

COMBINED ADJUSTMENT OF PHOTOGRAMMETRIC AND
NON-PHOTOGRAMMETRIC INFORMATION

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SUMMARY

The paper deals with photogrammetric point determination using general control and object information. It covers the question of suitable non-photogrammetric information as well as the combined adjustment itself. Major attention is directed to aerial triangulation, whereas close range triangulation is only briefly be treated. First a review is given of the state of the art which is followed by a section on future possibilities of generalized photogrammetric point determination. In the conclusion future tasks are specified.

1. INTRODUCTION

Photogrammetric point determination always needs some non-photogrammetric information. At least the rank deficiency of the resulting normal equation system with regard to three translations, three rotations and the scale factor has to be balanced by seven independent observation or condition equations. Classical photogrammetry makes use of control point coordinates to relate image or model coordinates to a given coordinate system. Usually this control results from geodetic measurements. Although both photogrammetric and geodetic data are processed simultaneously in the framework of a rigorous least squares adjustment the procedure is not understood as "Combined Adjustment". This term is only used if more general control information is used in addition to or instead of control point coordinates. Such control information can be given in the form of direct geodetic observations, auxiliary data or conditions; for example: a set of points measured photogrammetrically, belong to a curve or surface of known type and known or unknown parameters. The latter case may include object information, which has to be considered, but doesn't have a control function.

Consequently, the topic of this Invited Paper will be "Photogrammetric point determination with general control and object information". It will cover the question of suitable non-photogrammetric information as well as the combined adjustment itself. Point determination generalized in that way can lead to considerable improvements in accuracy, reliability and economy. Major attention will be directed to aerial triangulation, whereas terrestrial or close range triangulation will only briefly be treated.

The paper gives first a review of the state of the art which is followed by a section on future possibilities of generalized photogrammetric point determination. In the conclusion future tasks are specified.

2. STATE OF THE ART

2.1 REPRESENTATIVE EXAMPLES OF COMBINED ADJUSTMENT

Point determination with general control and object information goes back to at least 1972. Three developments shall be mentioned here, which have had significant influence on practice and further progress in the field.

The PAT-M43 program for block adjustment by models was extended to simultaneous consideration of stadioscope and APR data, to save height control points. Although such auxiliary data were successfully used before, it was the first time that this information was rigorously processed in a combined adjustment. For the same purpose of saving height control object information in the form of equal heights of points along the shorelines of lakes was taken into consideration (ACKERMANN et.al. 1972). The latter information was also used in the SPACE-M model block adjustment program (BLAIS, 1977). A combined adjustment of photogrammetric image data and geodetic observations was realized by the SAPGO program (WONG and ELPHINSTONE, 1972). It already offered the option of estimating the variances of adjusted object coordinates.

Till today quite a number of further contributions to generalized point determination by aerial and close range photogrammetry have been published. Representative examples shall be treated here, starting with investigations and developments which relate to aerial triangulation.

Block adjustment with auxiliary data, supplied by navigation systems is investigated in (ACKERMANN, 1984). Simulations and practical results are reviewed. Combined photogrammetric and Doppler adjustment is studied in (ANDERSON, 1982). The author suggests a rigorous adjustment in two steps. First the Doppler station coordinates are estimated with their full covariance matrix. This information then is used in photogrammetric block adjustment. A report on the use of auxiliary data in model block adjustments is given in (BLAIS, 1984). Results obtained with various types of simulated and real auxiliary data are discussed. The advantages of the use of existing geodetic observations in photogrammetric control densification are pointed out in (BURTCH, 1984). Bundle block adjustment with consideration of various geodetic observations is treated in (CORCODEL, 1984).

A concept for generalized adjustment by least squares, called GALS is presented in (ELASSAL, 1983). It allows for any number of types of observations with corresponding variances and covariances. In addition to the unknown parameters their covariance matrix is estimated. Further examples of using geodetic observations to control aerial

triangulation are given in (KENEFFICK, 1978). The connection of blocks and ground control is reconsidered in (MOLENAAR, 1984). Block and network shall be planned together, to get an integrated point determination system designed to meet uniform requirements for precision and reliability. For simultaneous adjustment of image coordinates and geodetic observations one author suggests the application of recursive methods (ROSCULET, 1980). An experiment on the use of different kinds of observations in block triangulation with models and bundles is presently being performed in the framework of Commission A of OEEPE (TALTS, 1984).

Moreover, program developments for applications in aerial as well as in close range triangulation have become known. GEBAT performs a bundle block adjustment and accepts a variety of geodetic observations. All equations relate rigorously to a three dimensional Cartesian coordinate system. Variances and covariances of adjusted object coordinates can be estimated (EL HAKIM and FAIG, 1981). A program for combined adjustment of model coordinates and geodetic data is presented in (DÜPPE, 1982). It is followed by a corresponding bundle block adjustment program. Optionally a free adjustment can be performed and variances of object coordinates can be computed (DÜPPE, 1984). The BINGO program is also based on bundle adjustment and considers various observations and conditions. Important parts of the inverse of the normal equation matrix can be computed and used to derive accuracy and reliability measures (KRUCK, 1984). A further program for combined adjustment of photogrammetric bundles and geodetic data is GENTRI (LARSSON, 1983). Finally in (SALMENPERA, 1984) a block adjustment program is mentioned which processes image coordinates, model coordinates and general control information.

Combined adjustment using direct geodetic observations instead of control point coordinates today is better established in close range than in aerial triangulation. On the one hand in close range photogrammetry often only relative control is required and available. On the other hand the realization of combined adjustment is much simpler here than in aerial triangulation where the blocks can be rather large. Several programs for close range triangulation are in practical use today, for instance (FUCHS and LEBERL, 1984), (HELL, 1979), (KAGER and KRAUS, 1976), (WESTER-EBBINGHAUS, 1984).

At the FIG Congress in Sofia 1983 a fine review of simultaneous adjustment of photogrammetric and geodetic observations was given (TORLEGARD, 1983).

2.2 NON-PHOTOGRAMMETRIC INFORMATION USED IN COMBINED ADJUSTMENT

The papers mentioned above show that a variety of observations can appear in the adjustment as, for example:

- image and model coordinates
- object coordinates and coordinate differences
- orientation parameters and parameter differences
- self calibration parameters
- slope and horizontal distances
- horizontal directions and angles as well as azimuths
- vertical angles and elevation differences

- astronomic longitudes and latitudes
- coordinates of points in a local coordinate system which has to be oriented to the global coordinate system by a number of free parameters.

The last case is rather general and allows e.g. the processing of statorscope - and APR data or local control point coordinates. At least for some of the mentioned observations full covariance matrices can be given.

Moreover, conditions may appear, e.g. in conjunction with general object information, particularly in close range photogrammetry.

2.3 FORMULATION AND COMPUTATIONAL REALIZATION OF COMBINED ADJUSTMENT

Not in all but in most programs observation equations are formulated. If necessary they are completed by a set of condition equations between unknowns, e.g. in case of a free adjustment. The consideration of general non-photogrammetric information in the adjustment usually leads to normal equation matrices of a rather general structure. If a direct solution with its general advantages is to be applied, special measures have to be taken to get a favourable structure and consequently, keep the computing time within reasonable limits. Three possible measures shall be mentioned briefly.

- The object points are separated into two groups. All points tied together by general non-photogrammetric information are arranged in the second group. The normal equation submatrix of the object coordinates has then block diagonal structure for the first group and is irregularly occupied in the area of the second group. If now the normals are reduced to the unknown orientation parameters the effort to invert the submatrix of the object coordinates is still acceptable as long as the number of points in the second group is not too high. This concept is used e.g. in (ROSCULET, 1980). Nevertheless, the structure of the reduced normal equations can become unfavourably general if images or models far away from each other are tied by non-photogrammetric information.
- The coordinates of object points, which are tied together by general non-photogrammetric information are arranged behind the orientation parameters and not reduced. This leads to a banded bordered structure of the reduced normal equation matrix without disturbance of the band. The resulting computing time can be considerably shorter than in the first case. This concept is used e.g. in (DÜPPE, 1984) and (LARSSON, 1983).
- A third possibility is suggested in (KRUCK, 1984). The author breaks up the strict separation of object coordinates and orientation parameters, which leads to a saving of fill ins during factorization of the normal equations. It would be interesting to compare this concept with the others mentioned above more in detail.

In conjunction with combined adjustment free parameters may appear as the result of the consideration of general control or object information. Examples are orientation parameters of sets of horizontal directions or parameters to orient a local to a global coordinate system. These unknowns can either be reduced together with the object coordinates or arranged behind the orientation parameters. Their coefficients then appear in the border of the reduced normal equation matrix.

3. FUTURE POSSIBILITIES

3.1 FUTURE INFORMATION SUITABLE FOR COMBINED ADJUSTMENT

For the future of aerial triangulation the use of data from inertial surveying systems (ISS) and from the NAVSTAR Global Positioning System (GPS) seems to be particularly promising. ISS determine coordinate differences between points in x, y and z as well as the three orientation angles (SCHWARZ, 1980). GPS allows for distance and distance difference (Doppler) measurements to the satellites, from which control point coordinates can be determined. Moreover, interferometric techniques can be applied. Using two instruments of the Macrometer type at two stations coordinate differences Δx , Δy and Δz between these stations can be derived (SEEBER, 1984). A possibility for the future is the use of ISS and GPS during the photo flight to determine the orientation parameters.

The ways of consideration of ISS and GPS data in the combined adjustment still have to be studied in detail. Full covariance matrices of the observations and free parameters to model the inherent systematic errors may appear here.

3.2 IMPROVEMENTS IN FORMULATION AND REALIZATION OF COMBINED ADJUSTMENT

In section 2.3 it was pointed out that special measures have to be taken to keep the computing times for combined adjustments within reasonable limits. Three ways to reach this goal have been described. Here, a new concept shall be proposed the properties of which are particular favourable. To show the principle we start from the following system of observation equations

$$\hat{v}_1 = A\Delta\hat{x} - l_1, P_{11} \quad (1a)$$

$$\hat{v}_2 = C\Delta\hat{x} - l_2, P_{22} \quad (1b)$$

(1a) represents a classical block adjustment. The term l_1 contains the image or model coordinates as well as the control point coordinates. The parameters $\Delta\hat{x}$ are the corrections to the initial values of the unknown object coordinates and orientation parameters. In (1b) general control information is considered. Possible observations contained in l_2 are e.g. slope distances. P_{11} , P_{22} are the weight matrices assigned to l_1 , l_2 and \hat{v}_1 , \hat{v}_2 are the residuals of the observations contained in l_1 , l_2 .

Now as a trick (1b) is split into two equations, which read

$$\hat{v}_2 = \hat{l}_2 - l_2, P_{22} \quad (1c)$$

$$C \Delta \hat{x} = \hat{l}_2 \quad (1d)$$

(1c) are observation equations in which new parameters \hat{l}_2 appear, whereas (1d) represents condition equations between the two groups of unknowns $\Delta \hat{x}$ and \hat{l}_2 . The combined least squares adjustment of (1a), (1c) and (1d) leads to the following system of normal equations

$$\begin{aligned} A^T P_{11} A \Delta \hat{x} + C^T \hat{k} &= A^T P_{11} l_1 \\ C \Delta \hat{x} - I \hat{l}_2 &= 0 \\ -I \hat{k} + P_{22} \hat{l}_2 &= P_{22} l_2 \end{aligned} \quad (2)$$

In (2) I is a unit matrix and \hat{k} are correlates, the number of which is equal to the number of observations contained in l_2 . From (2) the parameters l_2 can easily be eliminated. The remaining system of normal equations then reads

$$\begin{aligned} A^T P_{11} A \Delta \hat{x} + C^T \hat{k} &= A^T P_{11} l_1 \\ C \Delta \hat{x} - Q_{22} \hat{k} &= l_2 \end{aligned} \quad (3)$$

In (3) Q_{22} is the weight coefficient matrix assigned to l_2 and the inverse of P_{22} respectively. A system of equations similar to (3) also appears in (SCHMID, 1980) in conjunction with free adjustment.

A closer look at (3) shows that the normal equation matrix $A^T P_{11} A$ of the classical block adjustment remains unchanged. Additional general control information only extends $A^T P_{11} A$ to a bordered matrix. From (3) reduced normal equations can be derived containing the orientation parameters and the correlates alone. The resulting coefficient matrix is of banded bordered structure, with the band width of the classical adjustment.

It is interesting to compare this new concept with the one used in (DÜPPE, 1984) and (LARSSON, 1983) where general control information also leads to a banded bordered system. Whereas the band width is the same, the border, however, is wider. A geodetic observation connecting 2, 4 or 6 object coordinates results in 2, 4 or 6 columns in the border. Compared with this the new concept leads to only one column per observation. Another advantage is that errorfree observations can be considered by simply assigning weight coefficients zero to them, without the danger of numerical problems.

At the Chair of Photogrammetry of the Technical University in Munich a general orientation program for two images (STEPHANI, 1984) and a general bundle block adjustment program for applications in aerial

and in close range triangulation (MÜLLER and STEPHANI, 1984) have been developed, using this concept.

Further improvements may result from the use of sparse matrix techniques, although in (STEIDLER, 1980) it was shown for classical block adjustment that methods based on banded bordered systems are superior. A step in the direction of the new techniques is (KRUCK, 1984). The different concepts for formulation and computational realization of combined adjustment treated in this paper are not necessarily alternatives but can, of course, also be combined successfully.

4. CONCLUSION

The state of the art and the outlook on future possibilities show that combined adjustment has developed remarkably and will develop further. Moreover, it is interesting to see that aerial and closer range triangulation have come closer to each other. Combined adjustment today should allow for rigorous processing of all available and suitable control and object information, including the consideration of full covariance matrices of non-photogrammetric observations. Besides the unknown parameters at least the most important parameter variances and covariances should be estimated and used to derive accuracy and reliability measures. These goals are already reached to a considerable extent in close range triangulation. In aerial triangulation the task is more difficult, because of the much larger blocks to be treated. Basically, however, the tools are available to solve the problem.

In addition to this task comprehensive accuracy and reliability studies are necessary to show which non-photogrammetric information is most favourable for certain requirements. For aerial triangulation such investigations are of similar importance as previous studies on the effect of different control point distributions have been.

From such developments and investigations significant improvements of accuracy, reliability and economy of photogrammetric point determination can be expected, particularly in aerial triangulation.

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