INTERPOLATION AND SMOOTHING ALGORITHMS FOR THE AUTOMATIC PLOTTING OF CONTOUR LINES

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1. Introduction

Maps containing height information are required for all kinds of investment projects. It is necessary, therefore, that automatic mapping processes include the plotting of contours and the inscription of spot heights. That is why the DZT 90x120 Digital Plotting Table designed by VEB Carl Zeiss JENA has an electronic height counter with a scale selection facility. When connected on-line to a stereoplotting machine, the DZT will automatically write spot heights on the operator's command. In the TRACK mode, it plots contours in largely the same way as a mechanically connected analog plotting table would. Index and auxiliary contours can be drawn with lines of different type or thickness.

In off-line operation, the DZT 90x120 reads plotting commands off a magnetic tape. The tape may be written right during stereocompilation with a COORDIMETER-G data logger, the registration of contour coordinates in time and path intervals being released automatically. In many cases, however, users prefer the measurement of spot heights in the photogrammetric model, thus generating a digital height model, the data base of which are points in a random or grid arrangement. This method is used especially in hydraulic engineering projects where the terrain is relatively flat and precise height information is required. Height information is obtained also in the profile-by-profile control of the differential rectification (orthophotography) process, and contours can be derived from them.

The TOPAS software package for off-line orthophotography with the TOPOCART-D/ORTHOPHOT-E equipment system of VEB Carl Zeiss JENA includes a program, CONT, which computes the commands for plotting a contour map with the DZT 90x120. The CONT program is based on random point distribution, so that it can be employed even if the initial data come from sources other than off-line orthophotography.

2. Demands placed on a contour line program

Basically, the quality of a digital height model is characterized by the point distribution of the data base. The interpolation instruction for the densification of these height points is of secondary importance. In the CONT program, the height points of a rectangular grid are computed by the principle of the gliding tilted plane [2]. The contours are interpolated and sorted on the basis of that grid. The contour pattern has to satisfy several demands so that the landforms plotted represent the natural topography as closely as possible:

- The interpolated rectangular grid must not show up in the contour pattern.
- The arrangement of data base points (e.g. profile by profile in orthophotography) must not show up.
- Oscillations of the contours near break lines must be avoided.
- No saddle points must occur along valley bottoms.
- The number of saddle points on ridge lines must be minimized.

In order to satisfy these requirements it is essential that the reference points of the data base are spaced to suit the terrain curvature, and that additional reference points exist along valley bottoms, crests, fault scarps and other important landform lines. The CONT program, in addition, employs interpolation and smoothing algorithms which form a mathematical model largely adapted to the above requirements. For smoothing, the floating arithmetic mean is connected with a weighting function. This contains constants which are not interrogated in the dialog but may be varied by the user in case of need.

3. Smoothing and interpolation methods

3.1 Smoothing of measured values in dynamic profiling

Data acquisition for a digital height model is a dynamic process if it is performed during on-line orthophotography. To acquire data for later off-line orthoprojection, too, the photogrammetric model is usually scanned profile-wise with a floating mark moved continuously in a meandering pattern. With such a dynamic process, a multitude of measured data can be obtained, but their accuracy is significantly lower than that of static height measurements, viz. 0.3 to 0.5 per thousand of the projection distance (or flying height). Dynamically obtained measures can be corrected for systematic influences by means of filtering. Several filtering techniques have been reported which involve relatively much system outlay and time [1, 3].

If terrain heights are registered with the Digital Control accessory to the ORTHOPHOT-E of VEB Carl Zeiss JENA, model coordinates are formed by means of the BIDI program. This program includes data smoothing (especially normal to the scanning direction) as a function of terrain curvature, which is the criterion for weighting the measured points.

$$p_i = |z_{i-1} - 2z_i + z_{i+1}| \cdot P \qquad \text{with } p_i = 2,$$
$$\text{if } |z_{i-1} - 2z_i + z_{i+1}| \cdot P < 2 \qquad (1)$$

The resulting (cross-)smoothed value,

$$\bar{z}_i = \frac{z_{i-1} + p_i z_i + z_{i+1}}{2 + p_i} \qquad (2)$$

For a projection distance of 300 mm (as generally available in the TOPOCART-D) and a mean height error of 0.3 per thousand, $P = 5 \text{ mm}^{-1}$. By selecting a different constant P, the smoothing effect can be enhanced or reduced.

3.2 Interpolation within the grid meshes

For the interpolation of contour lines within a rectangular grid mesh, one uses the sides of the rectangle and one of the diagonals, which is selected depending on terrain curvature in the mesh. For that purpose, a test dimension T_i is established which is the vertical distance between the two diagonals in the mesh centre (Fig. 1):

$$T_i = 0{,}5\,(z_1 + z_3 - z_2 - z_4) \qquad (3)$$

If that test dimension is smaller than a specified value T (e.g. T = 0.1 R, R being the mesh size), the mesh is considered sufficiently flat and the diagonal having the greater height difference is used for interpolation. In the other case, an auxiliary point in the mesh centre is interpolated from the measured values by means of the gliding tilted plane, and the diagonal situated closer to that auxiliary point is chosen.

3.3 Smoothing of interpolated contour lines

After the supporting points of a contour have been sorted and thinned out, a smoothing is effected in order to diminish the influence of the interpolated grid with the mesh size R (RX, RY) on the shape of the contours. For the two neighbouring points of a point P_i to be smoothed, a distance-depending weight is established:

$$p_{i-1} = \frac{R}{K\,s_i} \qquad p_{i+1} = \frac{R}{K\,s_{i+1}} \qquad (4)$$

with $p = 1$ if $K \cdot s < R$ and $K = 10$.

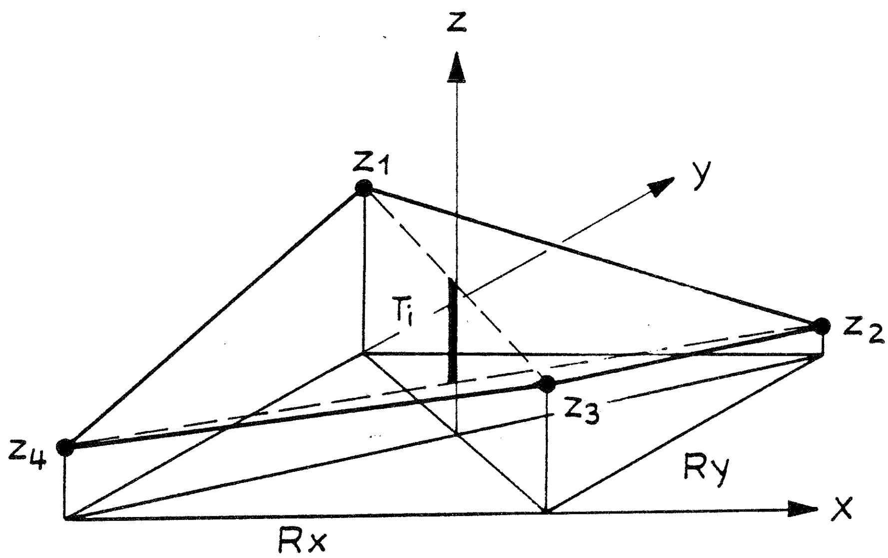

Fig. 1 Computation of the test dimension T for estimating terrain curvature within a grid mesh

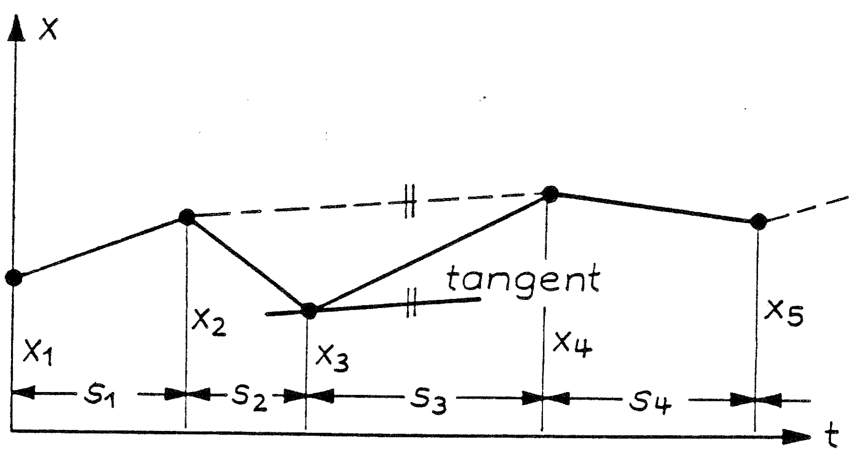

Fig. 2 Parameter function x = x(t)