SORA-MP: A PROGRAM FOR DIGITALLY CONTROLLED MAP TRANSFORMATION

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Abstract

Digitally controlled differential rectification allows a map drawn in any projection to be transformed into any other projection. For this purpose, the Institute of Photogrammetry of the Technical University Vienna has developed a program (SORA-MP) for Wild Heerbrugg Ltd.

When the equations of image formation for the projections concerned are known, strictly mathematical transformation is possible. If the transformation laws are unknown, a non-parametric interpolation method can be used in which a choice is possible between purely affine transformation, transformation based on a polynomial system or, in more complex cases, transformation by interpolation by the method of least squares.

Since the original map is not always available but may be a photographic reproduction, the program makes provision for taking into account affine (paper and/or film shrinkage) and perspective (photographic image) deformation. The software is modular in structure to allow the user to define map projections as required.

1 Introduction

Digitally controlled differential transformation opens the door to a large number of possible applications in the field of optical transformation. As regards the instruments available in this context, the Wild AVIOPLAN OR1 differential rectifier (Stewardson, 1976) occupies a paramount place. The control data required for this are provided by various SORA programs (Vozikis and Loitsch, 1982). Whilst the production of conventional orthophotographs (SORA-OP) and stereomates (SORA-OPS) is without doubt the most important field of activity, a number of other applications have provided further evidence of the outstanding flexibility of the OR1/SORA system.

These have included the rectification of plane objects (Vozikis, 1979; Vozikis and Loitsch, 1980), the development of regular surfaces (Kraus and Tschannerl 1976; Vozikis 1979), rectification of multispectral scanner photographs (Kraus, 1975; Jansa, 1980) and map transformation (Bormann and Vozikis, 1982), all of which have proved possible. This paper provides information about a new SORA-MP program whose name refers to its suitability for map transformation but which in fact can be used far more widely.

2 SORA-MP

The SORA-MP program (SORA Map Projection transformation) comprises three transformation modules. These are used for:

Direct transformation
Indirect transformation
Transformation by interpolation methods

In addition, several ancillary modules can also be used, the most important of which are:

Copy (of AVIOPLAN data from disk to tape)
Tapedump (output of AVIOPLAN data to printer)

2.1 Direct transformation

This method is used when relationships can be expressed by mathematical formulae (e.g. function f in fig. 1), by which each point of the result can be directly assigned to a corresponding point in the original document. The inverse function f^{-1} must also be defined, since it is necessary for the transformation of various parameters, such as the area to be transformed from the original to the resulting document.

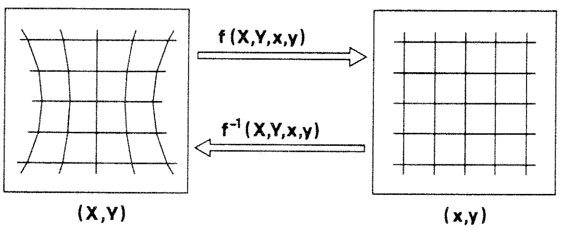

Fig. 1: Relationship between original document and product in direct transformation

The method used in producing the set of control data for the AVIOPLAN conforms to established principles (Otepka and Loitsch, 1976; Stewardson, 1976):

1: Definition of a regular grid in the resulting image area

2: Transformation of the points of intersection of this grid into the original document by means of fig. 1.

The distorted grid obtained by this means corresponds to the AVIOPLAN control data.

This paper includes two examples of direct transformation (section 7):

1: Transformation of a map using a projection in which the scale diminishes according to its distance from a given central point (Lichtner, 1982).

2: Transformation of a photograph taken with a panoramic camera into a conventional central-perspective photograph

2.2 Indirect transformation

When the mathematical relationships between the existing (X,Y) and new (x,y) projections are given only by geographic coordinates (φ λ), this method should be used.

As in the case described in 2.1, the relationship between the original docu~~ment and the~~ result must be known in both directions (fig. 2).

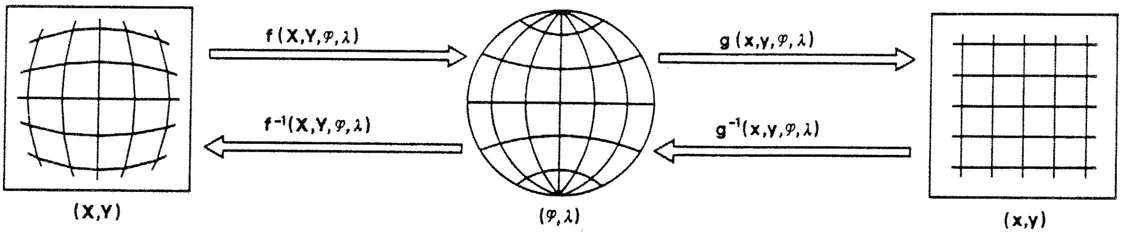

Fig. 2: Diagram of relationships in indirect transformation between original, geographic coordinates and result

An example of the use of indirect transformation is the transformation from azimuth to Mercator projection (Bormann and Vozikis, 1982).

In the indirect method, the points limiting the area or the AVIOPLAN orientation points may be specified either in rectangular map coordinates (X,Y) or geographic coordinates (φ,λ).

2.3 Interpolation method

Where there is no known image-forming law relating the original to the required result and provided that a sufficient number of points common to both can be identified, the interpolation method can be used. Initially, the coefficients for carrying out the interpolation are determined by means of the common points identified.

The interpolation function thus obtained acts as the first law I of image formation for the original and the result (fig. 3).

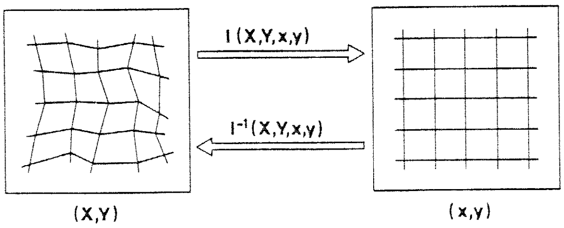

Fig. 3: In the interpolation method, the relationships between original and result are established by interpolation function I

Three initial interpolations are available:

1: Affine or similarity transformation:

$$\begin{aligned} x &= a_0 + a_1 \cdot X + a_2 \cdot Y \\ y &= b_0 + b_1 \cdot X + b_2 \cdot Y \end{aligned} \tag{1}$$

2: Polynomial not greater than of the second degree, in which the terms in X^2 and Y^2 can be deactivated to produce a bilinear transformation:

$$\begin{aligned} x &= a_0 + a_1 \cdot X + a_2 \cdot Y + a_3 \cdot X \cdot Y + (a_4 \cdot X^2 + a_5 \cdot Y^2) \\ y &= b_0 + b_1 \cdot X + b_2 \cdot Y + b_3 \cdot X \cdot Y + (b_4 \cdot X^2 + b_5 \cdot Y^2) \end{aligned} \tag{2}$$

3: Interpolation by least squares (Kraus, 1972; Kraus and Mikhail, 1972):

$$\begin{aligned} x &= F(X,Y) + UK \\ y &= F(X,Y) + UY \\ & \quad UX = cx' \cdot CX^{-1} \cdot lx \\ & \quad UY = cy' \cdot CY^{-1} \cdot ly \end{aligned} \tag{3}$$

where

$F(X,Y)$	systematic portion (here: affine trend)
CX, CY	Covariance matrix of distortions at control points for X and Y components
CX^{-1}, CY^{-1}	Inverse of CX, CY
cx, cy	x anf y components of the covariance vector of distortions between control points and a new point
cx', cy'	cx transposed, cy transposed
lx, ly	X and Y components of the distortion vector of control points

If in the first two of these methods more points are given than would be necessary to determine the coefficients, an adjustment is carried out. In interpolation by least squares,a filter can be used to take into account measuring inaccuracies and increase the mathematical stability of the method.

Whilst methods 1 and 2 suffice for simple transformation (e.g. elimination of affine distortion, rectification of LANDSAT images), the method using least squares is suitable for more complex rectification, e.g. the rectification of scanner images taken by air planes etc (Jansa and Zierhut, 1981).

3 Processing of large maps

If the original for transformation can not be placed directly in the AVIOPLAN, either due to excessive size (exceeding 230mm x 230mm) or because it is not available in the form of a transparent film, the map must first be photographed. In computing the AVIOPLAN control data, the perspective ratio for the input photograph relative to the original map has to be taken into account. Since the process is always, one of transforming a projection from one plane to another one, the formulae for projective transformation were used:

$$x = (a_0 + a_1 \cdot X + a_2 \cdot Y) / n$$

$$y = (b_0 + b_1 \cdot Y + b_2 \cdot Y) / n$$

$$n = (1 + c_1 \cdot X + c_2 \cdot Y)$$

Hence, the image and map coordinates of at least four points must be given. If indirect transformation is used, the control points in the map can also be given as geographic coordinates (φ,λ), i.e. the points of intersection of the geographic map grid can be used as control points.

Note that affine film shrinkage and map distortion is also taken into account in projective transformation. No data are required on inner and outer orientation of the camera. As a result, work is made a great deal easier, particularly for a user who is not very familiar with photogrammetry.

3.2 Determining paper shrinkage in a map

If information is required on paper shrinkage of a map, this can be obtained by control transformation. For this purpose, the theoretical coordinates must be well distributed over the map sheet. In indirect transformation, geographic coordinates can be used.

The effective coordinates are obtained by using a digitizer to measure these points on the map sheet. These are then compared with the theoretical coordinates by a control transformation.

The following three types of control transformation can be used:

Three-parameter transformation:
Orthogonal transformation with one rotation and two shifts, and no automatic scale adjustment. A known scale factor for both coordinate axes may be also specified

Four-parameter (= similarity) transformation:
Orthogonal transformation with one rotation and two shifts, automatic scale adjustment

Six-parameter (= affine) transformation:
Non-orthogonal transformation with two rotation and two shifts, separate scale adjustment in each of the two coordinate directions

All these types of transformation automatically take into account any mirroring that may occur, provided that more than two control points are given.

4 Program control and data input

The user communicates with the program as required either by using the command language or a menu technique. For this purpose, SORA-MP uses the DRE-X (Directive Recognition -X) software package for universal applications. The user can switch from command input to menu input and vice versa. This flexibility provides the user with detailed information on the input possibilities at any time.

Further, a menu user can change quickly from one menu level to another by using the command language, without having to go back through consecutive menus. This is particularly useful where data transfer is slow.

The command language is easy to learn, since the currently valid commands are listed in the menu table. If required, a summary in command form can be obtained following menu input, thus providing further help in learning the commands.

The following are the main features of DRE-X:

Fast input by commands in abridged form for advanced users

Input by command language also in batch operation and interactively where data transfer is slow

Input with explanatory information in menu mode for unskilled or infrequent program users

Menu mode useful for complex problems as a help to careful construction of input

Data editing possible in menu operation

Switchover possible from command to menu operation

Largely independent of computer-specific limitations

5 Data editing

Control-point and other data specifying the limiting and orientation points can be edited as required, either formatted via files or by direct tabulated input in menu mode. The standard basic editing functions (delete, insert, replace) are available. Command language can also of course be used for data input, but in this mode editing is not possible.

6 Integration of new type of projection

The structure of the SORA-MP software is designed in such a way that new direct and indirect types of projection can be integrated at any time with little effort. A sub-program must be provided for each new type of projection, which computes the projection functions and matches a defined interface. The effort involved in any consequential changes to adapt command and menu techniques to new projection types is negligible.

7 Practical examples

The reader is referred to the specialist literature for detailed information on examples of indirect transformation and the interpolation method, e.g.:

> Transformation of the 1:500 000 Swiss ICAO air traffic chart from a conformal conical projection (Lambert type 1) with equidistant latitude (λ_0 = 46°57'08".05 E) into a 1:1 000 000 flat chart in a rectangular Cassini projection with the additional condition $\Delta B/\Delta I = 1.5$ (Bormann and Vozikis, 1982)
>
> Transformation of the physical map of Europe from an equivalent azimuth projection at a scale of 1:15 000 000 (λ_0 = 20°E, φ_0 = 50°N) into an orthogonal cylindrical projection (φ_0 = 50°N) at a scale of 1:30 000 000 (Bormann and Vozikis, 1982)
>
> Production of the Austrian satellite from LANDSAT 1 and 2 images as a conformal conical projection (Lambert type 2) with two equidistant parallels 46°N and 49°N at a scale of 1:1 000 000 (Jansa and Zierhut, 1981)

Two examples of direct transformation are presented and briefly described below.

7.1 Intentional distortion of a map

This concerned a topographic map of the environs of Darmstadt (W Germany) at a scale of 1:25 000 which was to be distorted in such a way that the scale decreases continuously as the distance from a central point increases. Transformation was based on the following transformation formulae (Lichtner, 1982):

$$m_i = m_1 + c \cdot s_i' \qquad \text{using } c = (m_2 - m_1)/s_0' \qquad (5)$$

where:

m_i: scale number for any point P_i'
m_1: scale number for central point Z'
m_2: scale number at a control point P_2'
s_i: distance $\overline{Z\,P_i}$ before transformation
s'_0: distance $\overline{Z'P_2'}$ after transformation
s'_i: distance $\overline{Z'P_i'}$ after transformation

If the scale number m_i is defined as ds_i/ds'_i, by combination with (5) we obtain the differential equation:

$$ds_i = m_i \cdot ds_i' + c \cdot s_i' \cdot ds_i' \qquad (6)$$

Since distortion depends on direction a, the following relationships can be introduced:

$$x_i = s_i \cdot \cos a \;, \quad x_i' = s_i' \cdot \cos a$$
$$y_i = s_i \cdot \sin a \;, \quad y_i' = s_i' \cdot \sin a \qquad (7)$$

where:

x_i, y_i coordinates of a point before transformation

x_i', y_i' coordinates of the same point after transformation

When (6) and (7) are combined, the following transformation equations are obtained:

$$x_i' = \frac{-m_1 \pm \sqrt{m_1^2 + 4 \cdot c_x \cdot x_i}}{2 \cdot c_x} \;, \quad c_x = \frac{c}{2} \cdot \sqrt{\frac{x_i^2 + y_i^2}{x_i^2}} \qquad (8)$$

$$y_i' = \frac{-m_1 \pm \sqrt{m_1^2 + 4 \cdot c_y \cdot y_i}}{2 \cdot c_y} \;, \quad c_y = \frac{c}{2} \cdot \sqrt{\frac{x_i^2 + y_i^2}{y_i^2}}$$

In the example under consideration, the castle in the centre of Darmstadt was taken as the central point, with $m_1 = 1.0$, $m_2 = 1.40$, $s_0 = 9.0$ [cm], $c = 0.0444$ [cm-1]. Figure 4 shows an excerpt from the 1:25 000 topographic map of the environs of Darmstadt before and after transformation with the OR1/SORA-MP system.

7.2 Transformation from panoramic to perspective photographs

In addition to panoramic cameras used mainly for mapping from the air, there are also terrestrial panoramic cameras used in advertising, architecture, accident, forensic documentation etc.

In the case considered here, a panoramic photograph had to be transformed into a perspective photograph.

Fig. 4: A topographic map (original scale 1:25000) before (a) and after (b) the transformation with the system ORI/SORA-MP
The scale in (b) decreases continuously as the distance from a central point increases (Lichtner, 1982)
(original map: Staatsvermessungsamt Darmstadt)

The original photograph was produced with an ALPA-ROTOCAMERA 60/70 made by Optiphot Ltd, Zurich. Geometrically, this photograph may be defined as an image of three-dimensional space on the two- dimensional surface of a cylinder. Although a derivation of the requisite transformation formulae for standard conditions is available, for these examples the following assumptions were made to simplify the problem:

The cylinder axis is vertical

The projection centre is the same for the panoramic and perspective photographs and lies on the cylinder axis

The optical axis is horizontal for both photographs

The image plane of the perspective photograph is a plane tangential to the cylinder and contains the point of intersection of the optical axis with the cylinder surface.

If x, y are taken as the plane image coordinates of a point in the perspective photograph and u, v are the cylinder coordinates of the panoramic photograph, and subject to the assumptions stated above, the following transformation equations are obtained:

$$\begin{aligned} u &= c_z \cdot \arctan (x/c_p) \\ v &= c_z/c_p \cdot y \cdot \cos \left[\arctan (x/c_p)\right] \end{aligned} \qquad (9)$$

where c_z and c_p are the camera constants of the perspective and panoramic photographs respectively.

Figure 5 shows a terrestrial panoramic photograph before (a) and after (b, c) transformation.

8 Conclusions

The SORA-MP software is a system which offers a broad field of applications for the AVIOPLAN OR1. Particularly the modules for direct transformation and, for interpolation method offer outstanding new capabilities that are of interest not only for the transformation of maps. One of the examples quoted concerns the rectification of a panoramic photograph and transformation into a perspective photograph. This provides some indication of the wide range of applications. The interpolation method, on the other hand, is very suitable for the rectification of multispectral-scanner (MSS) images. By permitting the inclusion at a later stage of further types of projection, the range of applications of this software package is not limited by the present state of knowledge.

9 Acknowledgements

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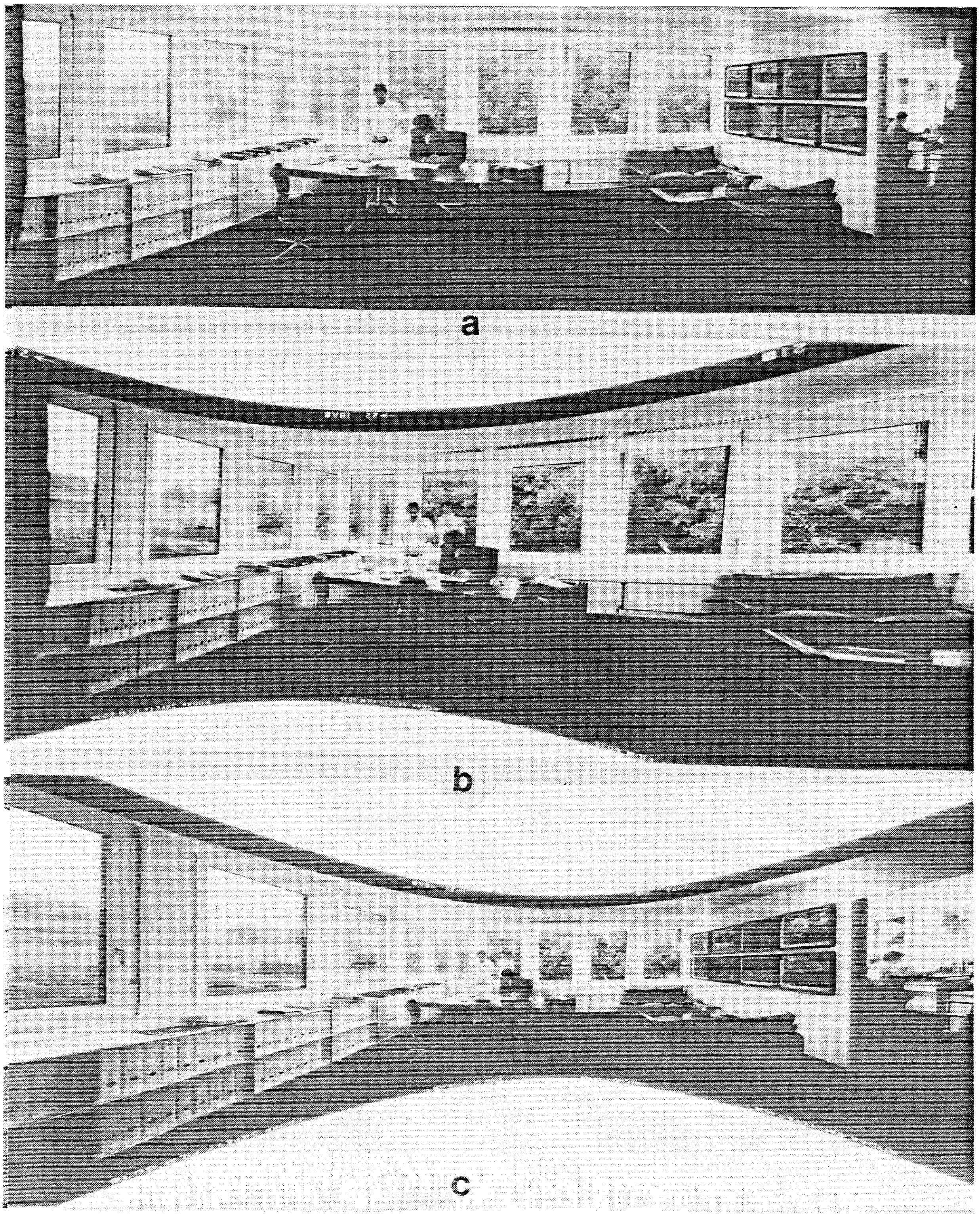

Fig. 5; a: Reduction of a photograph taken with the terrestrial panorama camera ALA-ROTOCAMERA
b: Transformation of a part of the photograph (a) into a perspective photograph
c: Transformation of the whole panorama photograph into a perspective photograph

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