A RIGOROUS PHOTOGRAMMETRIC ADJUSTMENT ALGORITHM BASED ON CO-ANGULARITY CONDITION

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ABSTRACT: As revealed by some investigations, using co-angularity condition to process photogrammetric data can lead to the fastest determination of the final results of the coordinates of groundpoints, and result in a block of the highest quality as compared with other conditions in photogrammetry. But up to now, the adjustment method(pyramid method) based on this condition has been yet an approximate one because two difficulties of finding essential conditions and computation work load stand in the way to rigorous method. They are thoroughly overcome in this, this results in a rigorous method based on co-angularity condition. The property of the method includes: finding essential conditions is very easy, fast, safe, and simple; Most impotantly, by means of condition transformation, the adjustment is resulted in an exact parameter one; The structure of normal equation matrix, the computation procedure and work load are all similar to that of bundle adjust-ment; Even the number of parameters are the same as that of bundle one, but many of them play a part of control data, then the condition of normal equation matrix is much better than that of bundle one. This method need not know original values of angular orientations.

KEYWORDS: Photogrammetry, Areotriangulation, Algorithm, Co-angularity, Crown Method, Condition Transformation

0. INTRODUCTION

Photgrammetric adjustment, also called analytical areo-triangulation or block adjustment, has been playing a very important part in photogrammetry. No matter what field the Photogrammetry is involved in, e.g. space ,areo-, Terrestal or close-rauge photogrammetry, it will have to include adjustment. No matter what way the photo-grammetry is based on, e.g., analog, analytical or digital photogrammetry, it will still cannot do without adjust-ment. The purpose of adjustment is to eliminate discran-pencies caused by redundent observations, estimate the probable values and evaluate the qualities(e.g. accuracy and reliability) of the coordinates of the object points and the orientation elements of the photographs.

In recent years, photogrammetric adjustment has made a great progress on aspects of compensating systematic errors, introducing non-photogrammetric information, detecting blunders and reliability theory, etc. (Ackermann 1983, 1988; Ebner. 1976; Faig, 1986; Foestner, 1983; Gruen, 1978, 1985; Kubik, 1982; Deren Li, 1983, 1987, 1988; Zhizhuo Wang, 1990; Yianmin Wang, 1988a, 1988b)

Up to now, three approaches, such as strip, independent models and bundle adjustment, have been widely used in photogiammctric adjustment. Of them, only bundle method is rigorous when image coordinates are directly observed. this method is based on well known co-linearity condition and derived model is Gauss-Markov one. This model pocesses many advantages which will be seen in section three of this paper. Therefore bundle adjustment is considered the best method.

Unfortunately, a well known strange phenomenon will apear when you practically use bundle adjustment, the minimum control(2 horizontal and 3 vertical control points, or corres-ponding geodetic data) can not effect(Faig, 1986), even in the case of computer similated block with only 2 photos and 9 ground points(the author tested, But if you use an approximate method such as relative-absolute orientation, the minimum control will fefct. This makes the bundle method debatable. Why is rigorous method inferior to approximate one? Is bundle method rigorous? The author thinks the bundle method is really rigorous in theory, though the over-parameterized model results in weak condition of normal equation matrix, then leads to the poor results of adjustment. This implies that co-linearity condition is not a strong one. We have to pursue other stronger one.

In fact, besides co-linearity condition, there exist other two fundamental conditions in photogrammetry, namely co-angularity and co-planarity. It can be proved that latters are included in former, though they take less parameters than former.

Because the models derived from both conditions are general forms instead of Gauss-Markoff one, two dif-ficulties stand in the way to the aim of rigorous adjustment, one is how to find essential or necessary condions, and another is how to reduce the computation work loads to an acceptable level.

Professor Faig(1974, 1975) tried to use co-planarity condi-tion to construct a rigorous adjustment. In his method, minimum control is effective, this result affirmes the validity of the above viewpoint. Because of the above two difficulties, his method has not be accepted widely by others.

Up to now, the two conditions are only applied widely to form approximate adjustment systems in photogrmmetry (Zhizhuo Wang, 1990), which can not campare favorably with bundle adjustment.

From above discussion, we know that the key to develop available rigorous adjustment methods based on the two powerfull conditions is to overcome the two difficulties. The paper will stress on this aspect.

As revealed by some investigations, the co-angularity con-dition can lead to the fastest determination of the final results of the coordinates of ground points, and if other factors being equal, it can result in a block of the highest quality as compared with other methods (Veress, 1982; Zhizhuo Wang, 1990). This presents that co-angularity condition is the strongest one, so we select it to construct our method.

In section 1, the concerned concepts of adjustment are presented. The process of rigorous adjustment is outlined. The approximations of some existing method are revealed. The concepts of condition transformation is presented.

section 2 deals with the general modelling of the adjustment based on co-angularity condition. The functional model is constructed. The formula to calculate the number of essential conditions is presented. Most importantly, two methods, called Sector and Crown respectively, of finding essential conditions are developed, the finding difficulty is thoroughly overcome by the methods.

Section 3 is the kernel of the paper. According to the favorable structure of the general equations obtained from crown method, the Gauss-Markoff model based on co-angularity, called Crown Model, is ingeniously obtained by the scheme of condi-tion transformation with elimination. The computation difficulty is overcome satisfactorily.

Section 4 presents the structure of the normal equation matrix based on crown model. The structure is similar to that of bundle method, but the condition of the matrix is much better than that of bundle one.

Section 5 is some conclusion remarks, and section 5 lists the relevant literatures.

1. CONCERNED CONCEPTS OF ADJUSTMENT

In this section, the concerned concepts of rigorous and approximate adjustment, and condition transformation are presented, these concepts are usefull for understading the contexts in next sections.

1.1 Rigorous

Supposing that an adjustment problem consists of n_l observations, n_x parameters, and n_t essential observations. The functional model can be written as:

$$F(\underset{n_x \times 1}{X}, \underset{n_l \times 1}{L}) = 0 \tag{1}$$

and stochastic model as:

$$P = \sigma_0^2 \, D^{-1} \tag{2}$$

where **X** and **L** are unknown and observation vectors repectively, **P** and **D** are weight and variance matrices of **L** respectively, σ_0 is the unit weighted standard deviation. The number n_r of equations in eq. (1) is determined by eq. (3)

$$n_r = n_l - n_t + n_x \tag{3}$$

Eg. (1) is non-linear formed and can be linearized with a Taylor expansion as

$$\underset{n_r \times n_l}{A} V + \underset{n_r \times n_x}{B} dX + W = 0 \tag{4}$$

where

$$A = \frac{\partial F}{\partial L}\bigg|_{X^0, L}, \quad B = \frac{\partial F}{\partial X}\bigg|_{X^0, L}, \quad W = F(X^0, L) \tag{5}$$

V is residual vector and dX is the correction vector of parameter vector X.

If the least square principle is used to solve eq. (4), then the following condition must be satisfied

$$V^T P V = \text{minimum} \tag{6}$$

Using constrained extreme value principle, we can obtain the normal equation as

$$\begin{aligned} AP^{-1}A^T K + B dX + W &= 0 \\ B^T K &= 0 \end{aligned} \tag{7}$$

where K is called link vector. if rank(A) = n_r, eq. (7) can be written as

$$B^T(AP^{-1}A^T)^{-1} B dX + B^T(AP^{-1}A^T) = 0 \tag{8}$$

then

$$dX = -(B^T(AP^{-1}A^T)^{-1} B^T(AP^{-1}A^T)^{-1} W \tag{9}$$

The above solusion procedure is then called rigorous adjustment. If a solusion does not adhere above procedure, then the adjustment is called the approximate adjustment. Usually, The approximation can be grouped into three cases:

1) **Residual Approximation**: The eq. is simplified as

$$V' + B dX + W = 0 \tag{10}$$

where $V' = AV$ and $A \neq E$ (unit matrix). Eq. (10) are solved for the unknown vector dX under the coudition

$$V'^T V' = \text{minimum} \tag{11}$$

The relative orientation, existing co-angularity condition (pyramid) method and co-planarity condition method (Zhizhuo Wang 1990) are all in this case.

2) **parameter Approximation**: in this case, the parameter vector dX is grouped into two or more sets. The solusion is performed with succesive approximation by alternating seting of parameters. That is, in each iteration, just one set of parameters are treated as unknown and others as known. The existing pyramid and co-planarity method are of this case.

3) **Weighting Approximation**: the weight matrix P is considered as digonal while the elements among L are relevant or considered as E while the elements among L are not equal weighted. The indepentent model adjustment, strip adjustment and Absolute orientation (Zhizhao Wang 1990) are all in this case. In fact, functional approximation can be merged into this case refer to eq. (11).

1.2 Particular Forms of Condition Equation

Eq. (4) is a general form of condition equations, and is called condition equation with parameters. It will be very inconvenient if eq. (4) is directly used in practical adjustment. Therefore, if it is not out of absolute necessity, no one uses the general form, but always use a particular form.

If A=E, then eq. (4) is simplified as

$$-V=BX+W \tag{12}$$

Eq. (12) is called error equation or observation equation, and eq. (7) is changed to

$$B^TPBdX+B^TW=0 \tag{13}$$
$$dX=-(B^TPB)^{-1}B^TPW \tag{14}$$

This procedure is called parameter or indirect adjustment. If B=0, then eq. (4) is reduced to:

$$AV+W=0 \tag{15}$$

then eq. (7) is changed to

$$APA^T+W=0 \tag{16}$$
$$K=-(APA^T)^{-1}W \tag{17}$$

eq. (15) is called condition equation, and the adjustment based on eq(15)-(17) is called condition adjustment.

1.3. Condition Transformation

Sometimes, it is very difficult to directly form the particular form of conditions(eq. (12) or (15)), in this case, for the covenience of computation, we can transform general form into particular one.

1.3.1 General Eq. to Observation Eq.

Assuming that matrix A and vecter V in eq. (14) can be divided into two sub-matrix and vectors respectively as follows:

$$A=(A_1\ A_2) \tag{18}$$
$$V^T=(V_1^T\ \ V_2^T) \tag{19}$$

where

$$\mathrm{rank}(A_1)=\mathrm{rank}(A)=n. \tag{20}$$

then eq. (4) can be written as:

$$[A_1\ A_2]\begin{bmatrix}V_1\\V_2\end{bmatrix}+B\,dX+W=0 \tag{21}$$

then

$$V_1+A_1^{-1}A_2V+A_1^{-1}B\,dX+A_1^{-1}W=0 \tag{22}$$

Eq. (22) can be written as

$$-V=B'dX'+W' \tag{23}$$

with

$$\left.\begin{aligned} &V^T=[V_1^T\ \ V_2^T\ ],\quad X'^T=[t^T\ \ X^T] \\ &B'=\begin{bmatrix}A_1^{-1}A_2 & A_1^{-1}B\\ E & 0\end{bmatrix}\quad W'=\begin{bmatrix}A_1^{-1}W\\ 0\end{bmatrix}\end{aligned}\right\} \tag{24}$$

refer to eq. (12), we can find that eq. (23) is just the observation equation.

1.3.2. General Eq. to Condition Eq.

matrices A and B in eq. (4) can be divided into two submatrices respectively as follows:

$$\left.\begin{aligned} A^T&=(A_1^T\ \ A_2^T)\\ B^T&=(B_1^T\ \ B_2^T)\\ W^T&=(W^T\ \ W^T)\end{aligned}\right\} \tag{25}$$

with

$$\mathrm{rank}(B_1)=\mathrm{rank}(B)=n_X \tag{26}$$

then eq. (4) can be written as

$$\left.\begin{aligned} A_1V+B_1X+W_1&=0\\ A_2V+B_2X+W_2&=0\end{aligned}\right\} \tag{27}$$

then

$$X=-B_1^{-1}A_1V-B_1^{-1}W_1 \tag{28}$$

substitute eq. (28) into the second equation of eqs. (27), we have

$$A'V+W'=0 \tag{29}$$

where

$$\begin{aligned} A'&=A_2-B_1^{-1}A_1\\ W'&=W_2-B_1^{-1}W_1\end{aligned} \tag{30}$$

refer to eq. (15), we obviwsly know that eq. (29) is just the condition equation.

2. GENERAL MODELLING

Having concepts described in preceding section, we can conveniently construct our adjustment system. We know that adjustment modelling includes functional and stochastic modelling, number of necessary conditions determination and condition seeking. We deal now with these problems.

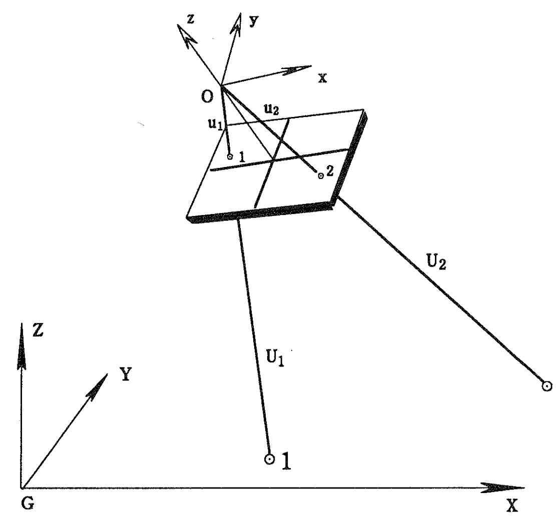

Fig. 1

2.1. Function Modelling

According to fig. 1, co-angulurity is that the vertex angle $\angle_{102}$ between the rays u_1 and u_2 in image space o-(x,y,z) is equal to that $\angle 1O2$ between the corresponding rays U_1 and U_2 in the object space O-(X,Y,Z). The function model can be expressed as

$$F(U,u)=\Phi_{12}-\phi_{12}=0 \tag{31}$$

where

$$\begin{aligned} &\phi_{12}=\cos\angle_{102}=\frac{x_1x_2+y_1y_2+f^2}{s_1\,s_2}\\ &\Phi_{12}=\cos\angle 1O2=\frac{\Delta X_1\,\Delta X_2+\Delta Y_1\Delta Y_2+\Delta Z_1\Delta Z_2}{S_1\ S_2}\\ &u_i=(x_i\ \ y_i\ \ -f),i=1,2 \qquad s_i=\sqrt{x_i^2+y_i^2+f^2}\\ &U_i=(\Delta X_i\ \ \Delta Y_i\ \ \Delta Z_i)=(X_i-X\ \ Y_i-Y\ \ Z_i-Z\ ),\ S_i=\sqrt{U_i^TU_i}\end{aligned} \tag{32}$$

Eq. (31) is a non-linear equation, therefore a Taylor expansion is used for linearisation

$$AV+BU+W=0 \tag{33}$$

Where

$$B=\left.\frac{\partial F}{\partial U}\right|_{U^0,u},\quad A=\left.\frac{\partial F}{\partial u}\right|_{U^0,u},\quad W=F(U^0,u) \tag{34}$$

eg. (33) is linearised co-anguiarity condition equation. Refer to eq. (32) and (34), we obtain:

$$
\begin{aligned}
&A(a_1\ a_2\ a_3\ a_4)\\
&a_1=-\frac{x_1}{s_1 s_2}+\phi_{12}\frac{x_2}{s_2^2} \qquad a_2=-\frac{y_1}{s_1 s_2}+\phi_{12}\frac{2}{s_2^2}\\
&a_3=-\frac{x_2}{s_1 s_2}+\phi_{12}\frac{x_1}{s_1^2} \qquad a_4=-\frac{y_2}{s_1 s_2}+\phi_{12}\frac{y_1}{s_1^2}\\
&V=(v_{x_1}\ v_{y_1}\ v_{x_2}\ v_{y_2})\\
&B=(b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ b_7\ b_8\ b_9)\\
&b_1=\frac{\Delta X_1}{S_1 S_2}-\Phi_{12}\frac{\Delta X_2}{S_2^2} \qquad b_2=\frac{\Delta Y_1}{S_1 S_2}-\Phi_{12}\frac{\Delta Y_2}{S_2^2}\\
&b_3=\frac{\Delta Z_1}{S_1 S_2}-\Phi_{12}\frac{\Delta Z_2}{S_2^2} \qquad b_4=\frac{\Delta X_2}{S_1 S_2}-\Phi_{12}\frac{\Delta X_1}{S_1^2}\\
&b_5=\frac{\Delta Y_2}{S_1 S_2}-\Phi_{12}\frac{\Delta Y_1}{S_1^2} \qquad b_6=\frac{\Delta Z_2}{S_1 S_2}-\Phi_{12}\frac{\Delta Z_1}{S_1^2}\\
&b_7=-\frac{\Delta X_1+\Delta X_2}{S_1 S_2}-\Phi_{12}\left(\frac{\Delta X_1}{S_1^2}+\frac{\Delta X_2}{S_2^2}\right)\\
&b_8=-\frac{\Delta Y_1+\Delta Y_2}{S_1 S_2}-\Phi_{12}\left(\frac{\Delta Y_1}{S_1^2}+\frac{\Delta Y_2}{S_2^2}\right)\\
&b_9=-\frac{\Delta Z_1+\Delta Z_2}{S_1 S_2}-\Phi_{12}\left(\frac{\Delta X_1}{S_1^2}+\frac{\Delta X_2}{S_2^2}\right)\\
&U^T=(dX_1\ dY_1\ dZ_1\ dX_2\ dY_2\ dZ_2\ dX_O\ dY_O\ dZ_O)\\
&W=\Phi_{12}\ \phi\ \phi_{12}
\end{aligned} \tag{35}
$$

2.2 Number and seeking of conditions.

In preceding subsection, just one condition has been presented. Eq. (33) must be expanded to construct an adjicstment system. Then, how many conditions like eq. (33) should be selected and how can they be found?

2.2.1 Number

Supposing that there are n_i image points on a photo, We know that each pair of points can form a co-anguiarity conditions. So the total number n' of conditions that can be formed must be:

$$n'=n_i(n_i-1)/2 \tag{36}$$

Though ma of them are functional relevant. From section 1, we know that the number n_{r_i} of cordition equations must be determined by eq. (3). It is obvious that there are 6 exterior orientation elements for a photo. In eq. (35), 3 of them are selected. All parameters for ground points are essential and selected. So the number of condition equations for a photo should be as

$$n_{r_i}=2n_i-3 \tag{37}$$

If a block consists of n_O photographes and n image points, then the total number of essential co-angularity equations should be as

$$n_r=\sum_{n=1}^{o}(2n_i-3)=2n-3n_O \tag{38}$$

Now we can expand eq. (33) to a block available form

$$\underset{n_r\times 2n}{A}V+\underset{n_r\times n_t}{B}U+\underset{n_r\times 1}{W}=0 \tag{39}$$

In matrix A, there are 2n elements for each line, but only 4 non-zero. In matrix B, there are nt elements for each line, and only 12 of them are non-zero. All of Them are also deterimed by eq. (35).

2.2.2 Find

The knottiest problem in codition adjustment is how to find essential independent conditions. For this reasen, people prefer to using parameter adjustment instead of using condition adjustment in their system, even though they know sometimes condition adjustment is more approprate than parameter one for their problem. Our photogrammetric adjustmevt is just of the case. We all know that the co-angularity candition is stronger than co-linearity one, but the latter is all along used for rigorous adjustment other than the former. For the purpose of improving photogrammetric adjuetment, the author studied co-angularity condition, and resulted that by the following two methods the essential co-angularity conditions can be easily found.

1) Sector Method

Because co-angularity conditions are the relations between the rays in the same photos, if the conditions of each photo in a block reachs essential ones, then that of the whole block will reach either. Therefore, the working unit is aiways the individual photo.

Assume that there are m image points in a photograph. (e.g. 9 points in fig.2), select one of them and reiate it to other m-1 points to generate m-1 co-angulority conditions, then let each of the m-1 points relate to its adjacent one to get other m-2 co-angularity conditions. the sum of conditions equal to m-3, the number of essential conditions.

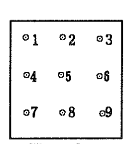

Fig.2

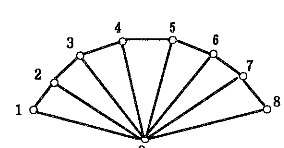

Fig.3

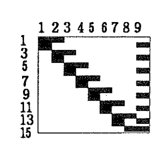

Fig.4

Fig.3 shows the results after the conditions obtained by this method. The line between two points implies they are related each other. This graph is just like a sector, so we call this finding method a Sector Method. Now, let's see whether the conditions are independent.

Fig.4 is the structure of matrix A of eq. (39) for the photo of fig.2. From eq. (4) we know that independence of eq. (39) means rank(A)=2m-3. fig,4, the first $n_r\times n_r$ matrix of matrix A is a quasi-triangular matrix, and according to eq. (35), each 2×2 main submatrix of it is inversable. So, rank(A)=2m-3. this represents that conditions resulted from the method are functional independent.

2) **Crown Method**

The working unit is also the individual photo. We still take the photo as show in fig.1 to explain the method. Select two points from m points in the photo as base points, let other m-2 points relate to the base pointswe obtain 2m-4 co-angularity conditions, and let base points relate each other to get the (2m-3)th condition. By this means to form co-angularity conditions results the relations between the points as shown in fig.5. The shape of fig.5 is like a crown of king, so we call our method as Crown Method.

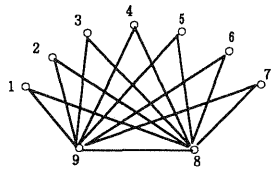

Fig.5

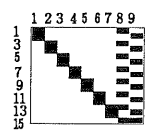

Fig.6

In fig.6, the column number presents the condition number, the line number stands for image point number. From fig.6 we know that the first (2m-3) × (2m-3) submatrix of matrix A is also a quasi-triangular matrix, according to the describtion of sector method, rank(A)=2m -3, so the conditions obtained from the Crown Method is also independent.

The base of the crown consists of the selected points and their relation, so the selected image points are called as base image points, the corresponding ground points as base ground points, or all called as base points for short, the relation is called base condition. The vertexes of the crown consists of the other points, so they are called the vertexes.

obviously, by the above two methods, the conditions are just formed one by one, needed no finding work, so the methods are simplest, safeest and easiest ones.

3. GAUSS-MARKOFF MODELLING

Refering back to section 1, we will find that eq. (39) is the same form as eq. (4), a general form of condition equations. If we directly use it to do adjustment, the computation work load will be very large, this is because the normal eguation matrix is very big. We must study further to get an available model.

3.1 Principle

To construct an available model, following three problems have to be solved.

3.1.1 Equation Form

The first problem is what equation form will be selected as our model. Both observation and condition equations are available to our model, though due to the following reasons, we will sellect observation one:

1) **Parameter vector** can be direclly estimated.
2) **It is convenient** to compensate syetematic errors, detect Blunders , estimate variance, include non-photogrammetric data and evaluate the quality of the result.
3) **It is easy** to construct and solve normal equation.
4) **almost all existing adjustment software** are based on parameter adjustment.

3.1.2 Finding

The second problem is what method for finding conditions we will select to construct our available model. In preceding section, we recomended two method, which one is appropriate? Let's comparing fig.4 with fig.6. In fig.4, the matrix A_1 is a 14×14 matrix, its structure is banded. In fig.6, the matix A_1 is a 14×14 quasi-diagonal matrix. Of course, to inverse a quasi-diagonal matrix is much easier than a banded one. Therefore we select crown method to construct our model.

3.1.3 Transformation

The last problem is how to transform the condition equations with parameters into observation equations. consulting into section 1 and fig.6, we know that if we directly using eqs. (18) ~ (24) to transform, sub-matrix A_1 in eq. (18) of our case in fig.2 will be a 15×15 margined quasi-diagonal matrix, its inverse will be full-occupied. This will make the transformation difficult. Therefore we use a transformation with elimination technique to transform general form to observation form instead of a direct transformation in section 1.

Above all. the principle of our modelling is that applying crown method to find conditions, using transformation with elimination technique transforms the general form into observation form.

3.2 Modelling

Acording to the above principle, we deal now with the observation modelling based on co-angularity condition. Because the observations related to each other by co-angularity condition are in the same photograph, we can take individual photograph as working unit. Furthermore, supposing that eq. (39) is for a photo with n image points, due to the crown finding method, from fig.6 we can see that the first 2(n-2)×2(n-2) sub-matrix A_1 of matrix A in eq. (39) is a quasi-diagonal matrix, we can transform the first 2(n-2) equations of eq. (33) into corresponding observation equations with one by one vertex scheme. Now, we take the ith vertex together with the base of the crown to inquire into the transformation of general equation to observation equation. Here, eq. (39) can be written as

$$\begin{bmatrix} A_{T_i} & A_{B_i} & A_{B_i} \\ & F_B & F_C \end{bmatrix} \begin{bmatrix} V_{T_i} \\ V_B \\ V_C \end{bmatrix} U + \begin{bmatrix} W_i \\ W_B \end{bmatrix} = 0 \tag{40}$$

where

$$A_{T_i} = \begin{bmatrix} a_{1im} & a_{2im} \\ a_{1in} & a_{2in} \end{bmatrix}, A_{B_i} = \begin{bmatrix} a_{3im} \\ a_{3in} \end{bmatrix}, A_{C_i} = \begin{bmatrix} a_{4im} & 0 & 0 \\ 0 & a_{3in} & a_{4in} \end{bmatrix}$$

$$F_B = a_{1mn}, \quad F_C = (a_{2mn}\ a_{3mn}\ a_{4mn}),$$

$$B_{Ti} = \begin{bmatrix} b_{1im} & b_{2im} \cdots & b_{6im} & b_{7im} & b_{8im} & b_{9im} & 0 & 0 & 0 \\ b_{1in} & b_{2in} \cdots & b_{6in} & 0 & 0 & 0 & b_{7in} & b_{8in} & b_{9in} \end{bmatrix}, \tag{41}$$

$$B_B = (b_{1mn}\ b_{2mn}\ b_{3mn}\ 0\ 0\ 0\ b_{4mn}\ b_{5mn} \cdots b_{9mn}),$$

$$V_{T_i}^T = (v_{x_i}\ v_{y_i}), \quad V_B = v_{x_m}, \quad V_C^T = (v_{y_m}\ v_{x_n}\ v_{y_n}),$$

$$W_i^T = (w_{im}\ w_{in}), \quad W_B = w_{mn}, \quad i=1,2,\cdots,(n-2), \quad m=n-1,$$

$$U^T = (X_o\ Y_o\ Z_o\ X_i\ Y_i\ Z_i\ X_m\ Y_m\ Z_m\ X_n\ Y_n\ Z_n).$$

About the subs of a_{jkl}, b_{jkl} and w_{kl} , j stands for the jth element ofthe matrix A and B in eq. (36), k meanses the hth point point in the plaoto, and l meanses that the the kth point in the photo, and l meanses that the kth point is related to the lth point with co-angularity consdition in the photo.

$$P = \sigma_0^2 D^{-1} = \begin{bmatrix} E_T & & \\ & E_B & \\ & & E_C \end{bmatrix} \quad (49')$$

where

$$n_T = 2(n-2n_B) = 2\sum_{i=1}^{n_B}(n_i-2) = \sum_{i=1}^{n_B} 2n_{T\,i}$$

$$n_U = 3(n_B+n_g), \quad n_C = 3n_B$$

$$V_T^T = ({}_1V_T^T \ {}_2V_T^T \ \cdots \ {}_{n_B}V_T^T),$$

$${}_iV_T^T = (V_{T_1}^T \ V_{T_2}^T \ \cdots \ V_{T_{n_{T_i}}}^T),$$

$$C_T = \begin{bmatrix} {}_1C_T & & & \\ & {}_2C_T & & \\ & & \ddots & \\ & & & {}_{n_B}C_T \end{bmatrix}$$

$${}_iC_T^T = (C_{T_1}^T \ C_{T_2}^T \ \cdots \ C_{T_{n_{T_i}}}^T \quad (50)$$

$$G_T^T = ({}_1G_T^T \ {}_2G_T^T \ \cdots \ {}_{n_B}G_T^T),$$

$${}_iG_T^T = (G_{T_1}^T \ G_{T_2}^T \cdots G_{T_{n_{T_i}}}^T)$$

$$V_B^T = ({}_1V_B^T \ {}_2V_B^T \ \cdots \ {}_{n_B}V_B^T)$$

$$C_B = \begin{bmatrix} {}_1C_B & & & \\ & {}_2C_B & & \\ & & \ddots & \\ & & & {}_{n_B}C_B \end{bmatrix}$$

$$G_B^T = ({}_1G_B^T \ {}_2G_B^T \ \cdots \ {}_{n_B}G_B^T)$$

$$Q^T = ({}_1Q^T \ {}_2Q^T \ \cdots \ {}_{n_B}Q^T)$$

$$U^T = ({}_1U_B^T \ U_B^T \cdots {}_{n_B}U_B^T \ {}_1U_g^T \ {}_2U_g^T \cdots {}_{n_g}U_g^T)$$

$$U_B^T = (X_o \ Y_o \ Z_o)$$

$$U^T = (X \ Y \ Z)$$

$$L_T^T = ({}_1L_T^T \ {}_2L_T^T \ \cdots \ {}_{n_B}L_T^T)$$

$${}_iL_T^T = (L_{T_1}^T \ L_{T_2}^T \cdots \ L_{T_{n_{T_i}}}^T)$$

$$L_B^T = ({}_1L_B^T \ {}_2L_B^T \ \cdots \ {}_{n_B}L_B^T)$$

In eq.(49) and (50), right subs are about photos or ground points, left subs are equivolent to that of eqs (43) -(48).

Eq.(49) is developed from co-angularity condition and crown finding method, and belongs to the Gauss-Markoff model, so we can call it Crown Co-angularity Gauss-Markoff Model(CCGMM) or simplely Crown Model(CM). The adjustment based on CCGMM is called Crown Adjustment.

Refer to eqs.(49)-(50) we know that the parameter vectors Q and U_B are both photo-invarriants, so we can place them together in practical adjustment. From eq.(49) we can easily find the properties of the model:

1) The normal equation can be composed one by one image point (vertex) and photo (base), and reduced one by one ground point (or photo) such as bundle adjustment.

2) The sum of observation equations and the sum of the parameters in crown method are both equal to that of bundle method.

3) The angular orientations are not any more parameters in this metod. This will make the computation convenient, and broaden the application range of photogrammetry, for example, in terrestral or close-range photogrammetry, we

For eqs.(40), we let

$$V_C = Q \quad (42)$$

The residual vector Vc has been transformed to parameter to parameter Q, we call it the first transformation.

The second equation of eqs.(40) left-handly multiplied by F_B^{-1}, and substitute eq.(42) into it, we obtain

$$V_B = C_B Q + G_B U - L_B \quad (43)$$

where

$$C_B = -F_B^{-1}F_C$$
$$G_B = -F_B^{-1}B_B \quad (44)$$
$$L_B = F_B^{-1}W_B$$

The second equatiopn is transformed to an observation one, we call this the second teansformution.

Substitute eqs.(42) and (43) into the first equation of eq.(40), and merge the identical terms, we obtain

$$A_{T_i}V_{T_i} + C'_{T_i}Q + G'_{T_i}U + W_{T_i} = 0 \quad (45)$$

where

$$C'_{T_i} = A_{B_i}C_B + A_{C_i}$$
$$G'_{T_i} = A_{B_i}G_B + B_{T_i} \quad (46)$$
$$W'_{T_i} = W_{T_i} - A_{B_i}L_B$$

At this time, we have eliminated the residual vector V_B from the first equation of eqs.(40).

Eq.(45) left-handly multiplied by matrix $A_{T_i}^{-1}$, we obtain

$$V_{T_i} = C_{T_i}Q + G_{T_i}U - L_{T_i} \quad (47)$$

where

$$C_{T_i} = -A_{T_i}^{-1}C_{T_i}$$
$$G_{T_i} = -A_{T_i}^{-1}G_{t_i} \quad (48)$$
$$L_{T_i} = A_{T_i}^{-1}W_{T_i}$$

Now that, the first equation of eq.(39) has been transformed into an observation equation, this is called the third and last transformation. Up to now, the general form of co-angularity condition eq.(39) has been thoroughly transformed into the expected observation form. The approach consists of three transformations and an elimination, so we call it Transformation with Elimination Technique.

From eq.(42),(43) and (47), we know that the observation form of co-angularity condition consists of three types of equations. eq.(42) plays a part of control, so is called the Control Equation; eq.(46) presents the relation between the base points of the crown, then is called the Base Equation; eq.(47) is for the vertex of the crown, therefore is called verTex Equation.

The above equations are about just one vertex with the base related to it. They can be conveniently expanded to fit a block. Assume that a block consists of n_B photos, n_g ground points and n image points, then the corresponding Gauss-Markoff can be written as

$$\begin{bmatrix} \underset{n_T\times 1}{V_T} \\ \underset{n_B\times 1}{V_B} \\ \underset{n_C\times 1}{V_C} \end{bmatrix} = \begin{bmatrix} C_T & G_T \\ C_B & G_B \\ E_C & 0 \end{bmatrix} \begin{bmatrix} \underset{n_C\times 1}{Q} \\ \\ \underset{n_U\times 1}{U} \end{bmatrix} - \begin{bmatrix} L_T \\ L_B \\ 0 \end{bmatrix} \quad (49)$$

need not measure the approximations of angular orientations, even more, the method is available to the block photographied with amateur cameras.

4) The parameters are more tightly linked in this method than in bundle one, and many of the parameters are directly observed, so the condition of normal equatons is much better than that in bundle one.

4. THE STRUCTURE OF NORMAL EQUATIONS

The relations among parameters are strongthened in crown adjustment, though the structure of normal equations is changed too. We all know that a favorable structure of normal equations can spare much time and space of computer.

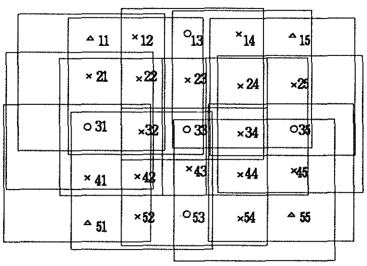

Fig.7

Now, let's take the block shown in fig.7 as an example to discuss the structure of normal equations by crown method. The photos and ground points are numbered in a direction perpendicular to that of the strip. The photo numbers adopted here are the same as those of the ground points near the principle points of the photos. Fig.8 is the structure of the observation equations. From fig.8 we can clearly see that, it is only base points that interferes the structure of normal equations, so we can teat them carefully to get a favorable structure. The following principles are helpfull to deal with the problem:

1) Select base points as few as possible, in other word, several photos can share the same base points. In our example, all groung points may be as base points, though we select just 4 points(number 31, 32, 34 and 35).
2) Select geodetic points as base points whenever possible, because the geodetic points will not be reduced in a reduced normal equations.
3) Remain the base points in reduced normal equations.

Fig.9 is the structure of normal equations for the block in fig.7 by following the above principles. It's clear that except the base point terms, the structure is the same as that of bundle method.

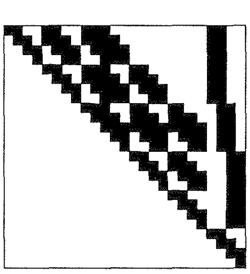

Fig.10

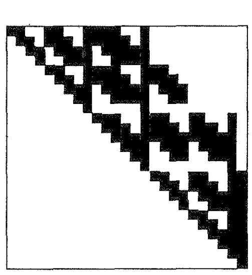

Fig.11

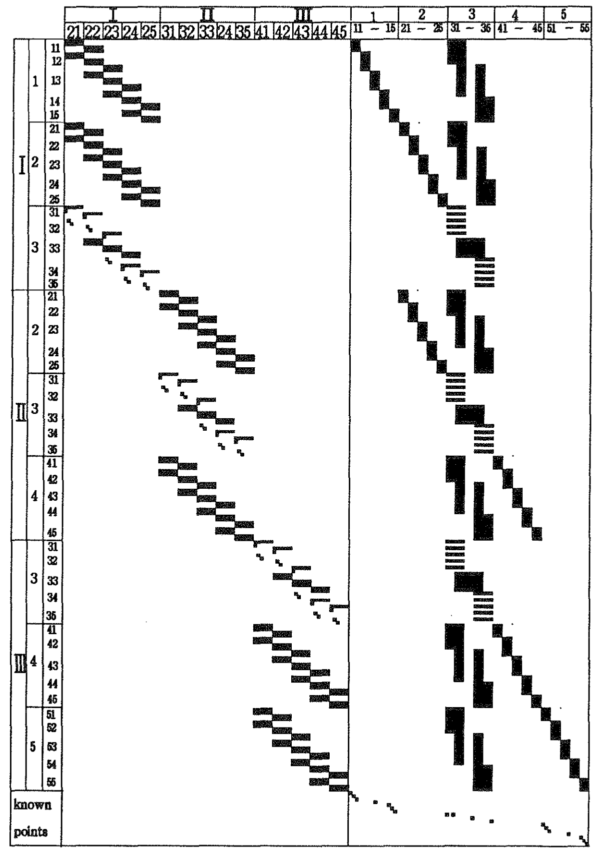

Fig. 8

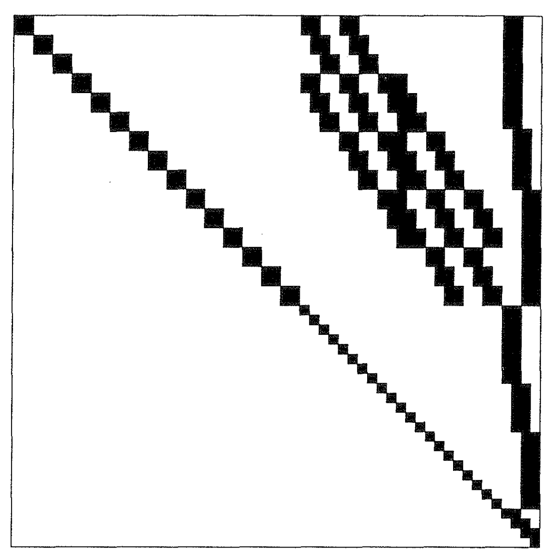

Fig.9

Fig.10 and 11 are the structure of the reduced normal equations in which the photo elements are reduced. In fig. 10, the base points are place at the end of the equation forming a margin of the matrix. This is suitable to the solution of banded banded matrix. In fig.11, the base points are placed just after the parameters which are related to them. Obviously, there is no margin now, and the profile of matrix is less than that of fig.10, but the matrix is not the equal-width banded one. This is suitable to the solusion of inequal-width banded matrix.

Above all, the structure of normal and reduced normal equations of crown method are similar to that of bundle method.

5. CONCLUSION REMARKS

Up to now, a new rigorous photogrammetric adjustment algorithm based on co-angularity condition—Crown Method has been developed. The situation that bundle method monopolizes the rigorous photogrammetric adjustment will be broken along with the spread of the new method.

comparing with bundle method, the new one processes the following properties:

1) The functional modelling is Gauss-Markoff one, so it inherits all advantages of Gauss-markoff modelling.

2) The process of forming observation, normal and reduced normal equations, solving the normal equation, the parameter quantities, the structure of normal equations are all same as or similar to that of bundle method.

3) The condition of normal equation matrix is much better than that of bundle method.

According to the principle of this paper, the rigorous algorithm based on co-planlarity condition can be similarly developed as well(Yianmin Wang, 1992 in press). Therefore, based on three different conditions, we can develop three different rigorous algorithm. Theoritically, the results should be the same, but due to the conditions of normal equation matrix, they will be different in practical adjustment. Which one is the best and most effective? This question need to be replied by comparing in practical adjustment.

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