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ACCURACY PREDICTION FOR DIGITAL ELEVATION MODELS

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Abstract

This paper presents a test of a practical method for predicting the accuracy of a digital elevation model (DEM).
The test is based on profile sampling carried out photogrammetrically by participants of the ISPRS, Commission III, W.G.3. Three different types of landscapes are investigated.
Giving a fixed sample spacing and the precision of the DEM-data points, the method yields an accuracy estimate for a DEM in the area considered.

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1. INTRODUCTION

The influence of the type of terrain on the accuracy of a digital elevation model is one of the elements of an international test, initiated by K. Torlegård, Stockholm. The test was organized under ISPRS, Commission III, Working group 3, and is described by Torlegård (1982).

In this test 6 different locations were chosen. Aerial photographs of each location was delivered to different photogrammetric institutions.
The test sites are listed in table 1.

Photoscale	Location	Country	Type of Terrain
1:30.000	Uppland	Sweden	Farmland
1:20.000	Stockholm	-	Urban area
1:30.000	Bohuslän	-	Granite bedrock
1:17.000	Drivdalen	Norway	Mountains
1: 4.000	Hannover	West Germany	Smooth terrain
1:10.000	Soehnstätten	-	Hills

Table 1.

On each location a specific area was chosen in which a digital elevation model was measured and calculated by the participants. In three of these locations: Uppland, Bohuslän and Drivdalen the participants of the tests were also asked to measure terrain profiles by photogrammetry in the same photos as the DEM. The profiles were defined by marking the end points of the profiles on aerial photographs.

Profiles measured by the following participants have been available for the authors:

A: Technical University of Norway, Trondheim, Norway.
B: Riverside County Flood Control and Water Conservation District, U.S.A.
C: Technical University, München, West Germany.
D: University of Technology, Budapest, Hungary.

In order to establish a "ground truth" in the WG.3 experiments, low altitude photographs were taken as shown in table 2.

Photoscale	Location
1:6.000	Uppland
1:5.300	Bohuslän
1:10.000	Drivdalen

Table 2.

Profiles were measured by the authors in these photos on a Zeiss Planicomp analytical plotter at the Geodetic Institute, Copenhagen. In the following tables these measurements are indicated by E.

2. METHOD

The statistical properties of the terrain surface are described by an "energy spectrum" compiled from measurements of profiles. The undulations of the terrain surface are plotted as function of the wavelengths of the undulations.

The numerical values are derived by a Fourier transformation of the heights in the profile, and are shown in a double logarithmic graph. The capability of this approach is only to show the stochastic undulations of the terrain. Surface features such as break lines cannot be described by this method and terrain types with many break lines have therefore been omitted. Non-random features such as a gradual slope or a bend in the profile must be removed as a trend from the height measurements before the Fourier transformation. In order to express the total energy of the profile, the Fourier coefficients must be multiplied by the length of the profile. (Frederiksen and Jacobi, 1982).

The spectra from the individual profiles are first smoothed, and the resulting spectrum is computed as an average of several profile spectra from the same area, as described by Frederiksen (1981). Some participants have measured 2, some 3 and other 4 profiles in each area.

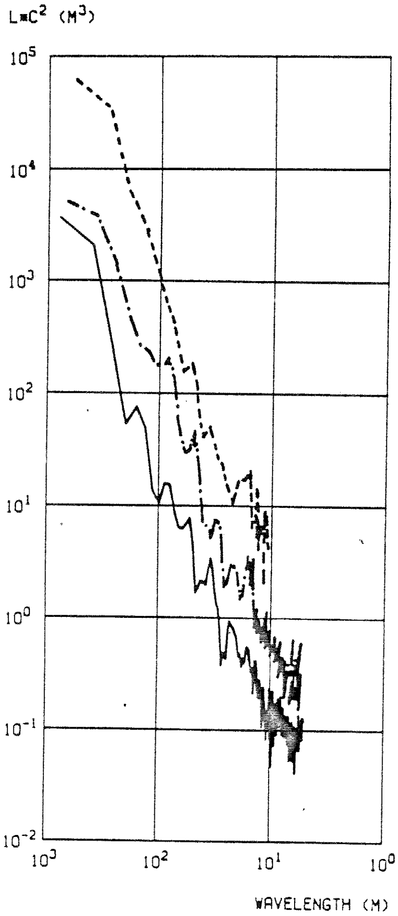


Fig. 1. Energy spectra from:

------- Drivdalen
—·—·— Bohuslän
——— Uppland

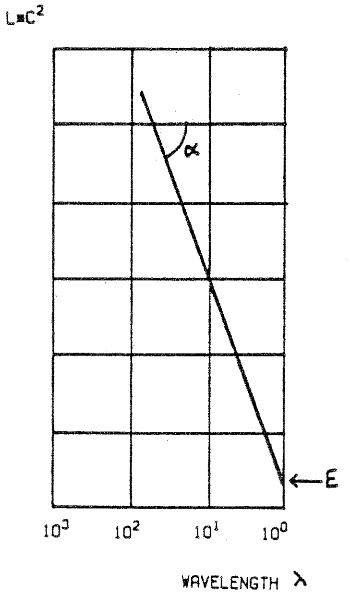
Each spectrum is estimated from 4 profiles measured in large scale photographs in an analytical plotter by E. The sampling interval was 5 meters in Drivdalen and 2.5 meters in the other landscapes.

3. ACCURACY PREDICTION

The energy-spectrum can be used to predict the accuracy of a digital elevation model. The error resulting from sampling a continuous surface by a finite number of discrete points can be described as a function of the surface-spectrum and the distance between the sampling points. If a surface with a known energy-spectrum is measured by points in a grid with the sample spacing, Δx, the standard deviation s_o due to the sampling process is given by:

$$s_o^2 = \sum_{\lambda=2\cdot\Delta x}^{o} P_\lambda \quad ; \quad P_\lambda = L\cdot C_\lambda^2$$

where P_λ is the spectral value corresponding to the wavelength λ (Frederiksen, Jacobi and Justesen, 1978). In order to estimate the standard deviation the energy-spectrum of the terrain must be extrapolated by an analytical function for wavelengths smaller than $2\cdot\Delta x$. As no theory is yet available to give an idea of a useful function, the most simple approach is used, i.e. a straight line in the log/log coordinate system:



$$\log(L\cdot C_\lambda^2) = \log E + \alpha \cdot \log(\lambda)$$

This approximation is equivalent to

$$L\cdot C_\lambda^2 = E \cdot \lambda^\alpha$$

Now the standard deviation can be expressed as:

$$s_o^2 = \frac{E\cdot(2\cdot\Delta x)^{\alpha-1}}{\alpha-1}$$

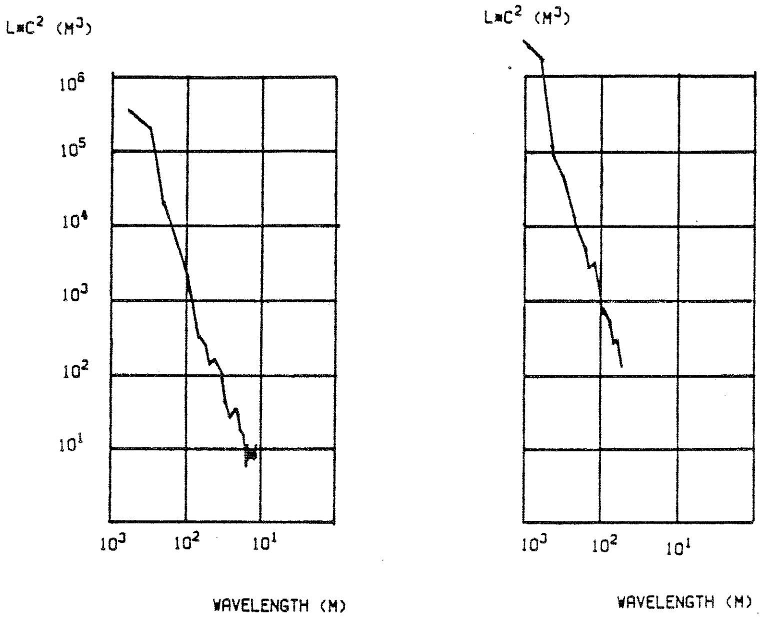
Fig. 2. Straight line approximation.

It is not easy to approximate or extrapolate the spectrum by a straight line. There are two contradictory considerations to take. The line should be placed according to the part of the spectrum with the shortest wavelengths since this has the greatest impact on the standard deviation. Unfortunately this particular part of the spectrum is deformed due to aliasing, so on the other hand, the spectral values corresponding to the shortest wavelengths must not influence the approximation too much. In the following examples the results are based on straight lines placed "by eye". However, the straight line approximation can be carried out applying a linear least squares fit to the spectral estimates (cf. Pike and Rozema, 1975), but the difficulties noted above can hardly be avoided.

4. THE ISPRS TEST AREAS

Drivdalen, Norway.

Profiles from the test area Drivdalen have been measured by 4 groups: A,B, D and E. In figure 3, spectra estimated from profiles measured by A and D are shown. The profiles from A were sampled with a point distance of 30 meters, while the profiles from D were sampled with a spacing of 5 meters. The profiles from A,B and D were measured in photos of scale 1:17.000, while the profiles from E were measured at scale 1:10.000.



Measured by D

Measured by A

Fig. 3. Spectra of Drivdalen, Norway.

The spectrum of Drivdalen has a rather steep slope between wavelength 100 meters and 1000 meters. This feature can clearly be seen in Fig. 1. Between 100 meters and 10 meters the inclination is somewhat less. From A, only the longwave part of the spectrum can be calculated, and because this part has a steep inclination, a straight line fit on to the spectrum will yield a higher value of α than a line fit on to the lower part of the spectrum from D.

In table 3, the coefficients for the straight line approximations of the spectra measured by the 4 participants are listed. The inclination angles are smaller for the two spectra calculated from dense profiles, and the value of E is greater. This is consistent with the form of the spectra discussed in connection with figure 3.

Participant	Sample spacing Δx meter	Coeff. for straight line approx. of terrain spectrum	
		α	E
A	30	2.80	$2.56\cdot10^{-3}$
B	30	2.95	$1.59\cdot10^{-3}$
D	5	2.58	$8.12\cdot10^{-3}$
E	5	2.64	$6.00\cdot10^{-3}$

Table 3. Drivdalen, Norway.

Predicted standard deviations for DEM at 4 different grid sizes

Participant	Δx=10 meters	Δx=20 meters	Δx=30 meters	Δx=50 meters
A	0.56	1.04	1.50	2.38
B	0.53	1.04	1.55	2.54
D	0.76	1.32	1.82	2.73
E	0.71	1.25	1.74	2.64

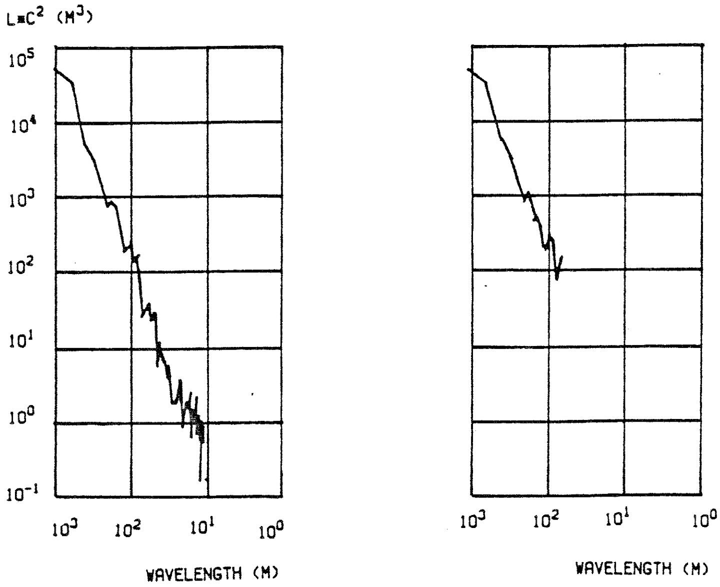
Table 4. Drivdalen, Norway.

If the terrain has a spectrum $L\cdot C^2 = 2.56\cdot10^{-3}\cdot\lambda^{2.80}$ as assumed in the case of participant A, and a digital elevation model was measured by points spaced 10 meters apart in a grid, the standard deviation between the terrain and the DEM equals 0.56 meters. It is, however, assumed that no measuring error is present in the data points. If the grid size is 20 meters, the standard deviation equals 1.04 meters, if $\Delta x = 30$ then $s_o = 1.50$ meters.

The standard deviation is directly related to the part of the spectrum having wavelengths smaller than twice the distance between the data points. In case A and B lower values for the spectrum are estimated for the shortwave part than in case D and E. Consequently a smaller standard deviation is predicted in the case for D and E.

Bohuslän, Sweden.

Profiles from the test area Bohuslän have been measured by 4 groups: A,B, D and E. In figure 4, spectra derived from profiles, measured by B and by D are shown. The profiles from B were sampled with a point distance of 30 meters, while the profiles from D were sampled with a spacing of 5 meters. The profiles from A,B and D were measured at scale 1:30.000, while the profiles from E were sampled at scale 1:5.300.



Measured by D

Measured by B

Fig. 4. Spectra of Bohuslän, Sweden.

Participant	Sample spacing Δx meter	Coeff. for straight line approx. of terrain spectrum	
		α	E
A	30	2.59	$1.14\cdot 10^{-3}$
B	30	2.51	$1.88\cdot 10^{-3}$
D	5	2.11	$4.06\cdot 10^{-3}$
E	2.5	2.15	$4.69\cdot 10^{-3}$

Table 5. Bohuslän, Sweden.

Predicted standard deviation for DEM at 4 different grid sizes

Participant	Δx=10 meters	Δx=20 meters	Δx=30 meters	Δx=50 meters
A	0.29	0.50	0.69	1.04
B	0.34	0.57	0.78	1.14
D	0.32	0.47	0.59	0.78
E	0.36	0.53	0.67	0.90

Table 6. Bohuslän, Sweden.

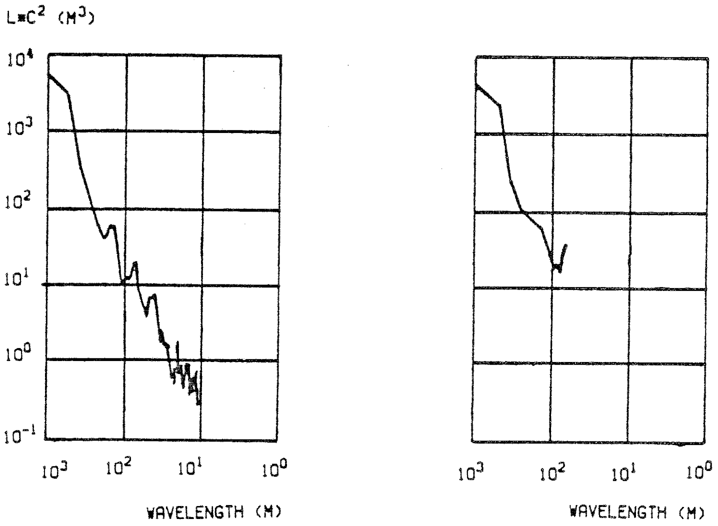
As was the case with Drivdalen, the spectra measured with a longer point-spacing Δx yield a somewhat steeper inclination and a smaller E.

Compared with Drivdalen, the standard deviation due to surface undulations is of only half the size. For small sample spacings no significant differences between the predicted values are observed. In contrast to the results of Drivdalen, the values for A and B increase more rapidly than is the case for D and E.

However, it only makes sense to state that for Δx = 5o meters the contribution from the terrain undulations to the resulting accuracy of a DEM is approximately 1.0 meter.

Uppland, Sweden.

Profiles from the test area Uppland have been measured by 5 groups: A,B, C,D and E. In figure 5, spectra estimated from profiles measured by C and by D are given. The profiles from C were sampled with a point distance of 30 meters, while the profiles from D were sampled with a spacing of 5 meters. The profiles from A,B,C and D were measured at scale 1:30.000, while the profiles from E were measured at scale 1:6.000.



Measured by D

Measured by C

Fig. 5. Spectra of Uppland, Sweden.

Participant	Sample spacing Δx meter	Coeff. for straight line approx. of terrain spectrum	
		α	E
A	30	2.21	$2.08 \cdot 10^{-3}$
B	30	2.35	$7.35 \cdot 10^{-4}$
C	30	2.28	$1.10 \cdot 10^{-3}$
D	5	2.14	$1.47 \cdot 10^{-3}$
E	2.5	2.20	$9.77 \cdot 10^{-4}$

Table 7. Uppland, Sweden.

Predicted standard deviation for DEM at 4 different grid sizes

Participant	Δx=10 meters	Δx=20 meters	Δx=30 meters	Δx=50 meters
A	0.25	0.39	0.49	0.67
B	0.18	0.28	0.37	0.52
C	0.20	0.31	0.40	0.56
D	0.20	0.29	0.37	0.50
E	0.17	0.26	0.33	0.45

Table 8. Uppland, Sweden.

No significant differences are found between the error estimates of the standard deviations. When a terrain is so flat as is the case with Uppland, all the spectra values are small and it is easy to fit a DEM to the terrain without any appreciable error.

5. COMMENTS

The standard deviation between a digital elevation model and the terrain is a sum of different errors in the measuring process. In this paper we only treat the standard deviation arising from the fact that a continuous surface is represented by discrete points. Only measurements in regular grids are considered, and we assume that the terrain consists of only random undulations and that no break lines or other systematic features are present. To this standard deviation we must add errors from the measuring process, errors from the instruments and from the mathematical interpolation algorithm (Tempfli, 1982), (Östman, 1983).

In a flat landscape like Uppland, the contribution from the terrain undulations between grid points is of little importance compared with errors from for instance photogrammetric measurements in high altitude photos. In a rough terrain like Drivdalen, a very dense grid must be applied in order to achieve a small standard deviation.

Our knowledge of true terrain spectra is limited. The profiles from which the spectra are derived are sampled by a limited number of points with a fixed sample spacing. We have therefore no precise information of the very shortwave part of the spectrum. By approximating the spectrum with a straight line, we make an extrapolation from the estimated part of the spectrum to the part, which has not been measured. This extrapolation is based on a supposition of self-similarity of the terrain (Frederiksen, Jacobi and Kubik, 1983).

6. CONCLUSION

Although there are some differences between the predicted standard deviations for a fixed Δx, an average s_o has been calculated for each terrain type. In figure 6 the standard deviations are plotted as functions of the grid size Δx for the 3 terrain types.

The graphs show that in a flat terrain like Uppland the standard deviation increases slowly, when the distance between the measured points is widened. With a grid size of 50·50 meters a better digital elevation model is obtained in Uppland than it is in Drivdalen with a 10·10 meters grid. We can increase the grid size to 62·62 meters in Uppland and still get the same result as in Drivdalen.

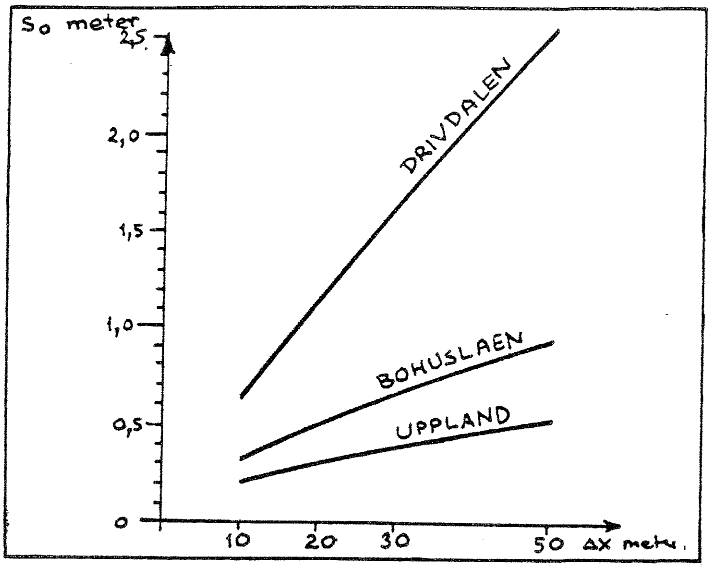


Fig. 6. Predicted standard deviations in the test areas as function of grid-size.

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