THE USE OF AUXILIARY DATA IN PHOTOGRAMMETRIC BLOCK ADJUSTMENTS OF STEREOMODELS

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Abstract:

Auxiliary airborne and terrestrial data of appropriate accuracy can contribute to improve the results of photogrammetric block adjustments while optimizing the field control requirements. In the context of SPACE-M block adjustments using independent stereoscopic models, different types of auxiliary data have been incorporated into the block adjustment procedure using coordinate difference methods. Such approaches are described with discussions of the results obtained with various types of simulated and real auxiliary data. The implications for aerial triangulation planning with SPACE-M are outlined for topographical mapping and related applications.

0. INTRODUCTION

Auxiliary airborne and terrestrial data have long been considered as complementary information in photogrammetric block adjustments. The principal objectives have always been in terms of constraining the adjustment in some desirable manner to optimize the results while minimizing the field control requirements.

Block adjustment methods using independent stereomodels are widely used for topographical mapping and related applications. There are numerous formulations for such adjustments which are based on various mathematical and statistical approximations. The SPACE-M approach [Blais, 1976, 1977 and 1979] is based on a spatial similarity formulation per stereomodel without any special consideration for the perspective centres which greatly facilitates the use of auxiliary airborne and terrestrial data.

With the rapidly changing technology, such auxiliary data exhibit a wide spectrum of characteristics and accuracies. Some examples of auxiliary airborne data include the measurements of Airborne Profile Recorders, statoscopes, inertial and satellite navigation systems. For auxiliary terrestrial data, local surveys using "total-station" surveying instruments, satellite receivers and inertial surveying systems provide station coordinates often affected by unknown translation and/or orientation and/or scale biases with respect to the geodetic reference system.

Such information about the relative positions of exposure and/or ground stations can clearly contribute to a photogrammetric block adjustment provided that accuracies are compatible. In the special case of known orientation and scale, single coordinate information can be used essentially as auxiliary lake or waterline data have been used for years in SPACE-M block adjustments. In general, however, coordinate difference techniques for triplets of stations are required to filter out the unknown orientation and/or scale biases per local net.

Coordinate difference methods are especially advantageous with spatial similarity formulations in terms of their generality and the numerical conditioning implications. No explicit modelling of profiles or trends is required with such an approach. Furthermore, the approach does not preclude any filtering or smoothing of the auxiliary data that could be carried out before or after a preliminary block adjustment.

1. OVERVIEW OF THE SPACE-M FORMULATION

The mathematical formulation used in the computer program SPACE-M is based on the spatial similarity relationship between a stereoscopic model and its terrestrial image as estimated by the method of least squares. The implicit assumptions in this approach are that compatible coordinate reference systems are used and that all known systematic errors have been corrected for in the observational information.

For the photogrammetric system, it is customary to use a right-handed Cartesian coordinate system (x,y,z) with x-axis along the base (or parallel to it) and the z-axis positive upward. For the terrestrial system, it is customary in topographical mapping applications to use the Universal Transverse Mercator (UTM) or any similar projection for (X,Y) and heights above Mean Sea Level (MSL) for Z. The reasons for choosing such coordinate systems are discussed in [Blais, 1977 and 1979].

Considering the linearized spatial similarity relating the photogrammetric measurements (x,y,z) to their terrestrial images (X,Y,Z), one has the transformation equations

$$X = ax + by - cz + e$$

$$Y = ay - bx - dz + f$$

$$Z = az + cx + dy + g$$

in terms of the seven unknown parameters a, b, c, d, e, f and g per stereomodel. In a SPACE-M adjustment, these seven unknown parameters per stereomodel are estimated simultaneously and directly using all the relevant observational information and statistical weights using the method of least squares. The perspective centres are treated in exactly the same manner as any other photogrammetric points in the stereomodels.

The rigorous similarity transformation of the photogrammetric stereomodel measurements (x,y,z) using the corresponding estimated parameters a, b, c, d, e, f and g per stereomodel is carried out in the following manner:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = k \begin{bmatrix} \kappa_1 & -\kappa_2 & 0 \\ \kappa_2 & \kappa_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega_1 & -\omega_2 \\ 0 & \omega_2 & \omega_1 \end{bmatrix} \begin{bmatrix} \phi_1 & 0 & -\phi_2 \\ 0 & 1 & 0 \\ \phi_2 & 0 & \phi_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} e' \\ f' \\ g' \end{bmatrix}$$

$$= k \begin{bmatrix} \kappa_1\phi_1+\kappa_2\omega_2\phi_2 & -\kappa_2\omega_1 & -\kappa_1\phi_2+\kappa_2\omega_2\phi_1 \\ \phi_1\kappa_2-\kappa_1\omega_2\phi_2 & \kappa_1\omega_1 & -\kappa_2\phi_2-\kappa_1\omega_2\phi_1 \\ \omega_1\phi_2 & \omega_2 & \omega_1\phi_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{bmatrix} e' \\ f' \\ g' \end{bmatrix}$$

in which

$$k = (a^2 + b^2)^{\frac{1}{2}}$$

$$\kappa_1 \equiv \cos\kappa = a/k$$

$\kappa_2 \equiv \sin\kappa = -b/k$

$\phi_1 \equiv \cos\phi = k/(a^2+b^2+c^2)^{\frac{1}{2}}$

$\phi_2 \equiv \sin\phi = c\ \phi_1/k$

$\omega_1 \equiv \cos\omega = k/(a^2+b^2+d^2)^{\frac{1}{2}}$

$\omega_2 \equiv \sin\omega = d\ \omega_1/k$

and e', f', g' are the modified translation parameters e, f, g so as to ensure unbiased residuals in X,Y,Z respectively [Blais, 1976, 1977 and 1979].

This approach to the least-squares adjustment of blocks of stereomodels is one of the simplest using the geometry of the stereomodels and all the available observational information in an optimal manner. The linearization of the spatial similarity for the least-squares estimation approach does not imply any convergence problems as in most topographical mapping applications, only one or two iterations are required with the usually small levelling angles and possible preadjustment rotations in azimuth of the stereomodels. With very large rotation angles, experience has shown that more than three iterations are very seldom required.

Such a straightforward mathematical formulation for the adjustment of stereomodels is most appropriate for coordinate difference techniques with auxiliary airborne and terrestrial data. As the perspective centres are treated in exactly the same manner as any other photogrammetric points, they can become control points in any absolute or relative sense with/without any additional constraints. These are most important considerations in the context of the following discussion.

2. AUXILIARY DATA WITH UNKNOWN TRANSLATION BIASES

When a local net of aerial and/or ground stations has been positioned up to unknown translations with respect to the adopted geodetic reference system, the corresponding coordinate differences of pairs of such stations can contribute to a block adjustment. The unknown translation biases are therefore eliminated by the formulation which is implemented with each coordinate independently.

Explicitly, considering two such stations with photogrammetric coordinates (x_i', y_i', z_i') and (x_j'', y_j'', z_j'') in the i-th and j-th models respectively, with corresponding ground coordinates (X', Y', Z') and (X", Y", Z"), the observation equations for the coordinate differences are simply

$$a_i x_i' + b_i y_i' - c_i z_i' + e_i - a_j x_j'' - b_j y_j'' + c_j z_j'' - e_j = X' - X''$$

$$a_i y_i' - b_i x_i' - d_i z_i' + f_i - a_j y_j'' + b_j x_j'' + d_j z_j'' - f_j = Y' - Y''$$

$$a_i z_i' + c_i x_i' + d_i y_i' + g_i - a_j z_j'' - c_j x_j'' - d_j y_j'' - g_j = Z' - Z''$$

in which the unknown parameters are subscripted by i or j according to the model index. When the two stations are in the same model then i is equal to j and the translation parameters cancel out. Using the statistical weights (W_X^*, W_Y^*, W_Z^*) associated with the coordinate differences (X' - X", Y' - Y", Z' - Z") respectively, the corresponding contributions to the normal equations can then be formulated directly from these observation equations and added to the normal equations.

Auxiliary data with unknown translation biases per net can therefore contribute in terms of scale and orientation to a photogrammetric block adjustment. The contributions are however restricted to those pairs of stations which do not imply an increase in the bandwidth of the normal equations. The situation is essentially the same as with lake information in the sense that all pairs of stations of a net with contributions within the bandwidth of the normal equations are considered. In all cases, the bandwidth is determined only by the tiepoint connections in the photogrammetric block of stereomodels [Blais, 1976, 1977 and 1979].

Another important point to notice is the fact that the field observations carried out to set up such local networks of aerial and/or terrestrial stations are not directly related to the observations used in the photogrammetric block adjustment with SPACE-M as described above. In fact, the number of contributions to the block adjustment normal equations usually exceeds the number of corresponding field measurements in spite of the bandwidth restrictions.

3. AUXILIARY DATA WITH UNKNOWN TRANSLATION AND ORIENTATION BIASES

When a local net of aerial and/or ground stations has been positioned up to unknown translations and rotations with respect to the geodetic reference system, linear combinations of the corresponding coordinate differences for triplets of such stations can contribute to a block adjustment. The isometric properties of the spatial triangles formed by triplets of stations can clearly provide scale and geometrical information to optimize the block adjustment results.

The explicit formulation for three (distinct) points P_1, P_2 and P_3 with known photogrammetric (x_i, y_i, z_i) and generally biased ground (X_i, Y_i, Z_i) coordinates ($i = 1,2,3$, respectively) is as follows:

$$\alpha\, P_1 + \beta\, P_2 + \gamma\, P_3 = P_o$$

for appropriate scalars α, β, γ subject to the condition that

$$\alpha + \beta + \gamma = 0$$

with P_o being a coplanar point of P_1, P_2 and P_3 defined by the scalars α, β, γ. The coefficients α, β and γ are defined in terms of the lengths of the triangle sides to ensure translation and orientation invariance:

$$\alpha = 2 \cdot |P_2P_3| - |P_1P_2| - |P_1P_3|$$

$$\beta = 2 \cdot |P_3P_1| - |P_2P_3| - |P_2P_1|$$

$$\gamma = 2 \cdot |P_1P_2| - |P_3P_1| - |P_3P_2|$$

in which $|\_\_|$ denotes the spatial distance using the corresponding terrestrial coordinates.

The implementation of such a relation between the coordinates of the three points P_1, P_2 and P_3 in the adjustment requires the availability of approximate terrestrial coordinates for those three points to avoid numerical and other complications. In the general situation, the points P_1, P_2 and P_3 are in three different stereomodels so that P_o can only be approximated using the above coefficients α, β and γ with the approximate terrestrial coordinates for P_1, P_2 and P_3, such as available from a preliminary SPACE-M block adjustment.

The corresponding observation equations can therefore be written as follows:

$$\alpha \begin{pmatrix} X_1' \\ Y_1' \\ Z_1' \end{pmatrix} + \beta \begin{pmatrix} X_2'' \\ Y_2'' \\ Z_2'' \end{pmatrix} + \gamma \begin{pmatrix} X_3''' \\ Y_3''' \\ Z_3''' \end{pmatrix} = \begin{pmatrix} X_o^* \\ Y_o^* \\ Z_o^* \end{pmatrix}$$

where

$$X_1' = a'x_1 + b'y_1 - c'z_1 + e'$$

$$Y_1' = a'y_1 - b'x_1 - d'z_1 + f'$$

$$Z_1' = a'z_1 + c'x_1 + d'y_1 + g'$$

and similarly for (X_2'',Y_2'',Z_2'') and (X_3''',Y_3''',Z_3'''). The unknown coefficients a', b', ..., a'', b'', ..., and a''', b''', ..., correspond to the stereomodels which contain P_1, P_2 and P_3, respectively. The right-hand-side coordinates X_o^*, Y_o^* and Z_o^* are obtained from the approximate terrestrial coordinates P_1, P_2 and P_3 as follows:

$$X_o^* = \alpha\ X_1^* + \beta\ X_2^* + \gamma\ X_3^*$$

$$Y_o^* = \alpha\ Y_1^* + \beta\ Y_2^* + \gamma\ Y_3^*$$

$$Z_o^* = \alpha\ Z_1^* + \beta\ Z_2^* + \gamma\ Z_3^*$$

respectively, using the coefficients α, β and γ obtained above.

One important implication of the condition that $\alpha + \beta + \gamma = 0$ is that the left-hand-side of the observation equations is translation invariant as

$$\alpha(P_1 + T) + \beta(P_2 + T) + \gamma(P_3 + T) = \alpha P_1 + \beta P_2 + \gamma P_3 = P_o$$

for any translation vector T. This situation is essential to avoid unrealistic constraints in the block adjustment as a consequence of using the approximate coordinates X_o^*, Y_o^* and Z_o^* in the observation equations for triplets of relative control stations.

Such triplets of relative control points are also subject to the bandwidth restrictions for the matrix of the normal equations in SPACE-M block adjustments. These restrictions are to the effect that only the triplets of relative control points with contributions within the bandwidth of the normal equation matrix are considered. Otherwise, the computational implications would not be acceptable.

The requirement for approximate terrestrial coordinates is not a serious restriction in practice as a block adjustment is seldom carried out in one pass or iteration. Using a scale factor check between the photogrammetric and terrestrial coordinates in the SPACE-M input data file, the program can take care of this restriction in a manner which is essentially transparent to the user except for a warning message when the scale factor is exceeded. In such a situation, the relative control information is simply ignored in the current adjustment iteration.

4. AUXILIARY DATA WITH UNKNOWN TRANSLATION, ORIENTATION AND SCALE BIASES

When a local net of aerial and/or ground stations has been positioned up to unknown translations, rotations and scale with respect to the geodetic reference system, linear combinations of the corresponding coordinate differences for triplets of such stations can contribute to a block adjustment. The Euclidean properties of the spatial triangles formed by triplets of stations can clearly provide geometrical information to optimize the block adjustment results.

As in the preceding case, the formulation for three (distinct) points Q_1, Q_2 and Q_3 with known photogrammetric (x_j, y_j, z_j) and generally biased ground (X_j, Y_j, Z_j) coordinates ($j = 1,2,3$, respectively) is as follows:

$$\lambda\ Q_1 + \mu\ Q_2 + \nu\ Q_3 = Q_o$$

for appropriate scalars λ, μ, ν subject to the condition that

$$\lambda + \mu + \nu = 0$$

with Q_o being coplanar point of Q_1, Q_2 and Q_3 defined by the scalars λ, μ and ν. The coefficients λ, μ and ν are defined in terms of the length ratios of the triangle sides to ensure translation, orientation and scale invariance:

$$\lambda = [2 \cdot |Q_2Q_3| - |Q_1Q_2| - |Q_1Q_3|] \ / \ |Q_1Q_2Q_3|$$
$$\mu = [2 \cdot |Q_3Q_1| - |Q_2Q_3| - |Q_2Q_1|] \ / \ |Q_1Q_2Q_3|$$
$$\nu = [2 \cdot |Q_1Q_2| - |Q_3Q_1| - |Q_3Q_2|] \ / \ |Q_1Q_2Q_3|$$

in which $|\_\_|$ denotes the spatial distance and $|\_\_\_|$ denotes the perimeter of the triangle using the corresponding terrestrial coordinates.

The implementation of this relation in the adjustment is carried out exactly as in the previous case of auxiliary terrestrial data with translation and orientation biases. The above definition of the coefficients λ, μ and ν makes them invariant to any scale biases in the corresponding terrestrial coordinates.

5. OVERVIEW OF TEST RESULTS

A number of experiments using simulated and real data sets have been carried out to confirm the applicability of various coordinate difference methods. Test results using auxiliary airborne data are documented in [Blais and Chapman, 1984a] while test results using auxiliary terrestrial data are documented in [Blais and Chapman, 1984b].

With auxiliary airborne data, three situations were investigated in details:

(a) The terrestrial coordinates (X,Y,Z) of the exposure stations are known and have associated statistical weights (W_X, W_Y, W_Z)

(b) The terrestrial coordinate differences $(\Delta X, \Delta Y, \Delta Z)$ for the stereomodel bases are known with statistical weights (W^*_X, W^*_Y, W^*_Z), respectively.

(c) The terrestrial coordinate differences $(\Delta X, \Delta Y, \Delta Z)$ for the stereomodel are only known in the relative sense (due to unknown biases) so that the corresponding second-order differences $(\Delta^2 X, \Delta^2 Y, \Delta^2 Z)$ for the adjacent stereomodel bases are reliable with statistical weights $(W^\dagger_X, W^\dagger_Y, W^\dagger_Z)$, respectively.

The first situation simply implies that the exposure stations can be used as control points and the corresponding test results confirm that in some contexts ground control points could practically be eliminated in multi-strip configurations. Unfortunately, such auxiliary airborne data are not often available in practice.

The second situation with first-order coordinate differences is a special application of the formulation for auxiliary data with unknown translation biases. The test results confirm that the block adjustment results can greatly improve with such auxiliary data. This is especially true in altimetry as can be expected from geometrical considerations.

The third situation with second-order coordinate differences is closely related to the case of unknown translation and orientation biases with triplets of points which are nearly equidistant and collinear. The test results clearly confirm that such auxiliary airborne data can significantly improve the block adjustment results.

With auxiliary terrestrial data, various situations corresponding to unknown biases in translation and/or orientation and/or scale per net have been investigated with simulated and real test data sets. Such auxiliary local nets of control points are obviously much easier to simulate than auxiliary airborne data.

The test cases with auxiliary terrestrial data affected by unknown translation biases give results that confirm the expected implications in terms of orientation and scale. In fact, in altimetry, the results are not much different from using lake data with the same configuration and accuracy. In planimetry, the improvements in the adjustment results are usually less outstanding because of the stronger geometry.

With unknown translation and orientation biases in the auxiliary terrestrial data, the test results show significant improvements, especially when the geometry of the photogrammetric block or subblock is relatively weak because of sparse control. The filtering out of the orientation unknowns per net requires that these data be spatial in nature and the SPACE-M formulation can only be implemented in a second iteration of the block adjustment to avoid numerical problems.

Auxiliary terrestrial data with unknown translation, orientation and scale biases per net have been shown to imply essentially the same order of improvements in test block adjustments as the previous type of auxiliary data. Also as the SPACE-M formulations are very similar with these two types of auxiliary data, the restrictions are exactly the same.

Very limited testing has also been done with auxiliary data which include exposure stations and relative control points on the ground. In the very near future, more extensive testing is going to be carried out in the context of the Kananaskis project wherein aerial photography has been collected over a 20×50 km rugged area (about 100 km North-West of Calgary) with simultaneous Litton LTN-051 inertial navigation, laser profiler and other auxiliary sensor data. These photogrammetric and auxiliary data should permit more extensive and comprehensive testing of all the options for auxiliary data with SPACE-M.

6. CONCLUDING REMARKS

The use of first and second-order coordinate difference formulations with auxiliary airborne and terrestrial data in SPACE-M photogrammetric block adjustments has been demonstrated to have promising potential for topographical mapping and other applications. Some of the characteristics of this approach are the minimal assumptions made about the nature of the errors in the auxiliary data, the linearity of the techniques and the compatibility with the SPACE-M normal equations.

Auxiliary airborne information can definitely contribute to improve the results of photogrammetric block adjustments and hence optimize the field control requirements. In most cases, however, the raw airborne data would require appropriate editing and filtering before optimal results can be expected with the preceding coordinate differences approach. More research and development work remains to be done on such questions.

Auxiliary terrestrial information in terms of local nets of relative control points with unknown translation, orientation and scale biases can also contribute to improve the results of photogrammetric block adjustments, especially in sparse control configurations. With only translation biases, single coordinate information can be used with appropriate statistical weights for all pairs of stations within the bandwidth of the normal equations. With translation and orientation or scale biases, spatial information can be used with appropriate statistical weights for all triplets of stations within the bandwidth of the normal equations, although only in a second SPACE-M iteration to avoid numerical problems.

The exposure stations can also be included in any local net of relative control points. These possibilities however require more detailed investigations for optimal combinations of auxiliary airborne and terrestrial data in SPACE-M block adjustments. In connection with topographical mapping and related applications, some of the outstanding research problems include optimal planning strategies, appropriate statistical weighting and comprehensive error analyses.

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