# SPECTRAL ANALYSIS APPLICATIONS IN DIGITAL IMAGE PROCESSING

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## ABSTRACT

Digital image filtering for restoration and enhancement is often advantageously carried out in the frequency domain. Estimates of two-dimensional spectra are usually required in the digital image spectral analysis. Among the various methods of two-dimensional spectrum estimation, the maximum entropy approach has advantageous characteristics in spite of the computational complexity. One iterative approach which utilizes fast Fourier transforms (FFTs) with adaptive support region and relaxation parameters using the entropy rate has been studied in comparison to conventional periodogram and parametric approaches. The principal features of the method are discussed along with practical filtering results using remote sensing and other similar imagery.

## 1. INTRODUCTION

Digital imagery, especially from Remote Sensing (RS) sensors, has long been recognized as a natural and cost effective source of data for Geographical and more generally Spatial Information Systems (GIS/SIS). Essentially, imagery data can provide a wealth of spectral information with a range of spatial and temporal resolutions. Advances in digital image processing and visualization offer new possibilities to analyze and model image distortions for restoration and enhancement purposes in order to facilitate the information extraction and processing.

Digital image processing for restoration and enhancement can be performed in both spatial and spectral domains or more specifically in terms of the grey values or the frequency components. For systematic noise filtering, known or modeled signal degradation and various other image processing purposes, spectral domain processing has definite advantages over the spatial domain computations.

In general, the image generation process can be simply described using the following linear model:

$$g(x,y) = h(x,y)*f(x,y) + n(x,y)$$

where f(x,y) denotes the ideal or true image function, g(x,y) is the corresponding degraded image function, h(x,y) is the point spread function of the imaging system and n(x,y) is the noise function for the image coordinates x and y. In the spectral domain, the preceding convolution equation becomes a product equation

$$G(u,v) = H(u,v)\cdot F(u,v) + N(u,v)$$

where G(u,v), F(u,v), H(u,v) and N(u,v) are the respective Fourier transforms of g(x,y), f(x,y), h(x,y) and n(x,y). It is worth noting that the linear modeling is often a simplification of the actual physical situation which simply reflects the availability of practical solutions.
The problems of filtering and restoration of digital images are often analyzed with filters designed using first and second statistical moment information. Sample autocovariance information needs to be extended to estimate the power spectrum. Such two-dimensional extensions are greatly more complicated than their one-dimensional equivalent as well known implications of factorization and interpolation difficulties in two and higher dimensions.

For least-squares restoration of a digital image, the restoration function is

$$M(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + [S_n(u,v) / S_f(u,v)]}$$

which is usually called the Wiener-Helstrom filter, where $S_f(u,v)$ denotes the spectrum of the ideal or true image and $S_n(u,v)$ is the noise spectrum [e.g., Castleman, 1979]. In the absence of noise, this filter reduces to an ideal inverse filter with $M(u,v) = 1 / H(u,v)$.

Another common approach to image deblurring and restoration is with the power spectrum equalization (PSE) filter for which the restoration function has the form:

$$M(u,v) = \left( \frac{1}{|H(u,v)|^2 + [S_n(u,v) / S_f(u,v)]} \right)^{1/2}$$

using the same notation as in the previous case [e.g., Castleman, 1979]. This linear transform filter is designed with the constraint that the spectrum of the restored image be equal to the spectrum of the ideal or true image, i.e. $S_f(u,v)$. This filter also reduces to the inverse filter in the absence of noise and has high frequency characteristics.

The design and implementation of the preceding and other similar filters for digital image processing require the estimation and analysis of the spectrum for the degraded images as well as the corrected images. In practice, information about the degradation processes affecting the digital images is often limited and hence, the use of spectral analysis becomes all the more important.

## 2. TWO-DIMENSIONAL SPECTRUM ESTIMATION

The estimation of the spectrum of a digital image is relatively complex in practice because of the usual spatial sampling, quantization and other limitations with images from processes which are not always stationary. Different frequency bands may also have very different signal-to-noise ratios and distortion characteristics which can imply complications with certain spectrum estimation approaches. Furthermore, the image degradation processes are rarely well specified and easy to model so that the estimation of the spectrum of degraded images tends to present difficulties of interpretation and analysis.

Fourier based methods are perhaps best known in terms of periodogram and correlogram approaches to spectrum estimation. In one dimension, they are well known for their lack of resolution which may be critical for spectral analysis of signal dominated imagery. The implied periodicities of the Fourier transforms have more serious implications in two dimensions and the resolution problems are also serious for interpretation purposes.

Parametric methods tend to be very popular for their resolution implications especially in one dimension. Autoregressive, moving average and autoregressive-moving-average modeling offer wide possibilities for empirical processes of all types. In two and higher dimensions, parametric methods are more complicated to formulate and justify in terms of the processes. Considerations of causality, realizability and other related questions can be very difficult to analyze in digital image processing applications.

The fundamental problem of spectrum estimation is the extension of the sample autocovariance sequence as the power spectrum is defined as the Fourier transform of the autocovariance function. With Fourier based techniques, an implied periodicity in the sample autocovariance function outside of the sampling domain is hardly justifiable in rigorous terms for most applications. With parametric methods, the modeling implies the extension of the sample autocovariance function outside of the sampling domain. Both of these implied assumptions are difficult to justify and tend to impose constraints on the spectrum estimation [e.g., Pendrell, 1979].

The maximum information or entropy approach seeks to formulate the estimation problem in such a way that no unaccounted supposition or assumption is included in the estimation procedures. In one dimension, it is well known to be in full agreement with the parametric autoregressive method of extending the sample autocovariance function. In two and higher dimensions, the situation is generally quite different because of the non-unicity of the parametric formulation and other complications related to the autocovariance function.

## 3. MAXIMUM ENTROPY APPROACH

Numerous researchers have investigated the maximum entropy approach to spectrum estimation in two dimensions. Among them are Burg [1975], Pendrell [1979], Wernecke and D'Addario [1977], Lim and Malik [1981] and Lang and McClellan [1982]. Woods [1976] has given two theorems which state that only the positive definiteness of the autocovariance functions needs to be checked in order to guarantee the existence and uniqueness of the power spectrum estimates regardless of the support region.

The approach of Lim and Malik [1981] which is based on the use of fast Fourier transforms (FFTs) has been further investigated and modified in terms of an adaptive support region and relaxation parameters to accelerate the convergence [Blais and Zhou, 1990]. Other research into the maximum entropy approach has led to a better understanding of the general situation and the difficulties in implementing the general approach of Lim and Malik [1981]. Related results and discussions of the maximum entropy approach to spectrum estimation can also be found in Blais [1992].

Given an estimated sample autocovariance function $C_z(k\Delta x, l\Delta y)$, k=0, ... , K, l=0, ... , L, with corresponding grid spacings $\Delta x$ and $\Delta y$, with the usual symmetry assumption

$$C_z(-k\Delta x,-l\Delta y) = C^*_z(k\Delta x,l\Delta y) \quad \text{for } k=0, \pm1, \pm2, \dots \text{ and } l=0, \pm1, \pm2, \dots$$

which is normally implied by the observational data (the complex conjugate may not be required with real digital image data), the required power spectrum is

$$S_z(u,v) = \mathbf{F}[C_z(k\Delta x,l\Delta y)]$$

with

$$-(2\Delta u)^{-1} \leq u < (2\Delta u)^{-1} \quad \text{and} \quad -(2\Delta v)^{-1} \leq v < (2\Delta v)^{-1},$$

that is, their respective Nyquist frequencies, where F[ ] denotes the Fourier transform. As the sample autocovariance $C_z(k\Delta x,l\Delta y)$ is only known for $|k| \leq K$ and $|l| \leq L$, the problem is to extend this sample autocovariance function for $k\Delta x$ and $l\Delta y$ with $|k| > K$ and $|l| > L$. In one dimension, this extension is readily achieved with an autoregressive model but in two and higher dimensions, the difficulties are more serious and the following discussion covers a very appropriate strategy for digital image applications.

The maximum entropy approach implies the existence of a dual autocovariance function $D_z(k\Delta x,l\Delta y)$ defined by

$$D_z(k\Delta x,l\Delta y) = \mathbf{F}^{-1}[1/\mathbf{F}[C_z(k\Delta x,l\Delta y)]\,]$$

with the properties that

$$D_z(k\Delta x,l\Delta y) = 0 \quad \text{for all } |k| > K \text{ and } |l| > L$$

and

$$D_z(-k\Delta x,-l\Delta y) = D^*_z(k\Delta x,l\Delta y)$$

for k=0, ±1, ±2, ... and l=0, ±1, ±2, ... (see Blais [1992] for details). The known sample autocovariance function values

$$C_z(k\Delta x,l\Delta y)=C^*_z(-k\Delta x,-l\Delta y),\quad k=0,\pm1,\dots,\pm K,\quad l=0,\pm1,\dots,\pm L,$$

and known dual autocovariance function values

$$D_z(k\Delta x,l\Delta y) = 0 \quad \text{for all } |k| > K \text{ and } |l| > L$$

provide a straightforward procedure for extending the sample autocovariance function. The approach of Lim and Malik [1981] consists in recursively using the following three equations:

$$S_z(u,v) = \mathbf{F}[1/D_z(k\Delta x,l\Delta y)]$$

$$C_z(k\Delta x,l\Delta y) = \mathbf{F}^{-1}[S_z(u,v)]$$

$$D_z(k\Delta x,l\Delta y) = \mathbf{F}^{-1}[1/\mathbf{F}[C_z(k\Delta x,l\Delta y)]\,]$$

This algorithm is initialized with values for $D_z(k\Delta x,l\Delta y)$ and then iterates over the three equations until convergence or termination in problem situations such as in cases of high SNR. Discussions of this algorithm can also be found in McClellan [1982] and Blais and Zhou [1990].

The preceding procedure for some estimated $C_z(k\Delta x,l\Delta y)$ values for $|k| \leq K$ and $|l| \leq L$ leads to a number of questions for practical implementation purposes:

(a) The difficulties in the estimation of the sample autocovariance function.

(b) The support region for $C_z(k\Delta x,l\Delta y)$ is not always obvious especially when the image characteristics are variable.

(c) The initial values for $D_z(k\Delta x,l\Delta y)$ have to be selected properly for convergence of the algorithm and reliability of the spectrum estimates.

(d) In cases of high SNR and in other cases of slow convergence, are there any possibilities of accelerating the convergence?

The following section will describe the results of some investigations into those implementation questions and some modifications to the original algorithm of Lim and Malik [1981].

## 4. PRACTICAL CONSIDERATIONS

The initialization question for the algorithm was first investigated. In the original publication of Lim and Malik [1981], the initialization of $D_z(k\Delta y,l\Delta y)$ was suggested as

$$D_z(k\Delta y,l\Delta y) \;=\; 1/C_z(0,0) \quad \text{at } k=l=0,$$
$$\qquad\qquad\;=\; 0 \quad \text{elsewhere},$$

but it was found that in cases of slow convergence due to a high SNR, the addition of some low level noise to the initial $C_Z(k\Delta y, l\Delta y)$ improves the convergence without affecting the estimate of the spectrum. Such addition of low level noise to the autocovariance function $C_Z(k\Delta y, l\Delta y)$ can be justified in different ways but it can be regarded as a regularization strategy for ill-conditioned applications, which is well known with ill-posed inverse problems.

The characteristics of the autocovariance function $C_Z(k\Delta y, l\Delta y)$ are obviously critical for the estimation of the corresponding spectrum. In practice, given some digital image data, the autocovariance function has to be estimated and in strictly stationary situations, the larger the data set, the more reliable the estimated autocovariance function. However, in cases of questionable stationarity, appropriately shaped and sized data windows have to be selected to arrive at reliable results.

The shape and size of the support region for the estimated autocovariance function are closely related to the characteristics of the autocovariance function and the lengths of the fast Fourier transforms (FFTs) [Kashyap and Chellappa, 1983]. In cases of highly correlated data sequences, the corresponding autocorrelation function decreases very slowly with lag and hence larger support regions and larger FFTs are required for proper estimation of the spectrum. With poorly correlated data sequences, small support regions are acceptable for good estimates of the spectrum. However, as these questions of optimal support regions are related to the SNR of the data sequence, the FFT length requirements and the convergence rates of the algorithm, further investigations into these problems are clearly warranted.

The algorithm involves step parameters which control the convergence rate of the algorithm and guarantee the positive definiteness of the extended autocovariance function. As the conditions are likely to change over a number of iterations, these parameters need to be adaptive to optimize the convergence of the procedure. As suggested in McClellan [1982], these parameters need to be adjusted so as to maintain the direction of steepest descent, which is a well known strategy in nonlinear optimization.

## 5. ANALYSIS OF EXPERIMENTAL RESULTS

A number of simulated data sets have been generated with different characteristics and the corresponding spectra have been estimated and analyzed. Limited experimentation has also been carried out with Landsat TM remote sensing imagery. One selected Landsat TM band one test image is a 256x256 pixels of a Calgary scene. The results of estimating the spectrum of samples of this test image are shown in Figures 1 to 4 with explanations in the following paragraphs.

The modified algorithm of Lim and Malik [1981] was experimentally used in different ways for comparison analysis purposes. The assumed stationarity of the test image was verified in subimages of size 64x64 pixels. Using the entropy of the grey level frequencies as an indicator of the information content in the subimages, the subimage with maximum entropy was selected for the sample autocovariance function computations. One interpretation of this procedure is that the most informative sample subimage was selected for spectral analysis. Figures 1 and 2 give the estimated spectra for the 256x256 and 64x64 pixel images, respectively, using a 5x5 pixel support region and a two percent noise variance added to the zero lag autocovariances. The similarities in these estimated spectra are quite pronounced.

The shape and size of the support region for the autocovariance function are well known to be important in spectrum estimation. With the TM test image and subimages, different shapes and sizes of support regions were experimented with. One important consideration with TM digital images is that due to the scanning procedures in the acquisition process, they tend to have higher correlation characteristics in the scanning direction than in the flight direction. It therefore follows that a rectangular support region with a larger dimension in the scanning direction should be more appropriate for spectrum estimation. Figure 3 shows the spectrum estimation results with a 5x7 support region used in comparison with the results in Figure 2 with a 5x5 support region. Some improvements in the scanning direction can be seen in the spectrum estimates.

The added noise to the zero lag value of the sample autocovariance function has been seen to accelerate the convergence of the algorithm without apparently altering the spectrum estimate results. Figure 4 shows the results corresponding to Figure 2 but without the addition of low level noise. In the latter case, some 40 iterations were required while in the former case, only 25 iterations were required for the same level of convergence. This computational improvement is obviously dependent on the SNR in the test image and cannot be generalized without taking the digital image characteristics into considerations.

Other aspects of spectrum estimation with this algorithm for digital image applications are still under investigation. Comparisons with Fourier based and parametric methods have confirmed the appropriateness of the maximum entropy approach although the computational aspects are more complex and the interpretation of the results are not always straightforward even with apparently simple digital images.

## 6. CONCLUDING REMARKS

The use of the maximum entropy approach to spectrum estimation in digital image processing has been confirmed in practice with a modified version of the Lim and Malik [1981] algorithm. Various aspects of the algorithm have been investigated and some variations in its implementation have shown some improvements over the original version. Further considerations of the dual autocovariance function are still under investigation to improve the convergence characteristics of this general approach to maximum entropy spectrum estimation.

Digital image processing for filtering, restoration and enhancement purposes can greatly benefit from more accurate, reliable and consistent spectrum estimates. The design of filters and quality control procedures generally require spectral information about the digital images. Considering the usual simplifications of stationarity and ergodicity in the sampling, more adaptive methods of spectrum estimation are definitely required for numerous applications.

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## REFERENCES

Blais, J.A.R., 1992. Spectrum Estimation Using Maximum Entropy. Manuscript under preparation, Department of Surveying Engineering, The University of Calgary.

Blais, J.A.R. and W. Zhou, 1990. Spectral Analysis of Digital Images for Filtering and Restoration. Proceedings of the ISPRS Commission III Symposium in Wuhan, P.R. China, Volume 28, Part 3/2, pp.48-63.

Burg, J.P., 1975. Maximum Entropy Spectral Analysis. Ph.D. Dissertation, Stanford University, Stanford.

Castleman, K.R., 1979. Digital Image Processing. Prentice Hall.

Kashyap, R.L. and R. Chellappa, 1983. Estimation and Choice of Neighbours in Spatial Interaction Models of Images. IEEE Trans. Information Theory, Vol. IT-29, pp.60-72.

Lang, S.W. and J.H. McClellan, 1982. Multi-Dimensional MEM Spectral Estimation. IEEE Trans. Acoust., Speech and Signal Processing, ASSP-30, pp.800-887.

Lim, J.S. and N.A. Malik, 1981. A New Algorithm for Two-Dimensional Maximum Entropy Power Spectrum Estimation, IEEE Trans. Acoust., Speech and Signal Processing, Vol. ASSP-29, No.3, pp.401-412.

McClellan, J.H., 1982. Multidimensional Spectral Estimation. Proceedings of the IEEE, Vol.70, No.9, pp.1029-1039.

Pendrell, J.V., 1979. The Maximum Entropy Principle in Two Dimensional Analysis. Ph.D. Dissertation, York University, Toronto.

Wernecke, S.J. and L.R. D'Addario, 1977. Maximum Entropy Image Reconstruction. IEEE Trans. Computer, Vol. C-26, pp.351-358.

Woods, J.W., 1976. Two-Dimensional Markov Spectral Estimation. IEEE Trans. Information Theory, Vol. IT-22, pp.552-559.

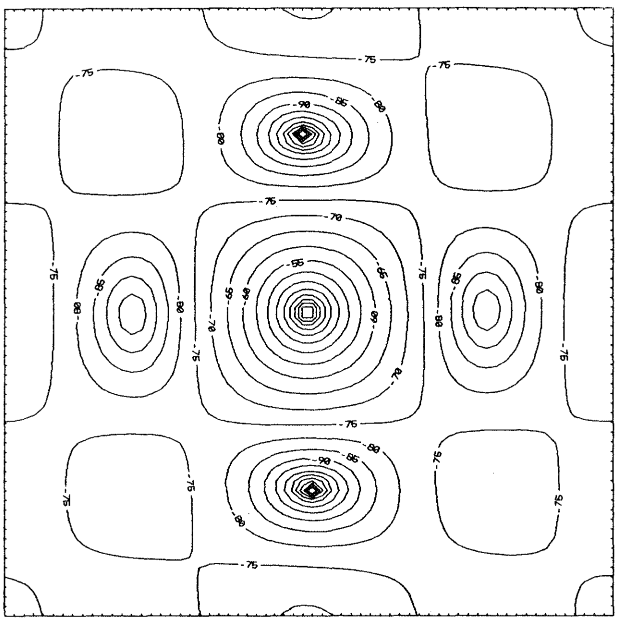

Figure 1: Contour Map of ME PSD Estimate for the TM Band 1 Test Image (Size 256x256, Support 5x5, with Noise)

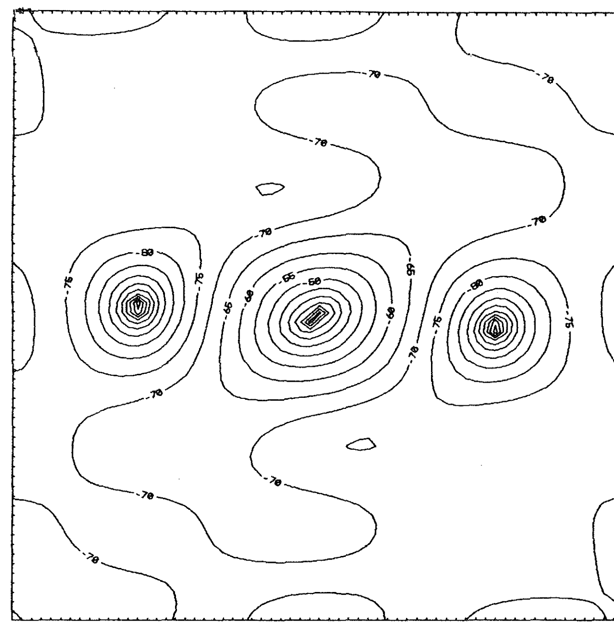

Figure 3: Contour Map of ME PSD Estimate for the TM Band 1 Test Image (Size 64x64, Support 5x7, with Noise)

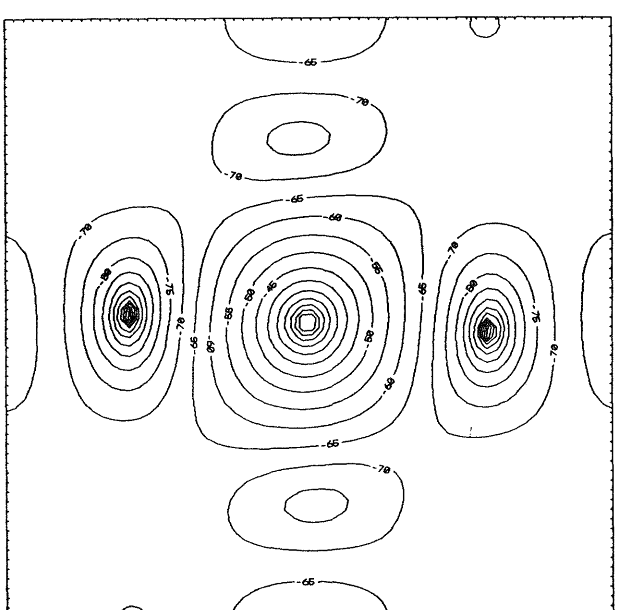

Figure 2: Contour Map of ME PSD Estimate for the TM Band 1 Test Image (Size 64x64, Support 5x5, with Noise)

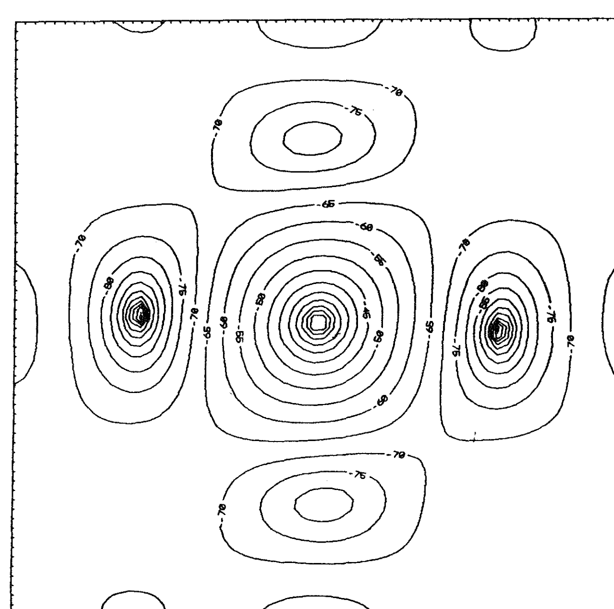

Figure 4: Contour Map of ME PSD Estimate for the TM Band 1 Test Image (Size 64x64, Support 5x5, without Noise)