THE CONNECTION OF AEROTRIANGULATION BLOCKS AND GROUND CONTROL RECONSIDERED

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ABSTRACT

This paper deals with the effect of the quality of terrestrial networks on the result of the adjustment of aerotriangulation blocks. Examples using a planimetric independent model block and different planimetric terrestrial networks show how the precision of terrestrial coordinates propagates to the precision of the final block coordinates. Furthermore, it is shown what effect an incorrect assumption for the precision of ground coordinates may have on the evaluation of the precision of the total system. Attention is also given to the effect of the reliability of ground control on the reliability of the final block coordinates. It is shown how undetected errors in the terrestrial network propagate to the block coordinates, and how these effects can be reduced by choosing better network structures.

1 INTRODUCTION

When looking through the literature on aerotriangulation published over the past 20 years, it is astonishing how little attention photogrammetrists have paid to the quality of ground control. In publications from the 1960s, we find directives for where ground control points should be fixed in a block and how dense this control should be (e.g., [1] and [7]). These directives, however, are based entirely on the analysis of the resulting precision of the aerotriangulation block, assuming that the given ground control is error-free and not stochastic. These directives were easily accepted by people working in applied photogrammetry because they were simple, i.e., control points for planimetry at the perimeter of the block plus several chains of height control points over the block. It was commonly accepted that the land surveyor could easily obtain a precision for coordinates superior to what photogrammetry could do. This was correct--but experience shows that superior precision does not mean that ground control is without errors.

In 1966, Ackermann [1] showed how difficult it is to detect errors in ground control in photogrammetric blocks. It was only at the end of the 1970s, however, that reliability parameters were used as criteria for the quality of geodetic networks--for which the basic theory had been published in the 1960s. Photogrammetrists used reliability studies to check and improve the quality of their own product, i.e., photogrammetric block coordinates. Eventually, they investigated the possibility of testing for errors in given ground control points (e.g., [6]), but the technique was not used for the formulation of criteria which should be satisfied by ground control. Nor did the increased precision of photogrammetric blocks lead to more detailed formulation of specifications for ground control.

Examples will therefore be given in this paper to demonstrate what kind of problems occur when networks are designed to supply ground control for aerotriangulation blocks and why photogrammetrists and land surveyors should get together to design an integrated point determination system. Some of the computation results presented here have been taken from the MSc thesis of J B Jorgensen [8], for which he was awarded an MSc degree in photogrammetry at ITC in July 1980.

2 DATA AND COMPUTING METHODS USED IN THE EXAMPLES

2.1 Data

One planimetric independent model block and two networks will be used in the examples. The block consists of three strips with six models each (Figure 1). Every model has four tie points, one in each corner.

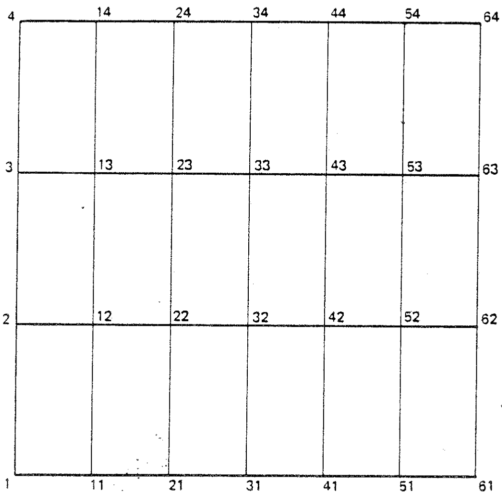

Fig. 1

The size of each model is 1000 m by 2000 m in terrain scale and the precision of the model coordinates is $\sigma x = \sigma y = 5.6$ cm; photo scale is therefore approximately 1:11,000 and $\sigma x = \sigma y \approx 5$ µm at photo scale. The model coordinates are considered as not correlated. From modern reliability studies, it is known that double points would give a better inner reliability in the block, but that is not our first concern here. Single points are sufficient to illustrate our discussion.

The terrestrial networks have the following structure: Net I: Each point defined in the block is also determined in the ground control network; the sides of the network coincide with the sides of the models. For every side in the network, a distance has been measured with a precision of σ_l = 1.5 cm/km and directions are measured from both end points with σ_r = 1 mgon.

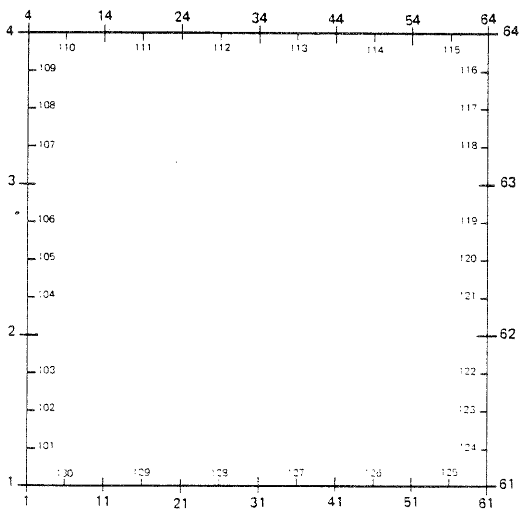

Fig. 2

Net II: Polygons are measured along the perimeter of the block with sides of 500 m (see Figure 2). For all sides, distances are measured with σ_l = 1.5 cm and from both end points directions are measured with σ_r = 1 mgon.

These two networks are extreme cases --one is very strong and the other is very weak. They are not realistic networks, but they have been chosen for illustration.

2.2 Computing methods

The strategy for connecting the block to ground control is derived from Baarda ([4] § 3.1). The block and the networks are first adjusted independently. The coordinate systems are defined by keeping the coordinates of points 1 and 64 fixed with the same values for the block and the networks. This means that block and networks are computed with respect to the same S-base.

For the second step of the adjustment, two different computations are made: (1) the block is connected to Net I and (2) the block is connected to Net II. In both cases, the procedure for the connection is as follows: Let $\underline{x}^i{}_p$ be the block coordinates of the points used for the connection and let $\underline{x}^i{}_n$ be the net coordinates of these points. Let $\underline{x}^k{}_p$ be the coordinates of other points in the block and let $\underline{x}^l{}_n$ be the coordinates of other points in the network.

On the assumption that block and network have not been distorted, we get as condition equations for the connection:

$$\tilde{x}^i{}_p = \tilde{x}^i{}_n \quad \text{with} \quad \tilde{x} = E\{\underline{x}\}$$

Because the two systems have the same S-base, there is no coordinate transformation involved here--so we have condition equations according to Tienstra's Standard Problem I in which coordinates serve as observations.

From the adjustment of the block, we get the coordinates $\begin{bmatrix} \underline{x}^i_p \\ \underline{x}^k_p \end{bmatrix}$ with the V.C. matrix $\begin{bmatrix} g_{ii} & g_{ik} \\ g_{ki} & g_{kk} \end{bmatrix}$

Similarly, from the adjustment of a network $\begin{bmatrix} \underline{x}^i_n \\ x^l_n \end{bmatrix}$ with the V.C. matrix we get $\begin{bmatrix} h_{ii} & h_{il} \\ h_{li} & h_{ll} \end{bmatrix}$

For both networks, three different methods are used to connect them with the block.

<u>Method 1</u>

The coordinates of both block and network are considered as stochastic; both sets will be corrected. The condition equations for the adjustment are:

$$(I\ 0\ -I\ 0)\begin{bmatrix} x^i_p \\ x^k_p \\ x^i_n \\ x^l_n \end{bmatrix} = 0 \rightarrow \tilde{x}^i_p - \tilde{x}^i_n = 0 \tag{1}$$

Introduction of the results of block and net adjustment gives:

$$(I\ 0\ -I\ 0)\begin{bmatrix} \underline{x}^i_p \\ \underline{x}^k_p \\ \underline{x}^i_n \\ \underline{x}^l_n \end{bmatrix} = \Delta\underline{x}^i \rightarrow \underline{x}^i_p - \underline{x}^i_n = \underline{\Delta x}^i \tag{2}$$

The V.C. matrix of $\Delta \underline{x}^i$

$$Q_{ii} = g_{ii} + h_{ii} \tag{3}$$

Least squares corrections to the input coordinates are:

$$\begin{bmatrix} \Delta \underline{X}^i_p \\ \Delta \underline{X}^k_p \\ \Delta \underline{X}^i_n \\ \Delta \underline{X}^l_n \end{bmatrix} = - \begin{bmatrix} g_{ii} & g_{ik} & & \\ g_{ki} & g_{kk} & & \\ & & h_{ii} & h_{il} \\ & & h_{li} & h_{ll} \end{bmatrix} \begin{bmatrix} I \\ 0 \\ -I \\ 0 \end{bmatrix} Q^{-1}_{ii} \; \Delta \underline{x}^i = \begin{bmatrix} -g_{ii} \\ -g_{ki} \\ h_{ii} \\ h_{ki} \end{bmatrix} Q^{-1}_{ii} \; \Delta \underline{x}^i \tag{4}$$

We see that the variates $\underline{x}^k_p$ and $\underline{x}^l_n$ appear with zero coefficients in the condition equations, which means that they play no role in these equations. They do get corrections, however, because they are correlated with the variates $\underline{x}^i_p$ and $\underline{x}^i_n$. The final corrected coordinates are:

$$\begin{bmatrix} \underline{X}^i_p \\ \underline{X}^k_p \\ \underline{X}^i_u \\ \underline{X}^l_n \end{bmatrix} = \begin{bmatrix} \underline{x}^i_p \\ \underline{x}^k_p \\ \underline{x}^i_n \\ \underline{x}^l_n \end{bmatrix} + \begin{bmatrix} \Delta \underline{X}^i_p \\ \Delta \underline{X}^k_p \\ \Delta \underline{X}^i_n \\ \Delta \underline{X}^l_n \end{bmatrix} \tag{5}$$

but because of the adjustment, we have $\underline{X}^i_p = \underline{X}^i_n$. Since the adjustment brought all coordinates into one system, we can omit the indices p and n. The V.C. matrix of these results is obtained from:

$$[G_{xx}] = \begin{bmatrix} g_{ii} & g_{ik} & & \\ g_{ki} & g_{kk} & & \\ & & h_{ii} & h_{il} \\ & & h_{li} & h_{ll} \end{bmatrix} - \begin{bmatrix} g_{ii} \\ g_{ki} \\ -h_{ii} \\ -h_{li} \end{bmatrix} Q^{-1}_{ii} \; (g_{ii} \; g_{ik} - h_{ii} = h_{il}) \tag{6}$$

Because $\underline{X}^i_p = \underline{X}^i_n$, we can omit in this matrix the rows and columns referring to $\underline{X}^i_n$; so we get

$$\begin{bmatrix} \underline{X}^i \\ \underline{X}^k \\ \underline{X}^l \end{bmatrix} \quad \text{with V.C. matrix} \quad \begin{bmatrix} G_{ii} & G_{ik} & G_{il} \\ G_{ki} & G_{kk} & G_{kl} \\ G_{li} & G_{lk} & G_{ll} \end{bmatrix} = (G_{xx})$$

where, with $Q_{ii} = g_{ii} + h_{ii}$

$$[G_{xx}] = \begin{bmatrix} g_{ii} & g_{ik} & 0 \\ g_{hi} & g_{kk} & 0 \\ 0 & 0 & h_{ll} \end{bmatrix} - \begin{bmatrix} g_{ii} \\ g_{ki} \\ -h_{li} \end{bmatrix} (g_{ii}+h_{ii})^{-1} \, [g_{ii} \quad g_{ik} - h_{il}] \tag{7}$$

Instead of omitting the rows and columns referring to $\underline{X}^i{}_n$, we could also have omitted those referring to $\underline{X}^i{}_p$.

Method 2

This is a pseudo least squares adjustment in which the terrestrial coordinates are not corrected after adjustment, but their V.C. matrix is taken into account when evaluating the precision of the adjustment results. This is done by replacing the matrices h in the correction equations by a zero matrix. When evaluating the precision of the final results, the matrix h is entered in the propagation laws. Hence instead of (4) we get:

$$\begin{bmatrix} \underline{\Delta X}^i_p \\ \underline{\Delta X}^k_p \\ \underline{\Delta X}^i_n \\ \underline{\Delta X}^l_n \end{bmatrix} = - \begin{bmatrix} g_{ii} \\ g_{ki} \\ 0 \\ 0 \end{bmatrix} \quad g_{ii}^{-1} \quad \underline{\Delta x}^i \tag{8}$$

With $\underline{\Delta x}^i = \underline{x}^i{}_p - \underline{x}^i{}_n$, we get:

$$\begin{bmatrix} \underline{x}^i_p \\ \underline{x}^k_p \\ \underline{x}^i_n \\ \underline{x}^l_n \end{bmatrix} = \begin{bmatrix} \underline{x}^i_p \\ \underline{x}^k_p \\ \underline{x}^i_n \\ \underline{x}^l_n \end{bmatrix} + \begin{bmatrix} -\underline{\Delta x}^i \\ -g_{ki}\, g_{ii}^{-1}\, \underline{\Delta x}^i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{x}^i_u \\ \underline{x}^k_p - g_{ki}\, g_{ii}^{-1} (\underline{x}^i_p - \underline{x}^i_n) \\ \underline{x}^i_n \\ \underline{x}^k_n \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ -g_{ni} \cdot g_{ii}^{-1} & 1 & g_{ki} \cdot g_{ii}^{-1} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \underline{x}^i_p \\ \underline{x}^k_p \\ \underline{x}^i_n \\ x^l_n \end{bmatrix} = \Lambda\, \underline{x} \tag{9}$$

Because $\underline{X}^i{}_p = \underline{X}^i{}_n = \underline{x}^i{}_n$ in this case, we can omit the rows referring to $\underline{X}^i{}_p$ in these matrices. Thus we get the matrix Λ, which we apply in the propagation law for V.C. matrices. This gives for the vector $(\underline{X}^i\ \underline{X}^k\ \underline{X}^l)$ the V.C. matrix:

$$(W_{xx}) = \begin{bmatrix} W_{ii} & W_{ik} & W_{il} \\ W_{ki} & W_{kk} & W_{kl} \\ W_{li} & W_{lk} & W_{ll} \end{bmatrix} = \bar{\Lambda} \begin{bmatrix} g_{ii} & g_{ik} & & \emptyset \\ g_{ki} & g_{kk} & & \\ & \emptyset & h_{ii} & h_{il} \\ & & h_{li} & h_{ll} \end{bmatrix} \bar{\Lambda}^r$$

$$= \begin{bmatrix} h_{ii} & h_{ii}\, g_{ii}^{-1}\, g_{ik} & h_{il} \\ g_{hi}\, g_{ii}^{-1}\, h_{ii} & \begin{pmatrix} g_{kk} - g_{ki}\, g_{ii}^{-1}\, g_{ik} \\ + g_{ki}\, g_{ii}^{-1}\, h_{ii}\, g_{ii}^{-1}\, g_{ik} \end{pmatrix} & g_{ki}\, g_{ii}^{-1}\, h_{il} \\ h_{li} & h_{li}\, g_{ii}^{-1}\, g_{ik} & h_{ll} \end{bmatrix} \tag{10}$$

Method 3

The network coordinates are considered to have so much better precision than the block coordinates that h is also replaced by a zero matrix in the evaluation of the final precision. The corrections are then computed according to (8), whereas the matrix W_{xx} in (10) reduces to:

$$(\overline{W}_{xx}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & g_{kk} - g_{ki} g_{ii}^{-1} g_{ik} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (11)$$

3 THE PRECISION OF THE ADJUSTMENT RESULTS

3.1 The individual networks and blocks

The precision of the coordinates computed from the individual networks and block is shown in Tables 1 and 2. The semi-major and minor axes of the standard ellipses for the networks are given. The radii of the circular standard ellipses generated by the block are also shown. Because for all these systems the same non-stochastic S-base 1,64 has been used, the results can be compared. The use of an S-base makes it possible to interpret the V.C. matrices of the coordinates, and thus the standard ellipses, as a measure for the precision of the inner geometry of the networks and the block (see [4], [10], [11] and [12]).

TABLE 1 Point standard ellipses. S-base (1, 64)

Point no	Point no	I.M.block σ_i = 5.6 cm R cm	Net I L1 cm	Net I L2 cm	Net II L1 cm	Net II L2 cm
63	2	14.02	2.13	1.90	4.30	3.23
62	3	20.70	2.77	2.50	7.90	4.38
61	4	27.92	3.51	3.14	10.10	5.11
54	11	9,54	1.45	1.32	2.39	1.91
53	12	12.82	2.15	1.60		
52	13	17.53	2.54	2.09		
51	14	24.00	2.92	2.87	9.18	4.62
44	21	14.19	2.09	1.77	4.30	3.23
43	22	13.42	2.25	1.58		
42	23	15.74	2.42	1.83		
41	24	20.81	2.71	2.44	7.90	4.38
34	31	17.73	2.47	2.10	6.26	3.97
33	32	14.48	2.34	1.67		

Because of the symmetric structure of the block and the networks, there are always two points which have the same precision.

Table 1 shows point standard ellipses. For reasons of symmetry, there are in these examples always two points with standard ellipses of the same size. Table 2 shows relative standard ellipses for several pairs of points. These relative standard ellipses are computed from the V.C. matrix of the coordinate differences of these point pairs.

From these tables, we see that the precision of Net I is by far the best. The radii of the standard circles from the block are five to eight times larger than the semi-major axes of the ellipses from Net I. The precision of Net I is also rather homogeneous. The standard ellipses of Net II are much larger than those of Net I and less homogeneous, but they are still much smaller than those of the block. So, both free networks give much better precision than the free independent model block. The implications of this for the connection of the block to ground control will be discussed below.

TABLE 2 Relative standard ellipses. S-base (1, 64)

Point no	Point no	Net I L1 cm	Net I L2 cm	Net II L1 cm	Net II L2 cm	I.M.block σ_o = 5.6 cm R cm
11	21	1.39	1.26	2.73	2.14	9.28
21	31	1.40	1.27	2.82	2.09	9.35
31	41	1.42	1.30	2.77	2.09	9.63
41	51	1.47	1.35	2.70	2.21	10.09
51	61	1.61	1.41	2.67	2.35	10.83
2	12	1.31	1.26			7.05
12	22	1.26	1.17			6.06
22	32	1.26	1.15			5.80
32	42	1.26	1.17			5.95
42	52	1.28	1.21			6.48
52	62	1.33	1.31			7.65
21	41	1.97	1.90	4.93	3.05	14.45
41	61	2.45	1.97	4.69	3.54	16.88
2	22	1.81	1.72			10.04
22	42	1.68	1.65			8.28
42	62	1.94	1.77			11.35
2	62	3.07	2.79	10.62	5.57	17.66
11	12	1.77	1.73			11.21
21	22	1.73	1.71			10.40
31	32	1.75	1.74			10.66
41	42	1.85	1.78			11.66
51	52	2.04	1.85			13.47
61	62	2.46	2.13			16.74
2	3	2.16	1.90	4.93	3.05	14.19
12	13	1.77	1.69	4.69	3.54	10.14
22	23	1.69	1.62			8.42
32	33	1.69	1.60			7.93
11	14	3.08	2.82	10.27	5.10	21.69
21	24	2.90	2.80	10.62	5.57	17.94
31	34	2.85	2.81	10.80	5.78	16.65
4	61	4.97	3.94	18.98	7.24	29.30

L1 and L2 are the half axes of an ellipse. R is the radius of a circle.

3.2 Precision of the independent model block after connection to ground control

To evaluate the precision of the independent model blocks, we will connect the block to each network, and the three methods described above will be used to study the effect of the different assumptions for the stochastic models.

3.2.1 Net I

Table 3 shows the precision of the block connected to Net I. This connection uses points at the perimeter of the block at intervals of two times the base length. The other points of the network and block have been treated as free points ($x^k{}_p$ and $x^l{}_n$).

Comparing the results under Method I (the rigorous adjustment) with those under Method 2 (the pseudo least squares adjustment), we see that the adjustment has very little effect on the precision of the network coordinates. The precision of the block coordinates improves considerably, however (compare Table 1). Under Method 3, the coordinates of the network are not considered as stochastic at all. It appears that for the free points of the block it makes very little difference which of the three methods is applied. This seems to justify Method 3--which is often used in practice. This statement should be considered with care, however, because the network used in this example is very well structured so that the precision is rather homogeneous and is much better than the precision of the free block. A problem is that by treating the network coordinates as not stochastic, we create discontinuities in the relative point precision between the perimeter and the interior of the block. These discontinuities may lead to problems when densification measurements are eventually made using the output of the block adjustment.

3.2.2 Net II

For Net II, the results are rather different (see Table 4). This is an extremely weak network. From the results given in the columns under Method 1 (M1) and Method 2 (M2), we see that the precision of the coordinates in this network certainly improves if they are corrected after adjustment. For the block coordinates, the precision after a rigorous adjustment (M1) is also much better than after a pseudo adjustment (M2). The columns under Method 3 show that we would make quite a large error in the evaluation of the final precision if the network coordinates were not considered as stochastic at all (compare M2 and M3). Hence the practical approach with fixed terrestrial coordinates is certainly not justified here.

The two networks used here are extreme cases; one has a very strong structure, the other one is very weak. The examples do give an impression, however, of what kind of errors we make when considering ground control as not stochastic. For good quality blocks, this is certainly not always justified. The second network also shows that the photogrammetric practice of using ground control for planimetry at only the perimeter of the block does not necessarily mean that the network should be measured only along the perimeter. Such a network does not give sufficient support to the block with regard to precision. A more dense network is required--not necessarily as dense as Net I, but extra connections between the opposite sides would strengthen Net II considerably.

4 THE RELIABILITY OF THE ADJUSTMENT RESULTS

The reliability of the networks and the connection of the block to each network is evaluated under the assumption that data snooping is performed according to Baarda [2, 3]. Hence tests are made in the different adjustment steps to detect possible observational errors. If the observations are tested with a level of significance α, then the reliability of these tests is expressed by the boundary value of the observations. A boundary value is the magnitude of an error which can just be detected with a specified probability β.

In our examples, we will use $\alpha = 0.001$ and $\beta = 0.80$. When evaluating the reliability of a network, we are interested in not only the magnitude of errors which can be found in the observations--the internal reliability--but also the effects which an undetected error may have on the final coordinates--the external reliability. This effect is therefore computed for each observation.

We will first consider the reliability of Net I and Net II individually and then the reliability of the connection of the block to each network. Computations have been made using the programs described elsewhere [5].

4.1 Net I

The strength of this network becomes apparent from the rather small boundary values for the observations. For direction measurements, these values are between 5.6 and 8.1 mgon. The effect of an undetected error on the coordinates is another indicator of the strength of the network. In a strong network, these effects will be small.

TABLE 3 Point standard ellipses

The precision of the connection between the free Independent Model block and the terrestrial Net I, which is composed of quadrilaterals.

Experiment		Method 1		Method 2		Method 3	
Point no		L1 cm	L2 cm	L1 cm	L2 cm	L1 cm	L2 cm
S-base points (1, 64)							
Common points used for the connection							
63	2	2.09	1.87	2.13	1.90	0.00	0.00
62	3	2.72	2.45	2.77	2.50	0.00	0.00
61	4	3.45	3.10	3.51	3.14	0.00	0.00
44	21	2.06	1.74	2.09	1.77	0.00	0.00
41	24	2.67	2.40	2.71	2.44	0.00	0.00
Other points in Net I							
54	11	1.44	1.31	1.45	1.32	0.00	0.00
53	12	2.13	1.59	2.15	1.60	0.00	0.00
52	13	2.50	2.06	2.54	2.09	0.00	0.00
51	14	2.88	2.82	2.92	2.87	0.00	0.00
43	22	2.22	1.57	2.25	1.58	0.00	0.00
42	23	2.40	1.82	2.42	1.83	0.00	0.00
34	31	2.44	2.07	2.47	2.10	0.00	0.00
33	32	2.31	1.66	2.34	1.67	0.00	0.00
Other points in the Independent Model block							
54	11	5.79	5.75	5.80	5.75	5.68	5.68
53	12	4.89	4.78	4.90	4.79	4.58	4.58
52	13	5.06	4.96	5.08	4.97	4.58	4.58
51	14	6.29	6.25	6.31	6.27	5.68	5.68
43	22	5.42	5.31	5.43	5.32	5.16	5.16
42	23	5.50	5.39	5.51	5.40	5.16	5.16
34	31	6.11	5.99	6.12	6.00	5.70	5.70
33	32	5.62	5.52	5.63	5.52	5.35	5.35

L1 and L2 are the half axes of an ellipse.

M1: The rigorous adjustment.

M2: The pseudo least squares adjustment of the Independent Model block onto fixed coordinates from Net I. The correct stochastical properties are considered in the propagation of variances and covariances.

M3: The coordinates from Net I are considered to be non stochastic in the adjustment and for the propagation of variances and covariances.

TABLE 4 Point standard ellipses

The precision of the connection between the free Independent Model block and the traverse, Net II.

Experiment		Method 1		Method 2		Method 3	
Point no		L1 cm	L2 cm	L1 cm	L2 cm	L1 cm	L2 cm
S-base points (1,64)							
Common points used for the connection							
63	2	3.42	2.82	4.30	3.23	0.00	0.00
62	3	5.37	3.88	7.90	4.38	0.00	0.00
61	4	6.42	4.86	10.10	5.11	0.00	0.00
44	21	3.44	2.83	4.30	3.23	0.00	0.00
41	24	5.39	3.89	7.90	4.38	0.00	0.00
Other points in Net II							
54	11	2.10	1.77	2.39	1.91	0.00	0.00
51	14	5.96	4.27	9.18	4.62	0.00	0.00
34	31	4.65	3.47	6.26	3.97	0.00	0.00
Other points in the Independent Model block							
54	11	5.93	5.86	6.01	5.89	5.68	5.68
53	12	5.18	5.02	5.39	5.10	4.58	4.58
52	13	6.00	5:45	7.29	5.64	4.58	4.58
51	14	7.82	6.86	10.02	7.00	5.68	5.68
43	22	5.57	5.50	5.60	5.54	5.16	5.16
42	23	5.95	5.71	6.56	5.81	5.16	5.16
34	31	6.86	6.39	7.72	6.54	5.70	5.70
33	32	5.83	5.74	5.99	5.79	5.35	5.35

L1 and L2 are the half axes of an ellipse.

M1: The rigorous adjustment.

M2: The pseudo least squares adjustment of the coordinates from the free Independent Model block onto fixed coordinates from Net II. The correct stochastical properties are considered in the propagation of variances and covariances.

M3: The coordinates from Net II are considered to be non stochastic in the adjustment and for the propagation of variances and covariances.

In this network, we find that the effect of direction measurements is not larger than 5.6 cm. We should keep in mind, however, that the coordinate system has been defined by keeping points 1 and 64 fixed as an S-base. The choice of another S-base will lead to other values for the distortion of the coordinates. In an S-system, each erroneous observation will give its own vector of distortions.

As a measure of distortion caused by a particular observation, we can use the quadratic product formed by the inverse of the V.C. matrix of the coordinates post-multiplied by the related vector of distortions and pre-multiplied by its transpose. For an observation $\underline{x_i}$, this product is called $\overline{\lambda}_i$. For example, let the effect of an undetected error in observation $\underline{x_i}$ with the magnitude of the boundary value, on a function F of the coordinates, be $\nabla_i F$. We then find:

$$\nabla_i F \leqslant \sqrt{\overline{\lambda}_i}\, \sigma_F \qquad \sigma_F{}^2 \text{ is the variance of } F$$

If λ_i is used as an indicator for the reliability of a network, then its advantage over the distortion vector of the coordinates is that this vector depends on the S-base, whereas $\overline{\lambda}_i$ is invariant after the transformation of the coordinate system from one S-base to another. A disadvantage is that the value of $\sqrt{\overline{\lambda}_i}.\sigma_F$ is in many cases far too high as an upperbound for ∇F. In Net I, we find for direction measurements that $2.6 \leqslant \sqrt{\overline{\lambda}_i} \leqslant 4.4$, which means that the effect of undetected errors is rather small.

The boundary values of distance measurements are between 11.4 cm and 18.9 cm. The effect on the coordinates (S-base 1,64) is as much as 15.3 cm, so even in this strong network the distortions caused by erroneous distance measurements can be considerable. The effect of an error in distance 1-11 is given as an example in Table 5. The distortions of the coordinates are given only for points of which at least one coordinate has been distorted by at least 5 cm.

For distances, we find $2.6 \leqslant \sqrt{\overline{\lambda}_i} \leqslant 11.5$; thus distortions can be considerable. We should keep in mind, however, that all these values have been computed under the assumption that the observations are tested with significance level $\alpha = 0.001$. For errors with the magnitude of the boundary value, the power of the test has been fixed here at $\beta = 0.80$. This means that in most of the cases in which errors of this magnitude occur, they will be detected. Thus when the measurements are performed carefully, the probability of such errors occurring is small and, if they occur, the probability that they will be detected is large. The network is therefore rather well protected against distortions caused by undetected errors.

In connecting the block to this network according to (rigorous) Method 1, we again assume that the block has been adjusted internally using points 1 and 64 as an S-base. The connection points are the same as used earlier. When data snooping is applied to the coordinates of these points after the connection, the boundary values of these coordinates are as follows:
- the coordinates of the corner points (1, 61, 4,64) have boundary values ofapproximately 43 cm
- the other coordinates used for the connection have boundary values of approximately 32 cm.

These values have been computed for $\alpha = 0.001$ and $\beta = 0.80$. If an individual control point--in either the block or the terrestrial system--has been distorted, errors of this magnitude can easily be detected. Test computations show that if such an error is not detected, the adjustment will compensate for it almost completely provided it occurs in the block coordinates. Then only a small local effect will be left. If it occurs in the ground coordinates, however, it will hardly be compensated for at all. Almost the full effect is felt in the final results of the adjustment. The effect is a local one, however, i.e., only the distorted point itself and some points near it are effected.

Undetected errors in the original network are more harmful. The results in Table 5 can be used as an example. These distortions were caused by an error of 18 cm in distance 1-11, the boundary value of that observation in the network adjustment. If the distorted coordinates are used for the block connection, then almost the full effect will propagate to the finally adjusted coordinates.

The distortion pattern of the final results is shown in Figure 3. Data snooping after adjustment--i.e., testing individual coordinates--with $\alpha = 0.001$ does not reveal this distortion, so the final results will be accepted without this serious defect being noticed.

For this particular network, however, it is very unlikely that such large errors would remain undetected; hence the risk that this deformation could actually occur is rather small.

4.2 Network II

This network is weak, as can be seen from the standard ellipses of the points and from the boundary values of the observations and their effect on the coordinates. The boundary values of the direction observations are between 20.1 and 25.8 mgon. The largest effect on the coordinates (S-base 1,64) is 44 cm, whereas $14 \leq \sqrt{\lambda_1} \leq 18$. This network is therefore much more sensitive to undetected errors in the direction observations than Net I. This is even worse for distance measurements; their boundary values are all approximately 158 cm and their possible effect on the coordinates can be as much as 155 cm.

Table 6 gives the effect of an error in distance 103-2 as an example; the effects on the coordinates are shown only for points where they are larger than 50 cm.

The connection with the block is as described for Net I. Although the precision of this network is much lower than the precision of Net I, the boundary values of the individual coordinates of the connection points are not much different. The coordinates of the corner points have boundary values of approximately 45 cm. The values for the coordinates of the other connection points are approximately 35 cm. If individual points are distorted with these values, the probability of detection is $\beta = 80$ percent when testing with $\alpha = 0.001$.

If we introduce an error of 40 cm in distance 103-2 in the network, then according to Table 7 some of the points will be distorted by more than 30 cm--values which are close to the boundary values for testing the

TABLE 5
Effect on the coordinates
if > 5 cm:

Point	X	Y
11	15.3	- 0.6 cm
21	13.6	0.9
31	12.2	2.5
41	10.9	4.1
51	9.8	5.6
61	8.6	7.1
2	8.5	- 2.4
12	9.8	- 2.5
22	9.5	- 1.2
32	8.7	0.3
42	7.7	1.8
52	6.7	3.3
62	5.6	4.7
3	9.1	- 4.9
13	8.0	- 4.2
23	6.9	- 3.2
33	5.9	- 1.9
4	7.4	- 7.2
14	6.1	- 6.3
24	4.8	- 5.3

$\sqrt{\lambda_1} = 11.5$

Net I, distance : 1-11
Boundary value : 18.5 cm
Effect on corrected
observation : 16.4 cm

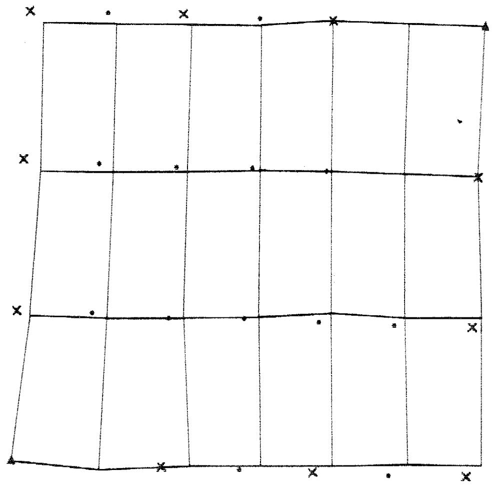

Fig. 3

Block ⟷ Net I

▲ S-base points
× Other connection points
• Other common points, not used for connection

Distortion 20 cm
Network 1 km

TABLE 6
Effect on the coordinates
if > 50 cm

Point	X	Y
2	0.5	144.4 cm
104	2.6	140.9
105	5.5	137.3
106	9.3	133.8
3	13.7	130.3
107	19.3	126.7
108	25.1	123.2
109	32.0	119.6
4	39.7	116.1
110	36.4	109.6
111	33.0	98.5
112	29.7	88.8
14	26.4	78.8
113	23.1	68.5
114	19.8	58.0

$\sqrt{\lambda_1} = 104.5$

Net II, distance : 103-2
Boundary value : 158.6 cm
Effect on corrected
observation : 158.3 cm

TABLE 7
Effect on the coordinates
if > 12 cm:

Point	X	Y
2	0.1	36.1 cm
104	0.7	35.2
105	1.4	34.3
106	2.3	33.3
3	3.4	32.6
107	4.8	31.7
108	6.3	30.8
109	8.0	29.9
4	9.9	29.0
40	9.1	26.0
111	8.2	24.6
112	7.4	22.2
14	6.6	19.7
113	5.6	17.1
114	4.9	14.5

Net II, error of 48 cm
in distance : 103-2

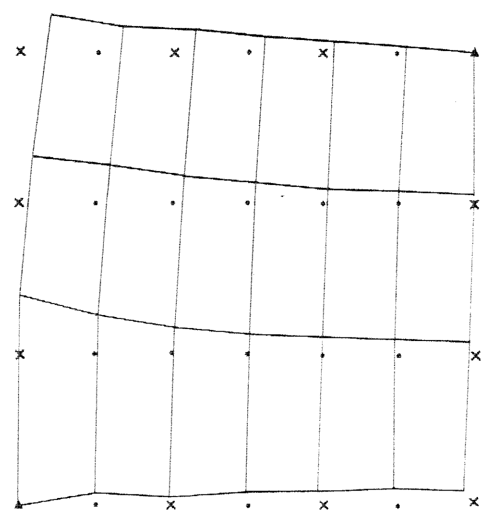

Fig. 4

Block ⟷ Net II

▲ S-base points
× Other connection points
• Other common points, not used for connection

Distortion 20 cm
Network 1 km

individual coordinates mentioned above. Test computations, however, show that data snooping applied to the coordinates of the connection points does not lead to any rejection when testing with $\alpha = 0.001$. The largest test value was 2.6 for the y-coordinate of point 2, whereas the critical value of the test is 3.3. The adjustment results show that here, too, nearly the full effect of this deformation propagates to the final coordinates (see Figure 4).

The situation for this network is much more dangerous than for Net I. Large distortions were not very likely to occur there because data snooping in the original network gave good reliability. Net II, however, is too weak to detect errors of the size used in this example ($\beta = 1$ percent), so they easily slip through to the second phase of the adjustment--in which it is not very likely that they will be detected.

5 DISCUSSION

At this point, a few observations should be made concerning the examples. The first observation concerns the computational procedure. The strategy followed here seems to be different from normal practice in the sense that the network and the block are first adjusted independently. For the network, that is a normal procedure; the coordinates computed after adjustment are introduced as ground control for the block. The block, however, is generally adjusted and connected to ground control in one step. The main reason for this is that height control especially is needed to stabilize the solution of this adjustment. This is merely a practical reason. In theory, however, it makes no difference for the evaluation of the quality of the connection of block and network whether this connection is performed in a separate adjustment step or not.

The conclusions of this paper are therefore equally valid for the practical approach. The approach used here was chosen to demonstrate clearly the effect of the different assumptions on the precision of ground control.

Method 3 is most often applied in practice; this is justified insofar as the terrestrial coordinates are not given a correction after adjustment. The reason for this is that the precision of the network is in general much better than the precision of the free block, so that the corrections applied to the terrestrial coordinates are negligible. The precision of the block after connection to ground control can be so good, however, that the V.C. matrix of the ground coordinates cannot be ignored for evaluating the precision of the total system. Then Method 2 is appropriate. If the V.C. matrix of the terrestrial coordinates is not available, we could use a substitute matrix as proposed elsewhere [4, 10]. This is preferred over a diagonal matrix because the structure of land surveying networks gives correlation among coordinates.

For evaluation of the reliability of the connection of the block to ground control, only Method 1 has been used. The reason is that if the connection were made by Method 2, testing should still be based on Method 1 to give proper tests in a pseudo least squares adjustment. If the connection were made by Method 3, the related test would show only a small difference with Method 1. With Method 3, the difference in reliability between Net I and Net II would vanish completely.

Because of undetected errors in the land surveying networks, the final coordinates of the block are distorted over the whole block (at some points as much as 15 cm for Net I and as much as 36 cm for Net II). The inner geometry of the block is distorted only where the error in the network occurs (in the vicinity of point 11 for Net I and near point 2 for Net II). The geometry of the rest of the block is hardly effected. The quality of the block therefore did not really deteriorate as much as the distortions of the coordinates suggest. Problems may arise, though, when the block is connected to neighboring point fields which are not effected by this distortion. This observation may indicate how careful we should be when evaluating the accuracy of blocks by comparing the block coordinates with another set of coordinates.

Discrepancies found in such a comparison do not necessarily indicate a poor block quality or serious systematic deformations. Judgement should be based instead on a comparison of the inner geometry of the two coordinate systems; measures to compare these should be defined (see [9]).

The networks used in these examples did not make use of any higher order points. Local coordinate systems were defined by means of an S-base. The coordinates in an S-system are a direct function of the geometry of the network. This was done to show the essential task of ground control points in a block adjustment. That task is not so much to define a coordinate system but rather to control the geometry of the block. Therefore, the internal geometry of the network should be well determined by the network structure itself. That means that the network should be structured so that it gives sufficient possibilities for testing for errors in the observations, i.e., have high internal reliability. Connection to higher order points could then be used to link-up with a national coordinate system, but this can be done safely only if the higher order points have not been distorted. The network should therefore be strong enough and there should be enough redundant higher order points to test these points while linking-up. This approach is in fact quite different from common practice, where higher order points are used to support or strenghten the network. In our philosophy, networks for the determination of ground control for aerotriangulation blocks should be self-reliant.

We see that the role of ground control points in a photogrammetric block adjustment is different from the role of higher order points in a network adjustment.

The two networks are extreme cases. Net I is very strong, but a structure like this will in many cases be difficult to realize. In general, the topography will make it impossible to measure all sides indicated and, moreover, this is too expensive and anyway not necessary. There are simpler structures which also give sufficient support to the block. Net II is too weak, but it has the advantage that polygon networks like this are, with some modifications, often easy to realize. In addition, no measurements are made in the interior of the block where, according to traditional photogrammetric specifications, no ground control is needed for planimetry.

The quality of the network can be improved considerably by measuring two polygons through the interior of the block, however, thus dividing the network into four closed loops (see Figure 5). If the side lengths of these extra polygons are also approximately 500 m, the boundary values of the measured distances reduce to approximately 50 cm, which is one-third of the original value.

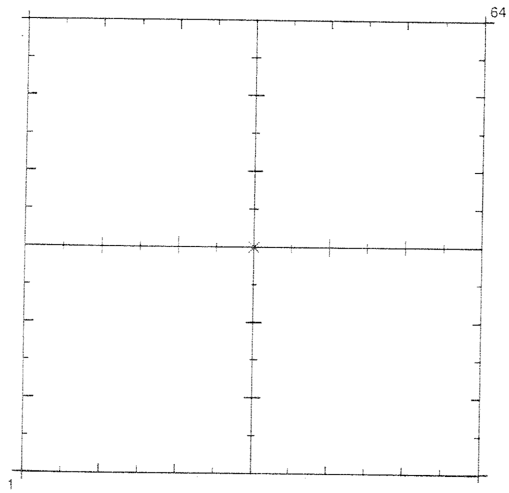

Fig. 5

The effect of an undetected error on the final coordinates reduces proportionally. After this densification, we have a network which is strong enough to support the block adjustment and will be feasible in real terrain. The short side lengths require many instrument stations, which make the measurement of the network expensive. If the topography of the terrain allows an average side lengths of 1 km, the number of stations is reduced by half and the boundary values of the distances reduce by two-thirds. This leaves boundary values of 110 cm for distances in a polygon at the perimeter of the block (as in Figure 2) and 37 cm if polygons through the interior are added (Figure 5). The possible distortions of the coordinates are reduced proportionally. Errors in the directional observations are less harmful; they are therefore not discussed here.

The traditional strategy in aerotriangulation--choosing control points for planimetry at only the perimeter of the block--has its main importance for controlling the precision of the adjusted block coordinates. This strategy was developed under the assumption that ground control can be considered as not stochastic. This assumption, however, is justified only as far as it leads to no correction of ground control coordinates after adjustment. For evaluating the precision of the total system after adjustment, maintaining this assumption depends very much on the quality of the network and the block. On the other hand, we saw that the reliability of the total system also depends largely on the quality of the network. Consequently, the block and the network should be planned together in a combined effort of photogrammetrist and land surveyor to obtain an integrated point determination system designed to meet uniform requirements for precision and reliability.

The fact that a terrestrial network will get sufficient strength only if measurements are made through the interior of the block area means that the selection of plan ground control points need not to be limited to the block perimeter. The argument that this will reduce the work of the land surveyor is no longer valid.

The examples in this paper referred to a rather small block with a simple block structure in which only planimetry was considered. When height is also taken into account, we should first define a local system for height or for the combination of planimetry and height. Then experiments should be made similar to the ones described here. It is about time that geodesists and photogrammetrists get together to come to an integration of their survey systems--certainly in an era when people are discussing the possibilities of setting up integrated land information systems.

REFERENCES

1 Ackermann, F E, 1966. On the theoretical accuracy of planimetric block triangulation. Photogrammetria 21, pp 145-170.

2 Baarda, W, 1967. Statistical concepts in Geodesy. Netherlands Geodetic Commission, Delft.

3 Baarda, W, 1968. A testing procedure for use in geodetic networks. Netherlands Geodetic Commission, Delft.

4 Baarda, W, 1973. S-transformations and criterion matrices.

5 Bouw, T, M Molenaar, 1980. On the evaluation of ground control. ITC Journal 1980-3, pp 421-433.

6 Förstner, W, 1980. The theoretical reliability of photogrammetric coordinates. ISPRS, Commission III, Hamburg.

7 Jordan, Eggert and Kneissl, 1972. Handbuch der Vermessungskunde, Band III a/3. Photogrammetrie, § 133, pp 1927-2020.

8 Jorgenson, J B, 1980. A comparison of the precision and reliability of terrestrial coordinates with those obtained from photogrammetry. MSc thesis, ITC.

9 Molenaar, M, and T Bouw, 1978. Essay on empirical accuracy studies in aerial triangulation. ITC Journal 1978-1, pp 81-97.

10 Molenaar, M, 1981. S-transformations and artificial covariance matrices in photogrammetry. ITC Journal 1981-1, pp 70-79.

11 Molenaar, M, 1981. A further inquiry into the theory of S-transformations and criterion matrices. Netherlands Geodetic Commission, Delft.

12 Molenaar, M, 1982. Recent developments in the criterion theory for precision of geodetic networks. "Forty years of thought", collection of papers at the occasion of Professor Baarda's 65th anniversary, Geodetic Institute, Delft.

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