A STUDY BEARING ON THE GEOMETRIC DEFORMATIONS OF THE VISUAL STEREOMODEL

DR H N NAGARAJA
Nanyang Technological Institute
Republic of Singapore
Commission I: Primary Data Acquisition

1. INTRODUCTION

In application fields such as architecture, archeology and many other fields, it often becomes necessary to generate the views of a given object as it is perceived by the human eye positioned at different distances and in addition when the scale and hence the size of the object is varied. When the object is physically not existing, and when the design details are still in the blue print stage, it may also be desirable to produce distortion-free views, preferably in three dimensions. It could be emphasized that the geometric fidelity of the representation should be an important guiding factor.

Accepting the human eye as a central system of spherical surfaces, it is possible and convenient if we are able to represent it in a simple form. Such simplified schematic eyes have been proposed by various writers, GULSTRAND being the most prominent. The "reduced eye" used in the field of visual optics can be conveniently adapted in any simulation study concerning stereo-vision. Where the simulated perspective data is in conformity with the various conditions of binocular vision, the validity of results could also be conveniently checked by the method of drawn anaglyphs. The study presented in the paper bears on the solution to the problem of producing an undistorted but scale reduced stereovision of a given object.

2. THE CONCEPT OF GEODETIC INTERSECTIONS

The human eye essentially consists of a convex lens in front known as crystalline lens and a sensitive membrance behind known as retina. In fact, the eye compares reasonably with a camera consisting of a shutter and a lens system. However, the way in which focussing is done in the human eye is different. The crystalline lens in the human eye is held in position by suspensary ligaments attached to ciliary muscles. With the contraction or relaxation of these muscles the shape and hence the focal length of the eye lens is changed. This process, which makes it possible to bring rays from objects at different distances to points on the retina, is known as accommodation. The retina contains hundreds of cones and rods whose main function is to receive light pulses and to transform the same to electrical signals. These electrical signals are translated by the brain into vision.

In the field of visual optics, a "reduced schematic eye" is considered as the geometric equivalent of the human eye. The overall power of this standard reduced eye is 69.72 diopters. Also, the refractive index for the acqueous humour of the eye is 1.34 and that for cornea or the crystalline lens is about 1.40. For extended studies, it is possible to calculate the relative three dimensional object and images positions by use of basic ray tracing formulae. However an immediate word of caution is in order. Sometimes, the human vision, which is often relied upon to convey the truth, can become quite unreliable. The whole group of "optical illusions" given in many a book illustrate the point.

The photogrammetrist is too familiar with the geometrical concepts of collinearity, coplanarity, inner, relative and absolute orientations. In viewing the stereoimages under the stereoscope or while viewing the anaglyph with the filter spectacles, apparently much the same sort of phenomena takes place. One important geometric concept currently used to explain this is termed "The Concept of Geodetic Intersections". Here the individual eye in its turn takes on the role of an observing theodolite used in measuring horizontal and vertical angles and the interocular distance provides the geodetic base. Thus, a mental triangulation surveying accompanied by a graphical or mental plotting takes place resulting in the three dimensional perception (1). In otherwords, this concept relies only on the relative parallax and relative convengence angle in building up the visual model. As this concept is geometric in nature, its validity can be easily checked by a suitable simulation study.

3. VISUAL GEOMETRIC MODEL DEFORMATION STUDY

For purposes of mathematical treatment, certain assumptions, none of which being exact in practice, are often made.

(a) Geometrically, the eye works as a camera/projector.
(b) The composite eye lens is distortion free.
(c) The retinae are truely spherical.
(d) The nodal points and centres of rotation coincide.
(e) There is true mathematical symmetry in the correspondence of the retina points or areas.
(f) Rotation of the eye globes around their visual axes is strictly geometric.
(g) The axes of the eye fix a single point in space and so for each model perceived by a given individual, there is a single "Horopter Surface".
(h) Imperfect position of the component images and the imperfect positioning of the eyes are effectively compensated by the brain after it receives messages from the occular, neck and other associated muscles.
(i) The retinal meridians are truely horizontal and vertical respectively.
(j) The physiological and mental processes make effective use of previous experience and so tend to convert the perspectively produced visual model to its equivalent "undistorted orthogonal digital terrain model".

Now, an attempt can be made to apply the geodetic intersection concept to the phenomena of binocular stereo-vision using a simplified approach. Table 1 contains the three dimensional coordinates in meters of the selected grid points and the exposure stations. For convenience, the grid points are situated in symmetric locations. Also, the data is representative of a common situation in architecture, where the appearance of such an object as perceived from different view points and with change of scale of the model would be of particular interest. Table 2 contains the photo-coordinates in millimetres as in left and right photographs. For convenience, the origin for both the photographs is shifted to the image position of point E4. A camera constant of 250 mm is used to correspond with the distance of normal vision. The formulae used are listed below:

TABLE 1: Three dimensional coordinates (m) of grid points

(origin: E4; X-axis +ve to right, Y-axis +ve down, Z-axis is +ve from E4 to H4; LE: left exposure station; RE: Right exposure station)

Pt. No.	X	Y	Z	Pt No	X	Y	Z	Pt No	X	Y	Z
A1	-0.25	0.75	-0.25	D1	-0.25	0.75	0	G1	-0.25	0.75	0.25
A2	-0.25	0.50	-0.25	D2	-0.25	0.50	0	G2	-0.25	0.50	0.25
A3	-0.25	0.25	-0.25	D3	-0.25	0.25	0	G3	-0.25	0.25	0.25
A4	-0.25	0	-0.25	D4	-0.25	0	0	G4	-0.25	0	0.25
A5	-0.25	-0.25	-0.25	D5	-0.25	-0.25	0	G5	-0.25	-0.25	0.25
A6	-0.25	-0.50	-0.25	D6	-0.25	-0.50	0	G6	-0.25	-0.50	0.25
A7	-0.25	-0.75	-0.25	D7	-0.25	-0.75	0	G7	-0.25	-0.75	0.25
B1	0	0.75	-0.25	E1	0	0.75	0	H1	0	0.75	0.25
B2	0	0.50	-0.25	E2	0	0.50	0	H2	0	0.50	0.25
B3	0	0.25	-0.25	E3	0	0.25	0	H3	0	0.25	0.25
B4	0	0	-0.25	E4	0	0	0	H4	0	0	0.25
B5	0	-0.25	-0.25	E5	0	-0.25	0	H5	0	-0.25	0.25
B6	0	-0.50	-0.25	E6	0	-0.50	0	H6	0	-0.50	0.25
B7	0	-0.75	-0.25	E7	0	-0.75	0	H7	0	-0.75	0.25
C1	0.25	0.75	-0.25	F1	0.25	0.75	0	J1	0.25	0.75	0.25
C2	0.25	0.50	-0.25	F2	0.25	0.50	0	J2	0.25	0.50	0.25
C3	0.25	0.25	-0.25	F3	0.25	0.25	0	J3	0.25	0.25	0.25
C4	0.25	0	-0.25	F4	0.25	0	0	J4	0.25	0	0.25
C5	0.25	-0.25	-0.25	F5	0.25	-0.25	0	J5	0.25	-0.25	0.25
C6	0.25	-0.50	-0.25	F6	0.25	-0.50	0	J6	0.25	-0.25	0.25
C7	0.25	-0.75	-0.25	F7	0.25	-0.75	0	J7	0.25	-0.75	0.25
LE	-0.15	0	-1.75					RE	0.15	0	-1.75

$$x_1 = C \frac{X - X(LE)}{Z - Z(LE)} - x_1(E4) \qquad (1)$$

$$y_1 = y_2 = C \frac{Y - Y(LE)}{Z - Z(LE)} \qquad (2)$$

$$x_2 = C \frac{X - X(RE)}{Z - Z(RE)} - x_2(E4) \qquad (3)$$

where,

(x_1, y_1): Photo-coordinates in the left picture
(x_2, y_2): Photo-coordinates in the right picture
$x_1(E4)$, $x_2(E4)$: x photo-coordinate of the chosen origin point in the left and right photos respectively

TABLE 2 : Photocoordinates (mm) with origin at E4

(x +ve to right and y +ve downwards. Normal case, camera axes parallel, C = 250 mm). (x_1, y_1) : coordinates in left photo, (x_2, y_2): coordinates in right photo.

Pt. No.	x_1	$y_1 = y_2$	x_2	Pt. No.	x_1	$y_1 = y_2$	x_2
A1	-38.096	125.000	-45.238	E4	0	0	0
A2	-38.096	83.333	-45.238	E5	0	-35.714	0
A3	-38.096	41.667	-45.238	E6	0	-71.429	0
A4	-38.096	0	-45.238	E7	0	-107.143	0
A5	-38.096	-41.667	-45.238	F1	35.713	107.143	35.715
A6	-38.096	-83.333	-45.238	F2	35.713	71.429	35.715
A7	-38.096	-125.000	-45.238	F3	35.713	35.714	35.715
B1	3.751	125.000	-3.571	F4	35.713	0	35.715
B2	3.571	83.333	-3.571	F5	35.713	-35.714	35.715
B3	3.571	41.667	-3.571	F6	35.713	-71.429	35.715
B4	3.751	0	-3.571	F7	35.713	-107.143	35.715
B5	3.571	-41.667	-3.571	G1	-33.929	93.750	-28.571
B6	3.571	-83.333	-3.571	G2	-33.929	62.500	-28.571
B7	3.571	-125.000	-3.571	G3	-33.929	31.250	-28.571
C1	45.238	125.000	38.096	G4	-33.929	0	-28.571
C2	45.238	83.333	38.096	G5	-33.929	-31.250	-28.571
C3	45.238	41.667	38.096	G6	-33.929	-62.500	-28.571
C4	45.238	0	38.096	G7	-33.929	-93.750	-28.571
C5	45.238	-41.667	38.096	H1	-2.679	93.750	2.679
C6	45.238	-38.333	38.096	H2	-2.679	62.500	2.679
C7	45.238	-125.000	38.096	H3	-2.679	31.250	2.679
D1	-35.715	107.143	-35.714	H4	-2.679	0	2.679
D2	-35.715	71.439	-35.714	H5	-2.679	-31.250	2.679
D3	-35.715	35.714	-35.714	H6	-2.679	-62.500	2.679
D4	-35.715	0	-35.714	H7	-2.679	-93.750	2.679
D5	-35.715	-35.714	-35.714	J1	28.571	93.750	33.929
D6	-35.715	-71.429	-35.714	J2	28.571	62.500	33.929
D7	-35.715	-107.143	-35.714	J3	28.571	31.250	33.929
E1	0	107.143	0	J4	28.571	0	33.929
E2	0	71.429	0	J5	28.571	-31.250	33.929
E3	0	35.714	0	J6	28.571	-62.500	33.929
				J7	28.571	-93.750	33.929

Using the simulated left and right photographs of the object obtained by plotting the coordinates of Table 2, an anaglyph could be prepared by coinciding the point E4 in both the pictures, then copying the left picture in red and the right picture in green. When this is viewed by matching filter spectacles, subject to the various conditions of binocular vision being fulfilled, a plastic deformed model of the object would be perceived. Now, using the geodetic intersection concept, the three dimensional coordinates of such points situated on the deformed stereomodel can be worked out. Let,

L : horizontal angle formed at the left eye measured from the nodal point of right eye R to the observed point on the model.

R : horizontal angle formed at the right eye measured from the nodal point of left eye L to the observed point on the model.

ϕ : angle of convergence formed at the model point

b : Eye base, taken as 70 mm

H : viewing distance, taken as 250 mm

r : radial distance from left eye to model point

r_1: radial distance from left eye to photo point

X, Y, Z : Three dimensional coordinates of the model point obtained by geodetic intersection.

Then, adopting a three dimensional coordinate system as previously used in Table 1, and with the origin at the model point E4, we have,

$$L = 90° - \mathrm{Tan}^{-1} \frac{x_1 + b/2}{H} \tag{4}$$

$$R = 90° + \mathrm{Tan}^{-1} \frac{x_2 - b/2}{H} \tag{5}$$

$$\phi = 180° - L - R \tag{6}$$

$$r = \frac{b \cdot \sin R}{\sin \phi} \tag{7}$$

$$r_1 = [\, H^2 + (x_1 + b/2)^2 \,]^{1/2} \tag{8}$$

$$X = r \cdot \cos L - b/2 \tag{9}$$

$$Y = y_1 \cdot r/r_1 \tag{10}$$

$$Z = r \cdot \sin L - H \tag{11}$$

The three dimensional coordinates so calculated are termed X(GI), Y(GI), Z(GI) and are tabulated in columns 5, 6 and 7 of table 3. As the viewing distance for the plastic model is only 250 mm compared with the viewing distance for the object of 1750 mm, coordinates corresponding to an ideal scale model, using a scaling factor of 250/1750 = 1/7 can be worked out. The three dimensional coordinates of selected grid points obtained thus is tabulated in columns 2, 3 and

TABLE 3 : Ideal scaled model coordinates, Geodetic intersection model coordinates and the corresponding model deformations (mm)

(scaling factor for col. 2, 3 and 4 = 250/1750 = 1/7)

1	2	3	4	5	6	7	8	9	10
Pt. No.	X (Id)	Y (Id)	Z (Id)	X (GI)	Y (GI)	Z (GI)	DX (2-5)	DY (3-6)	DZ (4-7)
A1	-35.71	107.14	-35.71	-37.81	113.43	-23.15	2.10	-6.29	-12.56
A2	-35.71	71.42	-35.71	-37.81	75.62	-23.15	2.10	-4.20	-12.56
A3	-35.71	35.71	-35.71	-37.81	37.81	-23.15	2.10	-2.10	-12.56
A4	-35.71	0	-37.71	-37.81	0	-23.15	2.10	0	-12.56
A5	-35.71	-35.71	-35.71	-37.81	-37.81	-23.15	2.10	2.10	-12.56
A6	-35.71	-71.42	-35.71	-37.81	-75.62	-23.15	2.10	4.20	-12.56
A7	-35.71	-107.14	-35.71	-37.81	-113.43	-23.15	2.10	6.29	-12.56
B1	0	107.14	-35.71	0	113.43	-23.15	0	-6.29	-12.56
B2	0	71.42	-35.71	0	75.62	-23.15	0	-4.20	-12.56
B3	0	35.71	-35.71	0	37.81	-23.15	0	-2.10	-12.56
B4	0	0	-35.71	0	0	-23.15	0	0	-12.56
B5	0	-35.71	-35.71	0	-37.81	-23.15	0	2.10	-12.56
B6	0	-71.42	-35.71	0	-75.62	-23.15	0	4.20	-12.56
B7	0	-107.14	-35.71	0	-113.43	-23.15	0	6.29	-12.56
C1	35.71	107.14	-35.71	37.81	113.43	-23.15	-2.10	-6.29	-12.56
C2	35.71	71.42	-35.71	37.81	75.62	-23.15	-2.10	-4.20	-12.56
C3	35.71	35.71	-35.71	37.81	37.81	-23.15	-2.10	-2.10	-12.56
C4	35.71	0	-35.71	37.81	0	-23.15	-2.10	0	-12.56
C5	35.71	-35.71	-35.71	37.81	-37.81	-23.15	-2.10	2.10	-12.56
C6	35.71	-71.42	-35.71	37.81	-75.62	-23.15	-2.10	4.20	-12.56
C7	35.71	-107.14	-35.71	37.81	-113.43	-23.15	-2.10	6.29	-12.56
D1	-35.71	107.14	0	-35.72	107.15	0	0.01	-0.01	0
D2	-35.71	71.42	0	-35.72	71.43	0	0.01	-0.01	0
D3	-35.71	35.71	0	-35.72	35.72	0	0.01	-0.01	0
D4	-35.71	0	0	-35.72	0	0	0.01	0	0
D5	-35.71	-35.71	0	-35.72	-35.72	0	0.01	0.01	0
D6	-35.71	-71.42	0	-35.72	-71.43	0	0.01	0.01	0
D7	-35.71	-107.14	0	-35.72	-107.15	0	0.01	0.01	0
E1	0	107.14	0	0	107.15	0	0	-0.01	0
E2	0	71.42	0	0	71.43	0	0	-0.01	0
E3	0	35.71	0	0	35.71	0	0	-0.01	0
E4	0	0	0	0	0	0	0	0	0
E5	0	-35.71	0	0	-35.71	0	0	0	0
E6	0	-71.42	0	0	-71.43	0	0	0.01	0
E7	0	-107.14	0	0	-107.14	0	0	0	0
F1	35.71	107.14	0	35.72	107.14	0.01	-0.01	0	-0.01
F2	35.71	71.42	0	35.72	71.43	0.01	-0.01	-0.01	-0.01
F3	35.71	35.71	0	35.72	35.72	0.01	-0.01	-0.01	-0.01
F4	35.71	0	0	35.72	0	0.01	-0.01	0	-0.01
F5	35.71	-35.71	0	35.72	-35.72	0.01	-0.01	0.01	-0.01
F6	35.71	-71.42	0	35.72	-71.43	0.01	-0.01	0.01	-0.01
F7	35.71	-107.14	0	35.72	-107.15	0.01	-0.01	0.01	-0.01
G1	-35.71	107.14	35.71	-33.84	101.52	20.72	-1.87	5.62	14.99
G2	-35.71	71.42	35.71	-33.84	67.68	20.72	-1.87	3.74	14.99
G3	-35.71	35.71	35.71	-33.84	33.84	20.72	-1.87	1.87	14.99

1	2	3	4	5	6	7	8	9	10
G4	-35.71	0	35.71	-33.84	0	20.72	-1.87	0	14.99
G5	-35.71	-35.71	35.71	-33.84	-33.84	20.72	-1.87	-1.87	14.99
G6	-35.71	-71.42	35.71	-33.84	-67.68	20.72	-1.87	-3.74	14.99
G7	-35.71	-107.14	35.71	-33.84	-101.52	20.72	-1.87	-5.62	14.99
H1	0	107.14	35.71	0	101.52	20.72	0	5.62	14.99
H2	0	71.42	35.71	0	67.68	20.72	0	3.74	14.99
H3	0	35.71	35.71	0	33.84	20.72	0	1.87	14.99
H4	0	0	35.71	0	0	20.72	0	0	14.99
H5	0	-35.71	35.71	0	-33.84	20.72	0	-1.87	14.99
H6	0	-71.42	35.71	0	-67.68	20.72	0	-3.74	14.99
H7	0	-107.14	35.71	0	-101.52	20.72	0	-5.62	14.99
J1	35.71	107.14	35.71	33.84	101.52	20.72	1.87	5.62	14.99
J2	35.71	71.42	35.71	33.84	67.68	20.72	1.87	3.74	14.99
J3	35.71	35.71	35.71	33.84	33.84	20.72	1.87	1.87	14.99
J4	35.71	0	35.71	33.84	0	20.72	1.87	0	14.99
J5	35.71	-35.71	35.71	33.84	-33.84	20.72	1.87	-1.87	14.99
J6	35.71	-35.71	35.71	33.84	-67.68	20.72	1.87	-3.74	14.99
J7	35.71	-107.14	35.71	33.84	-101.52	20.72	1.87	-5.62	14.99

4 of Table 3. Now, the differential quantities DX, DY and DZ tabluated in columns 8, 9 and 10 can be considered as the model deformations of the visual plastic model obtained under the assumption of geodetic intersections.

Having established the deformations of regularly spaced grid points, we may now consider the possibility of deriving the model coordinates in case of randomly spaced points, the three dimensional coordinates of the points on the object being considered unknown except for purposes of checking. In Table 4, the photocoordinates of selected test points, with origin at the image position of point E4, similar to the data of Table 2, are presented.

TABLE 4 : Photocoordinates (mm) of test points with origin at E4 (C = 250 mm)

Pt. No.	x_1	$y_1 = y_2$	x_2	Pt. No.	x_1	$y_1 = y_2$	x_2
1	-5.804	85.938	-9.821	13	-21.429	70.513	-17.033
2	-14.076	44.118	-15.336	14	-21.429	55.556	-20.238
3	-21.429	30.303	-24.026	15	-8.271	6.579	-4.887
4	-29.242	-7.813	-33.259	16	-14.672	-27.027	-12.355
5	-13.616	-46.875	-17.634	17	-27.839	-51.282	-23.443
6	-21.429	-106.061	-24.026	18	-21.429	-76.923	-17.033
7	15.336	80.882	14.076	19	20.238	97.222	21.429
8	33.259	62.500	29.242	20	5.598	60.811	7.915
9	9.821	31.250	5.804	21	19.112	6.757	21.429
10	31.601	-22.727	29.005	22	10.622	-25.641	15.019
11	18.894	-56.452	13.364	23	18.045	-59.211	21.429
12	31.601	-106.061	29.005	24	27.182	-90.278	28.373

As before using the potocoordinates, corresponding to the anaglyph positions, the three dimensional model coordinates as on the visual plastic model may be worked out based on the geodetic intersection concept. These values are tabulated in columns 2, 3 and 4 of Table 5. As these coordinates are affected by a perspective deformation, a correction to this could be worked out by an interpolation procedure using the known corrections at the grid points. For example consider point 6 located in the cubic wire frame A7 B7 D7 E7 A6 B6 D6 E6. Let,

$$\begin{bmatrix} X(B7) - X(A7), & X(A6) - X(A7), & X(D7) - X(A7) \\ Y(B7) - Y(A7), & Y(A6) - Y(A7), & Y(D7) - Y(A7) \\ Z(B7) - Z(A7), & Z(A6) - Z(A7), & Z(D7) - Z(A7) \end{bmatrix} = \begin{bmatrix} D1, & D2, & D3 \\ D4, & D5, & D6 \\ D7, & D8, & D9 \end{bmatrix}$$

and

$$\begin{bmatrix} (X(6) - X(A7) \;/\; (X(B7) - X(A7) \\ (Y(6) - Y(A7) \;/\; (Y(B7) - Y(A7) \\ (Z(6) - Z(A7) \;/\; (Z(B7) - Z(A7) \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

then

$$\begin{bmatrix} DX(6) \\ DY(6) \\ DZ(6) \end{bmatrix} = \begin{bmatrix} DX(A7) \\ DY(A7) \\ DZ(A7) \end{bmatrix} + \begin{bmatrix} D1, & D2, & D3 \\ D4, & D5, & D6 \\ D7, & D8, & D9 \end{bmatrix} \cdot \begin{bmatrix} L \\ M \\ N \end{bmatrix} \qquad (12)$$

Equation 12 is used to compute the corrections DX, DY, DZ for point 6. Similarly the corrections for all other points are worked out and entered in columns 5, 6 and 7 in Table 5. Now, columns 8, 9 and 10 are obtained by the addition of corresponding columns. It should be noted that the results of columns 8, 9 and 10 are only approximate. Corresponding exact values were obtained by scaling the original three dimensional coordinates of the points on the object. This was done and the maximum errors of DX = 0.8 mm, DY = 0.9 mm and DZ = 0.5 mm were found. This could have been reduced, if a closer interval for the control grid was used. The values in columns 8, 9, 10 may now be conveniently used for simulating pictorial views corresponding to selected eye base.

4. SOME AREAS OF APPLICATIONS

In the field of architecture and archeology, perspective views of the object under varying conditions of viewing need to be drawn. For this purpose methods of producing visually non-deformed equivalent scaled stereomodel, anaglyph being one such product, will be quite useful.

In the field of archeology, use of photogrammetry in preservation of monuments in technologically advanced countries is well known. In developing countries, the development of simple methods of mapping will help in providing a strong motivation to get all worthwhile archeological treasures mapped.

In the field of traffic accident studies and investigations, simulation of distortion free visual models of accident scenes can be quite useful.

TABLE 5 : Geodetic intersection model coordinates, corrections and corresponding Ideal Scaled model coordinates (mm)

Pt. No.	X (GI)	Y (GI)	Z (GI)	DX	DY	DZ	X	Y	Z
1	-7.39	81.27	-13.57	-0.46	-2.77	-7.37	-7.85	78.50	-20.94
2	-14.45	43.34	-4.42	-0.89	-0.71	-2.41	-15.34	42.63	-6.83
3	-21.91	29.22	-8.94	-0.07	-1.62	-4.86	-21.98	27.60	-13.80
4	-29.56	-7.39	-13.57	0.77	-0.46	-7.37	-28.79	-7.85	-20.94
5	-14.78	-44.33	-13.57	-0.05	0.73	-7.37	-14.83	-43.60	-20.94
6	-21.91	-102.27	-8.94	-0.07	1.81	-4.86	-21.98	-100.46	-13.80
7	14.45	79.45	-4.42	-0.80	-1.02	-2.40	13.65	78.43	-6.82
8	29.56	59.11	-13.57	-1.64	-2.41	-7.37	27.92	56.70	-20.94
9	7.39	29.56	-13.57	-0.41	-1.64	-7.37	6.98	27.92	-20.94
10	29.22	-21.91	-8.94	-1.62	-0.07	-4.85	27.60	-21.98	-13.79
11	14.95	-52.32	-18.30	-0.83	2.03	-9.94	14.12	-50.29	-28.24
12	29.22	-102.27	-8.94	-1.62	1.81	-4.85	27.60	-100.46	-13.79
13	-20.52	75.24	16.75	-1.52	3.02	12.12	-22.04	78.26	28.87
14	-21.19	56.52	4.33	-0.39	0.39	3.13	-21.58	56.91	7.46
15	-6.91	6.91	12.70	-1.15	0	9.19	-8.06	6.91	21.89
16	-13.98	-27.95	8.56	-0.77	-0.77	6.19	-14.75	-28.72	14.75
17	-27.36	-54.72	16.75	-1.52	-3.03	12.12	-28.88	-57.75	28.87
18	-20.52	-82.08	16.75	-1.52	-4.54	12.12	-22.04	-86.62	28.87
19	21.19	98.91	4.33	0	0.78	3.13	21.19	99.69	7.46
20	6.99	62.89	8.56	0	0.77	6.19	6.99	63.66	14.75
21	20.96	6.99	8.56	0	0	6.19	20.96	6.99	14.75
22	13.68	-27.36	16.76	0	-1.52	12.12	13.68	-28.88	28.88
23	20.74	-62.22	12.70	0	-2.29	9.18	20.74	-64.51	21.88
24	28.26	-91.84	4.33	0	-1.17	3.13	28.26	-93.01	7.46

In the field of audio-visual education, development of distortion free anaglyphs or other three dimensional viewing techiques would be another step in the right direction.

In the field of highway engineering the preparation and use of photo-montage for publicity purposes in highway development is well known. It would be helpful if such material could be made distortionless.

The field of visual optics is not supported on the same solid pedastal as geometric optics. In the former case, personal factors and interpretations provided by the human brain are brought into play. Anaglyphs with and without distortions, can be quite useful in a systematic study of the geometric aspects of human vision.

5. CONCLUSIONS AND RECOMMENDATIONS

(a) The deformations of the visual stereomodel are three dimensional in nature. The deformations are more serious in Z-direction than in the other directions.

(b) The deformations of the visual stereomodel also have another component due to the lens distortions introduced by the eye lens.

(c) The visual model deformations are mainly affine in nature and hence a three dimensional linear interpretation method is quite suitable.

(d) By suitable design and analyses of the geometrical concepts, it should be practicable to produce distortion-free but scale reduced stereovisions of a given object. Also there are many application fields in which the production of such distortion-free views are useful.

(e) The geodetic intersection concept provides a good first approximation in the geometric analysis of stereovision.

(f) Further extended studies of this nature are useful and practical.

(g) Geometric concepts prevalent in visual optics should be brought to bear on application studies in various fields. Production of undistorted but scale reduced visual stereomodel should be the basic motivation in such studies.

(h) One of the reasons for the use of single black and white photographs as the basis of photo-interpretations is that suitable simple and distortion-free viewing technics are lacking. This gap should be filled.

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