THE DIFFERENTIAL FORMULAS FOR ANALYTICAL PROCESSING OF PANORAMIC STEREO PHOTOS TAKEN BY FISH-EYE LENS

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Abstract

The rigid mathematical formulas for determining object-point coordinates of the panoramic photographs taken by a Fish-eye lens in stereo close range photogrammetry are deduced. That are general differential formulas and suitable to the modes of normal, parallel-averted, convergent and equal tilt photographs. The experiments are also presented.

KEY WORDS: Fish-eye photograph, Analytical processing, Differential formula.

1. Introduction

The application for panoramic photographs by fish-eye lens in close range photogrammetry has been expounded(cf.[2]). One group of the theoretic formula, in which, is the formula of FDLT(Fish-Direct Linear Transformation) and FDDLT (Fish Distortion Direct Linear Transformation) bearing the correction for additional parameter derived on the basis of DLT formula. Using the formula as stated, the layout can't be less than six control points in photographic object. It is difficulty for some close range photogrammetry to do so before or after the event, such as the processing for all of a sudden accident etc.. The principal advantage for the panoramic photography by fish-eye lens is able to take huge object at very near distance(even at 1-2m),having advantages of large area coverd,high resolution,large negative content and etc..Its photography use a simple and easy stereo close range camera and has a fish-eye lens attached to it. The camera is proceed with some calibration, adjustment or refitting by general camera(e.g.see gull 135). In order to make fish-eye lens applicable to stereo photogrammetry in any state. The rigid mathematical formulas are derived in this paper and the tests are conducted. In this way, the control points can be greatly decreased and they only need to layout according to control requirement of general terrestrial photogrammetry. The experiment cited in this paper uses the stereo-camera made by Shanghai 832 works subordinate to Ministry of Public Security. Its components consists of camera 135, base-bar, linker up with cameras,tripod and so on. Four frame-marker are carved into register plate of camera. There are three specifications of base-bar 40cm, 100cm, 120cm. The focusing installation of camera has been refitted.The fish-eye lens is attached to the lens of camera 135 in the test.The principal distance value f is kept from change during photo process. Carrying on stereo photography,the photographic cases may select at will and the synchronous photography may be performed.

2. General formula of fish-eye panoramic photograph

Proceed from the critical formula of determining ground point coordinate in terrestrial stereo photogrammetry, that is:
The ground point coordinate for left photo of stereo pair is:

$$\left.\begin{aligned}X_G&=X_{S_1}+N_1X'_1=X_{S_1}+N_1[a_1(x_1-x_0)+a_2f_1+a_3(z_1-z_0)]\\Y_G&=Y_{S_1}+N_1Y'_1=Y_{S_1}+N_1[b_1(x_1-x_0)+b_2f_1+b_3(z_1-z_0)]\\Z_G&=Z_{S_1}+N_1Z'_1=Z_{S_1}+N_1[c_1(x_1-x_0)+c_2f_1+c_3(z_1-z_0)]\end{aligned}\right\}\quad(1)$$

The ground point coordinate for right photo of stereo pair is:

$$\left.\begin{aligned}X_G&=X_{S_2}+N_2X'_2=X_{S_2}+N_2[a'_1(x_2-x'_0)+a'_2f_2+a'_3(z_2-z'_0)]\\Y_G&=Y_{S_2}+N_2Y'_2=Y_{S_2}+N_2[b'_1(x_2-x'_0)+b'_2f_2+b'_3(z_2-z'_0)]\\Z_G&=Z_{S_2}+N_2Z'_2=Z_{S_2}+N_2[c'_1(x_2-x'_0)+c'_2f_2+c'_3(z_2-z'_0)]\end{aligned}\right\}\quad(2)$$

where:
N_1 and N_2 are the coefficients of projective magnification; $a_ib_ic_i$ and $a'_ib'_ic'_i$ are direction cosines on left and right photo respectively, it can be determined from the formula in paper[1], $x_0z_0f_1$ and $x'_0z'_0f_2$ are elements of interior orientation on left and right photo respectively, x_1z_1 and x_2z_2 are measuring image coordinates on left and right photo respectively, $X_GY_G Z_G$ is geodetic coordinate of object point, $X_{S_1}Y_{S_1}Z_{S_1}$ and $X_{S_2}Y_{S_2}Z_{S_2}$ are coordinates of photographic station on left and right photo respectively. Eq.(1) and (2)are applied to photograph of central projection, whereas the photograph taken by fish-eye len is spherical central projection[2]:

$$x=f\frac{x'}{\sqrt{f^2-x'^2-z'^2}};\quad z=f\frac{z'}{\sqrt{f^2-x'^2-z'^2}}$$

Substitute into Eq.(1),(2), then get the coordinated formula for fish-eye panoramic photo pairs, the formula for left photo is:

$$\left.\begin{aligned}X_{G_1}&=X_{S_1}+N_1X'_1\\&=X_{S_1}+N'_1\left[a_1\left(f_1\frac{x'_1}{\sqrt{f_1^2-x'^2_1-z'^2_1}}-x_0\right)+\right.\\&\qquad\left.+a_2f_1+a_3\left(f_1\frac{z'_1}{\sqrt{f_1^2-x'^2_1-z'^2_1}}-z_0\right)\right]\\Y_{G_1}&=Y_{S_1}+N_1Y'_1\\&=Y_{S_1}+N'_1\left[b_1\left(f_1\frac{x'_1}{\sqrt{f_1^2-x'^2_1-z'^2_1}}-x_0\right)+\right.\\&\qquad\left.+b_2f_1+b_3\left(f_1\frac{z'_1}{\sqrt{f_1^2-x'^2_1-z'^2_1}}-z_0\right)\right]\\Z_{G_1}&=Z_{S_1}+N_1Z'_1\\&=Z_{S_1}+N'_1\left[c_1\left(f_1\frac{x'_1}{\sqrt{f_1^2-x'^2_1-z'^2_1}}-x_0\right)+\right.\\&\qquad\left.+c_2f_1+c_3\left(f_1\frac{z'_1}{\sqrt{f_1^2-x'^2_1-z'^2_1}}-z_0\right)\right]\end{aligned}\right\}\quad(3)$$

the coordinated formula on right photo of fish-eye panoramic photo pairs:

$$\begin{aligned}X_{G_2}&=X_{S_2}+N_2X'_2=X_{S_2}N'_2\left[a'_2\left(f_2\frac{x'_2}{\sqrt{f_2^2-x'^2_2-z'^2_2}}-x'_0\right)+\right.\\&\qquad\left.+a'_2f_2+a'_3\left(f_2\frac{z'_2}{\sqrt{f_2^2-x'^2_2-z'^2_2}}-z'_0\right)\right]\end{aligned}$$

$$Y_{G_2}=Y_{s_2}+N_2Y_2'=Y_{s_2}+N_2'\Big[b_1'\Big(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0'\Big)+ +b_2'f_2+b_3'\Big(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0'\Big)\Big]$$

$$Z_{G_2}=Z_{s_2}+N_2Z_2'=Z_{s_2}+N_2'\Big[c_1'\Big(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0'\Big)+ +c_2'f_2+c_3'\Big(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0'\Big)\Big] \quad (4)$$

where: $x_1'z_1'$ and $x_2'z_2'$ are image coordinates measuring on left and right photo respectively for fish eye panoramic photo pairs. Considering following formula:

$$N_1=\frac{B_XY_2'-B_YX_2'}{X_1'Y_2'-X_2'Y_1'}=\frac{B_1^*}{P^*};\quad N_2=\frac{B_XY_1'-B_YX_1'}{X_1'Y_2'-Y_2Y_1'}=\frac{B_2^*}{P^*}$$

$$B_X=B\cdot\sin A;\quad B_Y=B\cos A$$

where: A is directional angle of base B.
Differential all variations of Eq.(3),(4), we can get following Eq.(5)

$$\begin{aligned}dX_G=\;&dX_{s_1}+X_1'\frac{Y_2'}{P^*}dB_X-X_1'\frac{X_2'}{P^*}dB_Y+X_1'\frac{B}{P^*}(Y_2'\cos A+X_2'\sin A)dA-\\&-N_1X_2'\frac{(X_1')^2+(Y_1')^2}{P^*}d\varphi_1+N_2X_1'\frac{(X_2')^2+(Y_2')^2}{P^*}d\varphi_2-N_1X_2'G_1d\omega_1\\&+N_2X_1'G_2d\omega_2-N_1X_2'\frac{X_1^*Y_1'-Y_1^*X_1'}{P^*}dk_1+N_2X_1'\frac{X_2^*Y_2'-Y_2^*X_2'}{P^*}dk_2\\&+N_1X_2'D_1df_1-N_2X_1'D_2df_2-N_1X_2'E_1dx_0+N_2X_1'E_2dx_0'-\\&-N_1X_2'F_1dz_0+N_2X_1'F_2dz_0'+N_1X_2E_1dx_1'-N_2X_1'E_2dx_2'+\\&+N_1X_2'F_1dz_1'-N_2X_1'F_2dz_2'\end{aligned}$$

$$\begin{aligned}dY_G=\;&dY_{s_1}+Y_1'\frac{Y_2'}{P^*}dB_X-Y_1'\frac{X_2'}{P^*}dB_Y+Y_1'\frac{B}{P^*}(Y_2'\cos A+X_2'\sin A)dA-\\&-N_1Y_2'\frac{(X_1')^2+(Y_1')^2}{P^*}d\varphi_1+N_2Y_1'\frac{(X_2')^2+(Y_2')^2}{P^*}d\varphi_2-N_1Y_2'G_1d\omega\\&+N_2Y_1'G_2d\omega_2-N_1Y_2'\frac{X_1^*Y_1'-Y_1^*X_1'}{P^*}dk_1+N_2Y_1'\frac{X_2^*Y_2'-Y_2^*X_2'}{P^*}dk_2\\&+N_1Y_2'D_1df_1-N_2Y_1'D_2df_2-N_1Y_2'E_1dx_0+N_2Y_1'E_2dx_0'-N_1Y_2'F_1dz_0+\\&+N_2Y_1'F_2dz_0'+N_1Y_2'E_1dx_1'-N_2Y_1'E_2dx_2'+N_1Y_2'F_1dz_1'-N_2Y_1'F_2dz_2'\end{aligned}$$

$$\begin{aligned}dZ_G=\;&dZ_{s_1}+Z_1'\frac{Y_2'}{P^*}dB_X-Z_1'\frac{X_2'}{P^*}dB_Y+Z_1'\frac{B}{P^*}(Y_2'\cos A+X_2'\sin A)dA-\\&-N_1Z_1'\Big(\frac{X_1'X_2'+Y_1'Y_2'}{P^*}\Big)d\varphi_1+N_2Z_1'\frac{(X_2')^2+(Y_2')^2}{P^*}d\varphi_2+N_1(f_1^*-\\&-Z_1'G_1)d\omega_1+N_2Z_1'G_2d\omega_2+\frac{N_1}{P^*}[Z_1^*P^*-Z_1'(X_1^*Y_2'-Y_1^*X_2')]dk_1+\\&+N_2Z_1'\frac{X_2^*Y_2'-Y_2^*X_2'}{P^*}dk_2+\frac{N_1}{P^*}[c_2P^*+Z_1'(b_2X_2'-a_2Y_2')]df_1\\&-N_2Z_1'D_2df_2-\frac{N_1}{P^*}[c_1P^*+Z_1'(b_1X_2'-a_1Y_2')]\,dx_0+N_2Z_1'E_2dx_0'\\&-\frac{N_1}{P^*}[c_3P^*+Z_1'(b_3X_2'-a_3Y_2')]dz_0+N_2Z_1'F_2dz_0'+\frac{N_1}{P^*}[c_1P^*\\&+Z_1'(b_1X_2'-a_1Y_2')]\,dx_1'-N_2Z_1'E_2dx_2'+\frac{N_1}{P^*}[c_3P^*\\&+Z_1'(b_3X_2'-a_3Y_2')]dz_1'-N_2Z_1'F_2dz_2'\end{aligned}\quad (5)$$

where:

$$G_1=\frac{Z_1'}{P^*}(X_1\cos\varphi_1-Y_1'\sin\varphi_1);\quad G_2=\frac{Z_2'}{P^*}(X_2'\cos\varphi_2-Y_2'\sin\varphi_2);$$

$$X_1^*=a_3(x_1-x_0)-a_1(z_1-z_0);\quad X_2^*=a_3'(x_2-x_0')-a_1'(z_2-z_0');$$

$$Y_1^*=b_3(x_1-x_0)-b_1(z_1-z_0);\quad Y_2^*=b_3'(x_2-x_0')-b_1'(z_2-z_0');$$

$$Z_1^*=c_3(x_1-x_0)-c_1(z_1-z_0);$$

$$f_1^*=f_1\cos\omega_1-\sin\omega_1[(z_1-z_0)\cos k_1+(x_1-x_0)\sin k_1];$$

$$D_1=\Big(\frac{b_2X_1'-a_2Y_1'}{P^*}\Big);\ D_2=\Big(\frac{b_2'X_2'-a_2'Y_2'}{P^*}\Big);\ E_1=\Big(\frac{b_1X_1'-a_1Y_1'}{P^*}\Big);$$

$$E_2=\Big(\frac{b_1'X_2'-a_1'Y_2'}{P^*}\Big);\ F_1=\Big(\frac{b_3X_1'-a_3Y_1'}{P^*}\Big);\ F_2=\Big(\frac{b_3'X_2'-a_3'Y_2'}{P^*}\Big)$$

From Eq.(5),we can calculate the estimation of accuracy for coordinates of undetermined point and extract the permissive value of elements of interior and exterior orientation. The formula is also the general equation of fish-eye panoramic photograph,that is, applied to close range photogrammetry of arbitrary photographic case. The following sections are discussed respectively the mathematical formula for fish-eye panoramic stereo photogrammetry of normal, equally tilted, parallel-averted and convergent photography.

3. The formula of fish-eye panoramic photo pair at normal photography

At normal photography, two photographic optical axes of fish-eye photograph pair are horizontal, parallel each other and perpendicular to direction of photographic-base. The rotation matrix in this case is identity matrix. But the normal photo in reality still remains with elemental slender error of exterior orientation $d\varphi_1$, $d\varphi_2$, $d\omega_1$, $d\omega_2$, dk_1, dk_2. When the case of small angle existes, the approximation of the small value first order is applied. Thus the rotation matrix is:

$$R_1=R_2=R\approx\begin{bmatrix}1 & d\varphi & -dk\\ -d\varphi & 1 & -d\omega\\ -dk & d\omega & 1\end{bmatrix}\quad (6)$$

Substituting R_1, R_2 into Eqs. (3),(4) respectively and then get Eqs. (7):

$$\begin{aligned}X_{G_1}&=X_{s_1}+N_1X_1'=X_{s_1}+N_1\Big[\Big(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\Big)+\\&\quad+f_1d\varphi_1-\Big(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\Big)dk_1\Big]\\Y_{G_1}&=Y_{s_1}+N_1Y_1'=Y_{s_1}+N_1\Big[-\Big(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\Big)d\varphi_1+\\&\quad+f_1-\Big(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\Big)d\omega_1\Big]\\Z_{G_1}&=Z_{s_1}+N_1Z_1'=Z_{s_1}+N_1\Big[\Big(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\Big)dk_1+\\&\quad+f_1d\omega_1+\Big(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\Big)\Big]\\X_{G_2}&=X_{s_2}+N_2X_2'=X_{s_2}+N_2\Big[\Big(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\Big)+\\&\quad+f_2d\varphi_2-\Big(f_2\frac{z_2'}{\sqrt{f_2'^2-x_2'^2-z_2'^2}}-z_0\Big)dk_2\Big]\\Y_{G_2}&=Y_{s_2}+N_2Y_2'=Y_{s_2}+N_2\Big[-\Big(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\Big)d\varphi_2+\\&\quad+f_2-\Big(f_2\frac{z_2'}{\sqrt{f_2'^2-x_2'^2-z_2'^2}}-z_0\Big)d\omega_2\Big]\\Z_{G_2}&=Z_{s_2}+N_2Z_2'=Z_{s_2}+N_2\Big[\Big(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\Big)dk_2+\\&\quad+f_2d\omega_2+\Big(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\Big)\Big]\end{aligned}\quad (7)$$

4. The formula for fish-eye panoramic photograph when equally tilted photography.

Photography with equal tilt is that both two optical axes of camera for fish-eye lens are relative to horizontal direction to tilting jointly a same angle, right now: $\varphi_1=\varphi_2=k_1=k_2=0$, $\omega_1=\omega_2\doteq 0$. But in parctical operation, the value of small angle is existence throughout $d\varphi_1, d\varphi_2, dk_1$ and dk_2, therefore the rotation matrix of directional cosine R_1, R_2 has the following form:

$$R_1=R_2=R=\begin{bmatrix} 1 & d\varphi\cos\omega & -(dk+\sin\omega d\varphi) \\ -(d\varphi+\sin\omega dk) & \cos\omega & -\sin\omega \\ \cos\omega dk & \sin\omega & \cos\omega \end{bmatrix} \approx \begin{bmatrix} 1 & d\varphi & -dk \\ -d\varphi & \cos\omega & -\sin\omega \\ dk & \sin\omega & \cos\omega \end{bmatrix} \tag{8}$$

According to Taylor's formula to expand Eqs.(8), the first order terms are taken and substitute into Eqs.(3),(4), then get the formula of equally tilted photography (9):

$$\begin{aligned}
X_{G1}&=X_{s1}+N_1X_1'=X_{s1}+N_1\left[\left(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\right)+\right.\\
&\left.+f_1d\varphi_1-\left(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\right)dk_1\right]\\
Y_{G1}&=Y_{s1}+N_1Y_1'=Y_{s1}+N_1\left[-\left(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\right)d\varphi_1+\right.\\
&\left.+f_1\cos\omega_1-\left(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\right)\sin\omega_1\right]\\
Z_{G1}&=Z_{s1}+N_1Z_1'=Z_{s1}+N_1\left[\left(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\right)dk_1+\right.\\
&\left.+f_1\sin\omega_1+\left(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\right)\cos\omega_1\right]\\
X_{G2}&=X_{s2}+N_2X_2'=X_{s2}+N_2\left[\left(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\right)+\right.\\
&\left.+f_2d\varphi_2-\left(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\right)dk_2\right]\\
Y_{G2}&=Y_{s2}+N_2Y_2'=Y_{s2}+N_2\left[-\left(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\right)d\varphi_2+\right.\\
&\left.+f_2\cos\omega_2-\left(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\right)\sin\omega_2\right]\\
Z_{G1}&=Z_{s2}+N_2Z_2'=Z_{s2}+N_2\left[\left(f_1\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\right)dk_2+\right.\\
&\left.+f_2\sin\omega_2+\left(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\right)\cos\omega_2\right]
\end{aligned}\tag{9}$$

5. The formula of fish-eye panoramic photo pair in parallel-averted photography

Parallel-averted photography is that both two photographic optical axes of fish-eye photo pair are horizontal, perpendicular to the direction of photographic-baseline and avert a certain angle φ_0 this time, $\varphi_1=\varphi_2=\varphi\doteq 0, \omega_1=\omega_2=k_1=k_2=0$, but in reality, $\omega_1=d\omega_1$, $\omega_2=d\omega_2$, $k_1=dk_1$, $k_2=dk_2$, so that the rotation matrix form of directional cosine follows:

$$R_1=R_2=R=\begin{bmatrix} \cos\varphi & \sin\varphi & -(\cos\varphi dk+\sin\varphi d\omega) \\ -\sin\varphi & \cos\varphi & \sin\varphi dk-\cos\varphi d\omega \\ dk & d\omega & 1 \end{bmatrix} \approx \begin{bmatrix} \cos\varphi & \sin\varphi & -dk \\ -\sin\varphi & \cos\varphi & d\omega \\ dk & d\omega & 1 \end{bmatrix} \tag{10}$$

The first order term is still taken when derivated Eqs.(10). Substituting Eqs.(10) in Eqs.(3),(4), we can obtain the formula of parallel-averted photography, the derived meathod is the same as before and it is omitted here.

6. The formula of fish-eye panoramic photograph under convergent photography

The convergent photography is that both two photographic optical axes of fish-eye photograph pair are horizontal and rotated over a certain convergent angle γ.

According to reason, the angle should be: $\omega_1=k_1=\varphi_2=\omega_2=k_2=0$, $\varphi_1=\gamma$, but in practice it always retains error. $\omega_1\doteq d\omega_1$, $k_1=dk_1$, $\varphi_2=d\varphi_2$, $\omega_2=d\omega_2$, $k_2=dk_2$, $\varphi_1=\gamma$. So the formula of relative to Eqs.(10),(6) follows:

$$R_1\approx\begin{bmatrix} \cos\gamma & \sin\gamma & -dk_1 \\ -\sin\gamma & \cos\gamma & -d\omega_1 \\ dk_1 & d\omega_1 & 1 \end{bmatrix} \qquad R_2\approx\begin{bmatrix} 1 & d\varphi_2 & -dk_2 \\ -d\varphi_2 & 1 & -d\omega_2 \\ dk_2 & d\omega_2 & 1 \end{bmatrix}$$

Substituting R_1 R_2 in Eqs.(3),(4) respectively, then we can obtain Eqs.(11)

$$\begin{aligned}
X_{G1}&=X_{s1}+N_1X_1'=X_{s1}+N_1\left[\left(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\right)\cos\gamma+\right.\\
&\left.+f_1\sin\gamma-\left(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\right)dk_1\right]\\
Y_{G1}&=Y_{s1}+N_1Y_1'=Y_{s1}+N_1\left[-\left(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\right)\sin\gamma+\right.\\
&\left.+f_1\cos\gamma-\left(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\right)d\omega_1\right]\\
Z_{G1}&=Z_{s1}+N_1Z_1'=Z_{s1}+N_1\left[\left(f_1\frac{x_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-x_0\right)dk_1+\right.\\
&\left.+f_1d\omega_1+\left(f_1\frac{z_1'}{\sqrt{f_1^2-x_1'^2-z_1'^2}}-z_0\right)\right]\\
X_{G2}&=X_{s2}+N_2X_2'=X_{s2}+N_2\left[\left(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\right)+f_2d\varphi_2-\right.\\
&\left.-\left(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\right)dk_2\right]\\
Y_{G2}&=Y_{s2}+N_2Y_2'=Y_{s2}+N_2\left[-\left(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\right)d\varphi_2+\right.\\
&\left.+f_2-\left(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\right)d\omega_2\right]\\
Z_{G2}&=Z_{s2}+N_2Z_2'=Z_{s2}+N_2\left[\left(f_2\frac{x_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-x_0\right)dk_2+\right.\\
&\left.+f_2d\omega_2+\left(f_2\frac{z_2'}{\sqrt{f_2^2-x_2'^2-z_2'^2}}-z_0\right)\right]
\end{aligned}\tag{11}$$

After differential Eqs.(7)-(11) and deriving the local derivative of all coefficients, we can obtain the differential formula applied to the photographs of normal, parallel-averted, equal tilt and convergent. According to differential formula, we can calculate the valuation of coordinated accuracy to be located point of all photographic case and derive permiting value for elemental error of inner and exterior orientation, by the way, the meathod derived in this paper is also applied to aerophotogrammetry and space photogrammetry.

7. Experiment

The indoor and outdoor experiments are performed in order to examine that the theoretic formula of stereo photogrammetry carring on a variety of photographic cases using fish-eye lens is correct or not and has the possibility for using it. The stereo cameras developed by Shanghai 832 works subordinate to ministry of public security are adopted in this test. The focuses of fish-eye lens as 16mm are substitute for original lens. Since Oct.1989, aiming at the markers made by metal in the three dimensional indoor appraisal field are successively carried on photography of the photographic cases in normal and convergent.Its photographic distances Y used are about 2.2m, more then 60 marking points are laid out at corners of outdoor building.The three dimensional coordinates of marking points are determined by operating the methods of forward intersection and multi-angular height measurement using theodolite J6.The photo pairs of equally tilted and parallel-averted are taken using fish-eye lens. Its photographic distances Y used are about 7.5m and the photographic-baseline B=120cm. The accuracy comparison has been performed using the computed results of the formula mentioned above. For the sake of obtaining practical possibility and more experimental data, the normal photography is carried on for the car of analogizing traffic accident and bicycles collided on the spot. Theirs photographic-baseline are respectively: B=120cm and B=40cm, photographic distance Y=6.5m and Y=3.0m, meanwhile, the distances concerned are measured by steel rule on the spot and has been compared with the distances obtained by photogrammetric method.Image coordinates are measured by stereo comparator HCZ-1 made in China. More than 160 image points are collected in all in this test. According to the mathematical forlulas mentioned above, the Basic language has been programmed and has been run at IBM-PC/XT. All steps of running operation use Chinese promptings for using it conveniently. Photographic parameters and angular elements of exterior orientation are listed in Tab.1.

Tab.1

photo pair No.	name	φ_1	ω_1	k_1	φ_2	ω_2	k_2	photo-distance Y(m)	photo-base B(cm)
1	indoor appraisal field	0	0	0	0	0	0	2.2	120
2	indoor appraisal field	30.30°	0	0	0	0	0	2.2	120
3	outdoor marking	20.30°	0	0	20.30°	0	0	7.5	120
4	outdoor marking	0	+10.30°	0	0	+10.30°	0	7.5	120
5	car and bicycle	0	0	0	0	0	0	6.5	120
6	car and bicycle	0	0	0	0	0	0	3.0	40

The accuracy estimation was taken the pointing coordinates measured by theodolite as true value and was compared with the results solved by corresponding formula above mentioned using fish-eye photographic method and the comparative difference is obtained. The mean square error was calculated by $m=\pm\sqrt{\frac{[\Delta\Delta]}{n}}$, see Tab 2

Tab.2

pair No.	quantity of checking point	m_X (mm)	m_Y (mm)	m_Z (mm)	Remarks
1	39	1.1	1.5	0.7	
2	42	1.3	1.9	1.4	
3	45	3.1	5.4	3.1	
4	45	3.5	5.2	3.2	
5	12 sides	m_S = 9.7mm	ms is the mean square error of length of line segments		
6	15 sides	m_S = 14.3mm			

The relative error of relative to photographic distance is $m_Y/Y\approx 1/1400$.Through the theoretical deriving and proving and experiments, the conclusion can be reached as follows: The differential formulas derived in this paper applying to all photographic cases using fish-eye lens are exact and precise. From Eqs.(5), the accuracy of undetermined points coordinates and elemental errors of exterior orientation can be estimated. Proceeding with all sorts of engineering photogrammetry using fish-eye lens is efficient. It has not any restriction for condition and field and can take huge object at near distance under assuring accuracy.

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