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INVITED PAPER

OH, GROSSERORORS!

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Abstract

This paper gives a review of the concept of "gross errors", their causes
and the methods of elimination. In particular, the interrelationship of
"gross errors" with the method of least squares is discussed. Test theory,
variance estimation and robust estimation techniques are reviewed as reme-
dies for the phenomenon "gross errors".

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1. WHAT ARE GROSS ERRORS?

What are gross errors? There are mainly the following explanations, which combined give the answer.

Often, in adjustment, one is using very simple probability distributions for the errors, such as normally distributed errors. Classical statistics derives results under the assumption that these models are strictly true. However, these models are never exactly true.

Deviations from these models may be due to the occurance of blunders in measuring, wrong decimal points, wrong point numbers, errors in copying or because just "something went wrong". These "gross errors" may occur as clean outliers or as "hidden contamination" which usually cannot even be detected. Their frequency depends of course greatly on the quality of the data, but some figures may be reveiling.

The raw data in photogrammetric blocks involve typically about 1-5% of the beforenamed gross errors, mainly errors in point numbering, and errors in the identification and measurements of tie points. The Geodetic Institute of Denmark reports for small-scale (1:140.000) bundle adjustments in Greenland that the gross error frequency is 2%, with o.5% point numbering errors, o.5% ground control errors and 1.0% sloppy or errorness transfer of tie points. In the mainland of Denmark the error frequency is much lower, with practically only point numbering errors occuring. Among the reasons for the lower frequency in the mainland are larger scale and presignaled, high quality ground control.

The error model (normal distribution) may also be only an approximation anyway. Even high-quality samples in astronomy and geodesy, consisting of several thousands measurements each, which should be prime examples for the "normal law of errors", proved to be mildly but definitely leptocurtic (longer-tailed). This means that large errors occur more frequent than explainable by the normal distribution.

There are a number of examples known showing these longer-tailed distributions, including the large ones in Romanowski and Green (1965) and the old ones by Bessel (1818). The Geodetic Institute of Denmark reports for typical geodetic triangulation nets with 10.000 observations and 5.000 unknowns, that large residuals occur 10 times as frequent as explainable by the normal law of errors (50 large residuals versus 5, after elimination of blunders). The deviation of the distribution from its simple approximation tends to be explained by "gross errors", which then are rejected. Gross errors due to film distortion and refractive influences are popular in photogrammetry.

Gross errors, causing even slight and harmless-looking deviations from the distribution model render classically "optimal" procedures such as least squares adjustment rather inefficient and bad. The first basic reference here is Tukey's (1960) survey paper on early research at Princeton University. A lot of experimental evidence is also contained in Andrews et.al. (1972), who shows how bad even the arithmetic mean might be for distributions slightly deviating from the normal.

The question arises, how practical people could get along until now with least squares estimators such as the arithmetic mean, or least squares block adjustment procedures in photogrammetry. Small avoidable losses by the inadmissible use of these procedures will have gone entirely unnoticed.

Also, the habit of "throwing away" any strays before doing least squares adjustment is rather common.

However, there is an increasing danger in that more and more data are automatically processed on a computer without being looked at by a competent person. Indeed, some users of least squares adjustment have recognized this danger. Feeling the urgent practical need to suppress unwanted outliers in their computations, they robustify their programmes against gross errors on their own, using intuitive, trial and error procedures. (Masson d'Autumn, 1971).

Sometimes, it is objected, that results of mathematical statistics, like the Gauss-Markov theorem, yield the arithmetic mean as optimal even if nothing is known about the error distribution. In such cases the fallacy lies in assuming, that we always want to estimate the expected value of the observations, however wrong they might be. (We rather want to estimate roughly some central value in the bulk of the data). And even if we impose symmetry of the distribution function and existence of its mean and variance, then the conditions under which least squares is an optimal method, linearity and unbiasedness, are far too severe restrictions and of doubtful value anyhow.

2. THE DOGMA OF NORMALITY

The dogma that measuring errors should be distributed according to the normal law is still widespread amongst users of the method of least squares. We should remember, however, that the whole theory of estimation, and in particular the least squares method, originates with problems where almost all of the statistical variability is due to measurement errors only.

Statistical variability is regarded as just a nuisance to get rid of, and one is mainly interested in finding that combination of the measurements which lies on the average nearest to the true value.

It is illuminating to witness how the normal, or Gaussian, distribution was introduced by Gauss himself. We quote Gauss (1821):

The author of the present treatise, who in the year 1797 first investigated this problem according to the principles of the theory of probability, soon realized that it was impossible to determine the most probable value of the unknown quantity, unless the function representing the probability of the errors is known. But since it is not, there is no other recourse than to assume such a function in a hypothetical fashion. It seemed most natural to him to take the opposite approach and to look for that function which must be taken as a base in order that for the simplest of all cases a rule is obtained which is generally accepted as a good one, namely that the arithmetic mean of several observations of equal accuracy for one and the same quantity should be considered the most accurate value. This implied that the probability of an error x must be assumed proportional to an exponential expression of the form e^{-hhxx}, and that then just the same method which he had found by other considerations already a few years earlier, would become necessary in general. This method, which afterwards, in particular since 1801, he had almost daily opportunity to use in diverse astronomical computations, and which in the meantime also Legendre had happened upon, now is in general use under the name method of least squares.

Note that Gauss here introduces the normal distribution only to suit the sample mean. Moreover, one can hardly claim that the method of least squares was universally accepted, as Gauss did. Laplace (1793) and others proposed the method of least sum (see chapter 5) instead, and there is a charming contemporary paper (Anonymous, 1821), which states:

> "... The popular method (of least squares) has neither generally been followed nor has it been used without restrictions. For example, there are certain provinces of France where, to determine the mean yield of a property of land, there is a custom to observe this yield during twenty consecutive years, to remove the strongest and the weakest yield and then to take an eighteenth of the sum of the others".

The author then continues to remark that a considerable arbitrariness is involved here: Why should one not exclude the two greatest and the two smallest observations? But nevertheless he does not believe that all observations should end with the same weight into the determination of the mean.

The concept of "gross errors" is thus closely related to the assumption of normally distributed errors and the method of least squares. Also the concept of "systematic errors" is related to the abovementioned concepts, and all these concepts are interchangeable, as can be seen from the following historical example:

In early photogrammetric strip triangulations, coordinate errors or "strip deformations" seemingly showed a linear dependence on strip lenght, but with unexplainable sudden changes roughly after every 6th model. These sudden changes were explained as "gross errors" of the operators. It was von Gruber (1935), who later explained these strip deformations as due to "systematic errors" in the triangulation instruments, superimposed by some random effects; the era of polynomial strip adjustment and the search for systematic instrument errors was started. The era lasted until Vermeir (1954) and Ackermann (1965) demonstrated in a convincing manner, that strip deformations may also be explained by a double summation of normally distributed random errors. This started the development of the "Anblock " method (Hout, 1966) and made the earlier concepts of gross errors and systematic errors in strip triangulation redundant.

Also in recent times, we notice similar developments. In the early 1960's photogrammetric bundle adjustment was introduced and propagated as the final solution to photogrammetric triangulation. But quickly it was realized that corrections to the assumption of normality had to be allowed, and "systematic image errors" were introduced (Schmidt 1971, Masson d'Autumn, 1971). These systematic errors were originally valid for the whole block, later varied from strip to strip and even from image to image. Finally, the concept of "gross errors" was necessary to explain the remaining discrepancies from normally distributed errors (Grün, 1980). Extrapolating history, we may expect as next step totally novel photogrammetric adjustment methods, based on other models.

Modern theoretical developments in statistical estimation methods can guide us ahead, leaving behind us classical least squares and the assumption of normal distribution. After treating the classical "random errors" of "equally good" observations (only due to small measurement

errors or to inherent variability of the material), and after, to some
extend, controlling "systematic errors", theory finally started to deal
explicitly with "gross errors" and contaminated distributions. The mathe-
matical tools have improved, and owing to the development of computers
and Monte Carlo simulation methods, statistics now is a part of the ex-
perimental sciences, where different (non-least squares) adjustment methods
may be studied experimentally as to their efficiency, as evidenced by the
book of Andrews (1972).

But we are only at the beginning. We know a lot about modern and new
estimation techniques, and we still discover many open questions there.
We badly need more and better techniques for finding accuracy parameters,
to make more estimation methods practically useful. Much still has to be
done.

3. REJECTING GROSS ERRORS - THE CONSERVATIVE APPROACH

Photogrammetric engineers safeguard themselves against "gross errors"
since long. Knowing from experience about the devastating effects of
gross errors on least squares adjustment, they use arial triangulation
methods, which enable the detection of deviations from normal distribu-
tion ("gross errors" and "systematic errors") in an early stage of tri-
angulation. At first, photogrammetric models and strips are formed, which
enables the detection of a large part of the undesireable effects. Then,
strips are adjusted and joined together into a block assembly by simple
means, again controlling the discrepancies found and rejecting or re-
viewing suspecious data.

Finally, the block adjustment is performed using conservative ground
control arrangements to allow the detection of errors in the ground con-
trol. In this manner, praxis robustified the compulations against deviat-
ions from normality. The problem of blunders and systematic errors became
again enhanged by the introduction of high accuracy instruments, high per-
formance least squares block adjustment methods without precedings model -
and strip formations, and by the use of - theoretically justifyable - sparce
ground control arrangements.

One way of tackling gross errors is to reject all errors, which do not
fit into the assumption of normal distribution. The matter is complicated
by the fact, that we only know estimates of the errors after least squares
adjustment, where gross errors have been smoothed and distributed over
many observations, and thus are difficult to recognize. Also, the largest
residuals do not necessarily indicate the correct location of the blunders
(for demonstrative examples see Crüger et al, 1984 in this volume).

To localize a gross error, the estimated magnitude, $\tilde{e}_i$, of each error e_i
is tested whether it deviates significantly from 0. The test statistic
used is the normalized residual

$$t = \frac{|\tilde{e}_i|}{\sigma_{\tilde{e}_i}} \tag{1}$$

which is Student t-distributed with 1 degree of freedom. The symbol $\sigma_{\tilde{e}}$
denotes the standard deviation (mean square error) of the error estimate
$\tilde{e}$ and it may be calculated in the course of least squares adjustment. On-
ly the (locally) largest significant errors are rejected as probable gross

errors and the adjustment and test is repeated, until no further gross errors are located.

The test (1) is a well-known tool of classical statistics (cf. Wilks 1962), and it has been propagated and modified for geodesy by Baarda (1968) and Pelzer (1976). Today, many variants exist, cf. El Hakim (1981), Stefanovitz (1980) a.o.

A simple example may illustrate the method:

Consider the mean value to be estimated from the sample Z_i = (10,11,11,12,100) with standard deviation σ_{Z_i} = 1 unit. (2)

The computations for this example are summarized in table (1). This example shows the importance of only rejecting the most significant residual, since all residuals are significantly larger than the treshold value in the first computation cycle.

$1°$ cycle	sample Z:	10	11	11	12	100	$\bar{z}$ = 29	
	residuals $\|\tilde{e}\|$:	19	18	18	17	71	$\sigma_{\tilde{e}}^2 = 4/5 \cdot \sigma_Z^2$	
	normalized residuals $\|\tilde{e}\|/\sigma_{\tilde{e}}$	21	20	20	19	79	$t_{5\%,1}$ = 6.7	$\max (\frac{\|\tilde{e}\|}{\sigma_{\tilde{e}}}) > t_{\alpha=5\%,1}$ indicates blunders
$2°$ cycle	sample Z:	10	11	11	12	·	$\bar{z}$ = 11	
	residuals $\|\tilde{e}\|$:	1	0	0	1	·	$\sigma_{\tilde{e}}^2 = 3/4 \cdot \sigma_Z^2$	
	normalized residuals $\|\tilde{e}\|/\sigma_{\tilde{e}}$	1.2	0	0	1.2	·	$t_{5\%,1}$ = 6.7	$\text{all} (\frac{\|\tilde{e}\|}{\sigma_{\tilde{e}}}) < t_{\alpha=5\%,1}$

Table 1: Test for gross errors.

Pitfalls of the method are:

- the test is one-parametric, i.e. only one gross error is detected within each computational cycle, and wrong decisions about the exclusion of observations increase in probability with the number of gross errors in the data;

- gross errors remain unnoticed with relatively high probability. Wrong decisions in the rejection/acceptance of gross errors seriously effect the mean estimates;

- no simple accuracy expressions are known for the resulting mean estimates, including the risk of wrong decisions. The concept of reliability was introduced by Baarda (1968) in addition to the conventional accuracy parameters. Reliability describes, with which probability gross errors of different magnitudes still may be present in the adjusted results.

Praxis, however, requires unique and simple accuracy statements and confidence limits for the results instead of two or more unrelated quantities;

- the accuracy σ_z should be known a-priori in order to make testing possible.

Extension of the test theory to a multiparametric theory and the deviation of unique and simple accuracy statements is very difficult and complicates the usefulness for photogrammetry. It is our feeling that test theory has reached its practical limits of applicability and other ways have to be followed to tackle the phenomena "gross errors".

4. ARE GROSS ERRORS NORMALLY DISTRIBUTED?

Another approach of tackling gross errors regards all observations z_i as normally distributed, but with different variances. In this model, a "gross error" is interpreted as normally distributed observation with a very large variance. We assume the variances of the observations to be unknown, and they are estimated together with the other conventional unknown in the adjustment. The variance of the individual observation z_i may in principle be estimated from the residuals ($-e$) after adjustment, using the following formula:

$$\hat{\sigma}^2_{z_i} = \tilde{e}^2_i \qquad\qquad (3)$$

The least squares adjustment is repeated with the newly estimated variances of the observations and weights

$$p_i = \frac{1}{\hat{\sigma}^2_{z_i} + \varepsilon} \qquad (\varepsilon \text{ small constant})$$

until the results converge. Samples with very low weight are then interpreted as "blunders". Estimation formula (3) gives highly biased estimators for $\hat{\sigma}^2_{z_i}$ and should preferably be modified before use (cf.Kubik, 1967).

This method of variance estimation was for the first time proposed by Helmert (1924) and further elaborated by Kubik (1967 and 1970) and Ebner (1972). The method was independently proposed in statistical literature (Rao, 1977) and since then periodically rediscovered (Förstner, 1979, Li, 1983, a.o.). The method was successfully applied for variance estimation in geodesy, estimating the weights for different types of observations, but very little experience is gathered with the location of gross errors.

Table 2 summarizes the application of the method to example (2). The mean estimate stabilizes at 11.97 after 8 iterations.

Critical points in the method are:

- the measuring accuracy should be known a-priori, and the weights should not be allowed to increase above the corresponding a-priori weights, otherwise the method proceeds to exclude observations until only a minimum number of samples remain, necessary for an unique determination of the problem.

- confidence limits are not known for the solution. Maximum likelihood theory provides us only with asymptotic confidence limits, which are equal to the accuracy results of conventional least squares. But these confidence limits are only approximately valid for many repetitions of the individual measurements and are not useful for actual cases of photogrammetric adjustments (cf. Kubik, 1970).

Further work on these methods seems possible and useful, in particular on improvement of the variance estimate (3) and on confidence intervals.

Sample no.	weights					weighted mean	residuals				
	1	2	3	4	5		1	2	3	4	5
1°cycle	1	1	1	1	1	29	19	18	18	17	71
2°cycle	0.03	0.03	0.03	0.03	0.0002	12.4	2.4	1.4	1.4	0.4	87.6
3°cycle	0.17	0.50	0.50	5.4	0	11.8	1.8	0.8	0.8	0.2	88.2
4°cycle	0.3	1.5	1.5	19.8	0	11.8	1.8	0.8	0.8	0.2	88.2
5°cycle	0.3	1.4	1.4	28.4	0	11.9					

Table 2: Variance estimation ($\varepsilon = 0.01$)

5. ROBUST ESTIMATION - REJECTING NORMALITY

It would be ideal to have estimation methods, which would always yield nearly optimal results, independent of the actual error distribution. In particular, the estimators should be uninfluenced by large errors and the estimators should be constructed as central values of the bulk of data.

This concept of robust estimation was formally defined by Kendall (1948), although it was used already by Laplace in 1793, who proposed to use the median (central value of observations ordered according to size) as an estimator of the mean. The concept of robust estimation was especially developed at Zürich University by Huber (1964) and Hampel (1973) and at Princeton by Andrews et al (1972). For application of these methods to geodesy refer to Carosio (1979) and Borre et al (1983).

The characteristic for these methods is, that not the square of the errors is minimized, but another properly chosen function

$$\Sigma\varphi(e) \to min \qquad (4)$$

A simple example is $\varphi(e) = abs(e)$ (least sum method), leading to the estimation of the mean by the median. At present, there exist many proposals for the choice of the function $\varphi(e)$, (Andrews, 1972 ; Rey 1978), but a unifying theory for classification and comparison of these methods is still lacking. Only one thing is quite clear: the method of least squares is inferior to all robust estimation methods for errors deviating from normal.

The numerical solution of the adjustment principle (4) may be done iteratively, by successive application of the method of weighted least squares.

The iteration starts with the apriori given weights of the observations and a conventional least squares adjustment (assume for the moment all observations have the same weight). In the next iteration new weights p are computed for the individual observations from their residuals in the foregoing adjustment, and the least squares method is repeated with these new weights. The weights p of the individual observations follow from (4) to

$$p(e) = \frac{\partial \varphi}{\partial e} / e \qquad \text{*)} \qquad (5)$$

The iteration continues until convergence is achieved (usually 3-20 iterations are necessary). For our example $\varphi(e) = |e|$ and for Hubers method the weights are:

least sum method: (6a)

$$p(e) = \frac{1}{|e|+\varepsilon} \qquad \varepsilon \text{ small constant}$$

Hubers method: (6b)

$$p(e) = \begin{cases} 1 & \text{for } |e|/\sigma \leq a \\[2em] \dfrac{a}{|e|/\sigma} & \text{for } |e|/\sigma > a; \quad a = 1.5 \end{cases}$$

with σ denoting the estimated or apriori standard deviation of the observations.

The least sum method was applied to estimation problem 2, the results are summarized in table (3). The mean estimate is 11.01 after 20 iterations.

Sample no.	weights 1	2	3	4	5	weighted mean	residuals 1	2	3	4	5
1° cycle	1	1	1	1	1	28.8	19	18	18	17	71
2° cycle	0.05	0.06	0.06	0.05	0.01	16.2	6.2	5.2	5.2	4.2	83.8
3° cycle	0.16	0.19	0.19	0.23	0.01	12.4	2.4	1.4	1.4	0.4	87.6
4° cycle	0.4	0.7	0.7	2.2	0.01	11.7	1.7	0.7	0.7	0.3	88.3
5° cycle	0.6	1.4	1.4	3.5	0.01	11.5	1.5	0.5	0.5	0.5	88.5
6° cycle	0.6	1.7	1.7	2.4	0.01	11.4	1.4	0.4	0.4	0.6	88.6

Table 3: Least sum estimation of mean ($\varepsilon = 0.01$).

Note, that blunders are successively weighted down, while correct observations equally contribute to the final result, although they may have been weighted down temporarily in some of the iterations. Weights should not be

*) By rewriting the "normal equations", $\Sigma \frac{\partial \varphi(e)}{\partial x} = 0$; x unknowns; in form of the conventional normal equations $\Sigma p(e) \cdot e = 0$ these weights p(e) follow to $p(e) = \omega(e)/e = \frac{\partial \varphi(e)}{\partial x} / e$

allowed to increase beyond the initial values to avoid preferance of a few accidental sample values.

In the Danish Method (Krarup 1967, Krarup et al 1980, Kubik 1982) the unknowns are estimated from the bulk of consistent observations. Outliers which (hopefully) are in minority, are rejected. The Danish Method may be regarded as robust estimation method with weights

$$p(e) = \begin{cases} 1 & \text{for } |e| < 2\sigma \\ \exp\left(-\dfrac{|e|^2}{2\sigma^2}\right) & \text{for } |e| > 2\sigma \end{cases}$$

Table (4) shows the application of the Danish Method for our estimation problem (2).

Sample no.	weights 1	2	3	4	5	weight. mean	residuals 1	2	3	4	5
1° cycle	1	1	1	1	1	28.8	19	18	18	17	71
2° cycle	$1.7 \cdot 10^{-77}$	$1.6 \cdot 10^{-69}$	$1.6 \cdot 10^{-69}$	$5.1 \cdot 10^{-62}$	0	12.0	2	1	1	0	88
3° cycle	1	1	1	1	0	11.0	1	0	0	1	89

Table 4: Mean estimation by the Danish Method.

The Danish Method has favourable properties as compared to other robust estimation methods, both with regard to blunder detection and speed of computation. It is used as standard adjustment method both at the Danish Geodetic Institute and Aalborg University. At the Geodetic Institute the method is regarded as a mean to produce a list of candidates for further inspection and/or remeasurements. Actual rejection is only done after human inspection. After this screening rigorous least squares adjustment is performed.

For a detailed review of the robust methods and examples of applications refer to the paper of Crüger et al (1984) in this volume and to Crüger and Kubik (1983).

Critical points of robust methods are:

- the measuring accuracy should be known a-priori, and the weights should not be allowed to increase above a pregiven maximum, otherwise the method proceeds to exclude more and more observations;

- no simple confidence limits are known for the solution. The theory for confidence limits is given in Andrews et al (1972), but this theory must still be further elaborated and expanded in order to obtain simple accuracy parameters suited for praxis.

Robust estimation seems the most promising alternative for tackling "gross errors". There is already a wealth of theoretical and experimental experience available with the use of these methods, and large efforts are made by statisticians to further elaborate the theory of the methods.

6. WHICH WAY TO GO?

Having visited the various aspects of gross errors, we may conclude that gross errors do not exist as independent entities. They are indivisibly connected with the dogma of normal distribution and the method of least squares. In future we may go two ways: either accepting the method of least squares and thus also gross errors, or chosing new adjustment methods, in which case both the concepts of least squares and gross errors vanish. The choice may be simplified in that formally both ways have many aspects in common, although they are based on quite different theories.

In any case, the question of proper confidence limits has to be solved. For all methods, we lack proper and practically applicable confidence limits for the results of photogrammetric adjustment. This leaves enough chances to a new generation of photogrammetrists to become reknown.

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