roots may disappear and new roots appear in G as a result of the DEM pass. The process continues while all internal roots disappear, i.e. only nodes laying on the boundary of i,j remain as roots of trees. In the result of this process we obtain the DEM satisfying the "structural accuracy". The isolines of the "structurized DEM" is shown on fig.5. The mapping of root nodes FAA boundaries and nodes where $S(i,j)>25$ is shown on fig.6. The mappings of the DEM and G central part of 32×29 size before and after "structurization" are shown correspondingly on fig.7 and 8. This part is chosen to clear up the meaning of the process as the picture of the segment has changed largely. The heights of some points became almost 1,5m higher. We have shown the curve of the "structurization procedure" convergence on fig.9. 157 passes of the DEM were needed for obtaining the "structurized DEM" when $\Delta h=0,25m$. The fact of recovering of the surface structure adequate to the real one speaks about the robustness of the process.

3. THE FUNCTIONS VALUES ON THE GRAPH
AND SOME INTERPRETATIONS OF THE MODEL

As it was noted before, $p(i,j)$ points the direction of maximum decrease rate of $Z(X,Y)$ in (i,j) knot. $K(i,j)$ and $S(i,j)$ values play role of memory about surface "behaviour up the flow" of $Z(X_i,Y_j)$ point. We may interpret $S(i,j)$ as a square of $Z(X_i,Y_j)DB$ and $K(i,j)$ as a flowinglines merging number in the given basin or a measure of the horisontal curving of $Z(X_i,Y_j)DB$.

The model describes a liquid movement along the surface without friction and infiltration. If we put a drop of water to every point of the DEM, i.e. to cover it with water skin, then $p(i,j)$ points a drop's removal along the surface per time unit and $S(i,j)$ points the time period when $Z(X,Y)$ point becomes dry. If we water the surface continuously then in some period of time $t>N\times M$, $S(i,j)$ equals to the value of the flow passing through the $Z(X_i,Y_j)$ point per time unit.

Now let's define notions of the thalvegs network and watershed in the framework of discussed model. Watershed points or ridges are points, which have not any flow of external water. So, for construction of the map of ridges it's sufficient to mark the nodes where $S(i,j)=0$.

A notion thalveg is related with notions valley, ravine etc., for forming up of which there must exist some water accumulation area. The DEM reflects the present forms. Thus, the derivation of the drainage network comes to separation the subgraph E of graph G on nodes of which $S(i,j)>S_0$ and $K(i,j)>K_0$ as $K(i,j)$ values define the places of concentration a surface flow into a linear one. The separated subgraph with values of functions K and S on its nodes describes the thalvegs network. We call it as a numerical model of
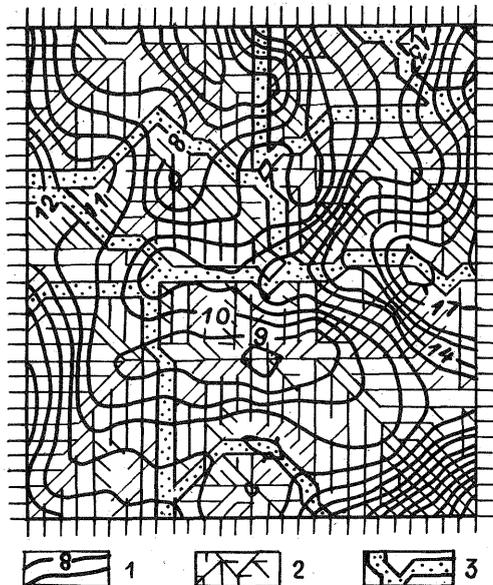
fig.7. Description picture of DEM central part before "structurization"
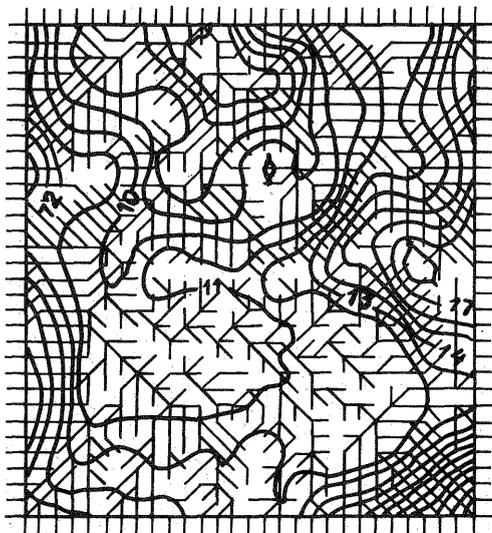

fig.8. The picture after "structurization"
    (1) - heights isolines
    (2) - edges of graph
    (3) - roots FAA boundaries.

drainage network /NMDN/.

On fig.10.1-10.6 we have shown in black nodes where $S(i,j)>S_o$ and on fig.11.1-11.6 nodes where $K(i,j)>K_o$ correspondingly for S =0,1,50,100,200,500 and for K =1,2,3,5,10,20. R means ratio of marked nodes to their common number in per cents. The present pictures show that the more a threshold is the higher is a level of drainage network generalization. Hence, if we say "thalveg of first order" we have to point input data scale, discretization step and tolerance of DEM points measuring, K and S values in initial points /leaf nodes/ of drainage network.
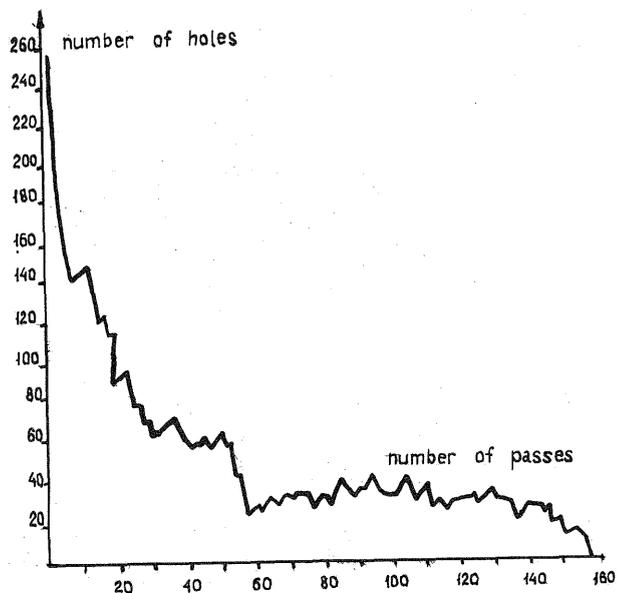
fig.9. "Structurization procedure" convergence graph.

Values of the surface describing functions may serve as measure of drainage net - work stability and drainage basins in the whole as closed hierarchical systems. Preliminary analysis of the functions distribution allows to evaluate the level of the development of the earth surface and its washing up of water erosion.

The pictures can be interpreted as temporal images of the drainage network formation process. In this case the functions values can determine the distinction measure of the substrat properties which the drainage network has been formed on. If we consider these images in the reverse order we can interpret them as process of the network development. The K/S ratio determines the measure of the flowlines merging accumulated from the definite area and characterizes the internal surface geometry.

4. SOME APPLICATIONS OF THE MODEL

It seems obvious that in the frame of the suggested mathematical model the most of tasks of the morphometrical analysis can be reduced to the simple algorithms on trees. For example, to define the lengths of thalvegs sections it's sufficient to calculate the paths lengths between the corresponding nodes. The corresponding slopes square value is determined as the difference of S values on these nodes minus the path length. The same procedure of the K-values gives the approximate length of the slopes limiting watersheds.

The definition of the Strahler's orders of thalvegs is reduced to the definition of the nodes ranks and arcs orders in the NMDN. As we can define the DB boundaries for any point of DEM, so we can construct the complete map of the drainage basins boundaries of any order. The hierarchy of these boundaries is the same as the one of the drainage network. Now we can easily
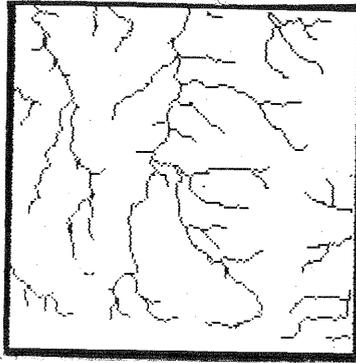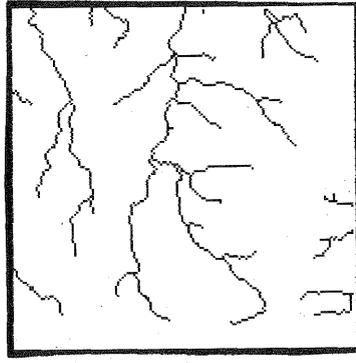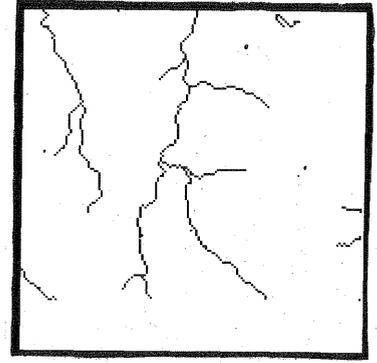
$S_o = 0; \ R = 70.34$ $S_o = 1; \ R = 51.36$ $S_o = 50; \ R = 9.23$
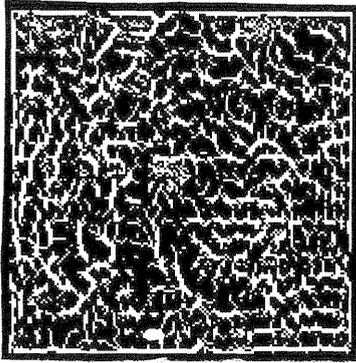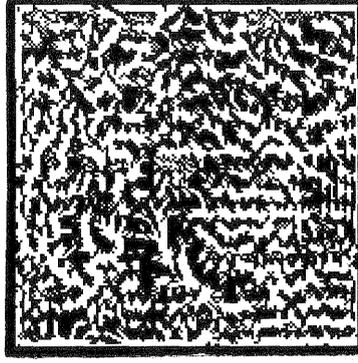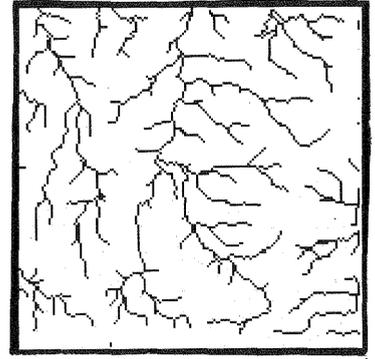
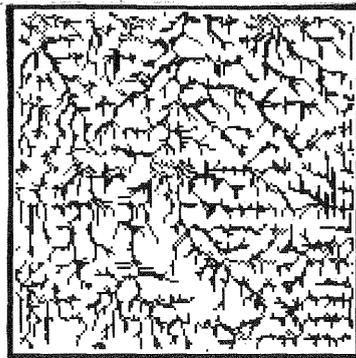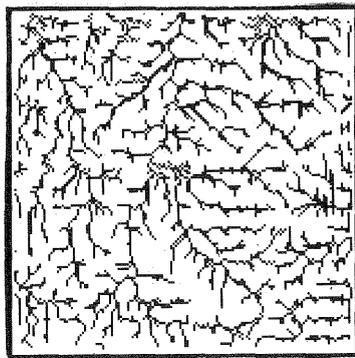$S_o = 100; \ R = 5.56$ $S_o = 200; \ R = 3.87$ $S_o = 500; \ R = 2.09$
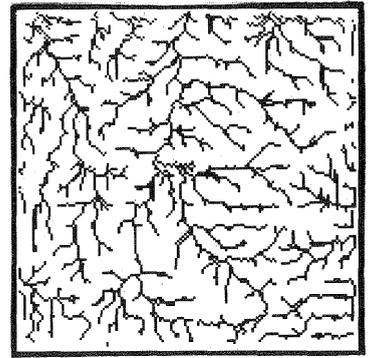
fig.10. Mappings in black nodes of graph where $S(i,j) > S_o$

$K_o = 1; \ R = 25.94$ $K_o = 2; \ R = 18.75$ $K_o = 3; \ R = 14.93$
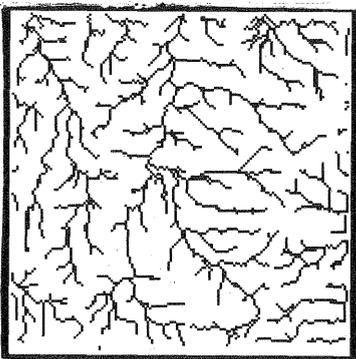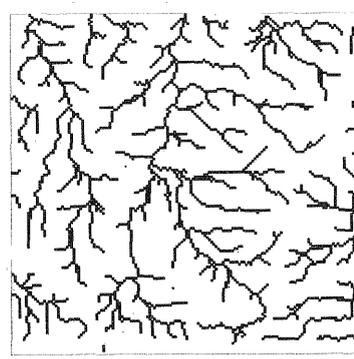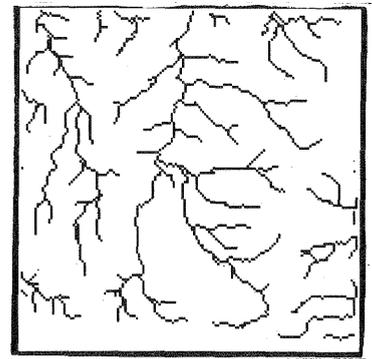
$K_o = 5; \ R = 11.85$ $K_o = 10; \ R = 9.24$ $K_o = 20; \ R = 6.85$

fig.11. Mappings in black nodes of graph where $K(i,j) > K_o$

formulate the procedure of the data pre-
paration for the apexes and basic surfa-
ces construction.

We have described above the DEM set on
the regular grid. The similar description
can be used for digital models in the
vector presentation, i.e. for the isoli-
nes maps processing. In this case we deal
with Z-axis ordering data. The construc-
tion of the relief sceleton lines frame-
work before the raster presentation per-
mits to make up the DEM satisfying not
only the topographical accuracy but keep-
ing the shape and the structure of the
initial surface.

The suggested mathematical model can be
considered as the static one. The surface
description in the form of the weighted
graph permits us to construct a mathema-
tical model of the water masses transfe-
rence and particle transportation along
the surface. If we put infiltration ratio
and external water power on nodes and
channel capacity on arcs of the graph we
can build a mathematical model serving
for the flow modeling and erosion proces-
ses investigation using simple discrete
calculation technique.

REFERENCES

Antipov,M.V., and Kireytov,V.R., 1979.
Relief surface T(W) set construction. Ge-
ologia i geofizika, 11:90-97 (in Russian).

Antoshchenko-Olenev,I.V.,and Golda,Ya.V.,
1986. Methodology of the automatization
of geological interpretation. Sovietskaya
geologia, 8:7-15 (in Russian).

Golda, Ya.V., and others, 1986. Working
out of methods, algorithms and computer
software for automatical construction of
geological interpretations map. Report by
theme I A.VIII/627 during the period of
1984-1986, Kazgeofizika, Alma-Ata, state
register No 26-84-21/31 (in Russian).

Horton,R.E., 1945. Erosional development
of streams and their drainage basins: hy-
drophysical approach in quantitative mor-
phology. Geol. Soc. America Bull.,
56:1117-1142.

Kuznetsov,B.M., and others, 1979. The de-
vice of inputting of coordinates and
heights of stereo terrain model points
into ES computer /"Format-130"/. In: Aeri-
al and space automatical data processing
methods and technical means. Leningrad,
pp.42-55 (in Russian).

Strahler,A.N., 1952. Hypsometric (area-
altitude) analysis of erosional topogra-
phy. Geol. Soc. America Bull.,
62:1117-1142.

Shreve,R.L., 1966. Statistical law of
stream numbers. The Journal of Geology,
(1):17-37.