STATISTICAL ANALYSIS OF RELEVANT TERRAIN FEATURES THROUGH DIGITAL ELEVATION MODELS

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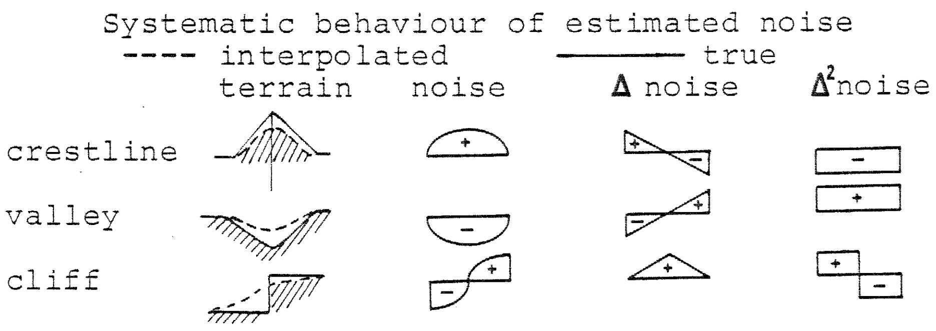
1. Introduction

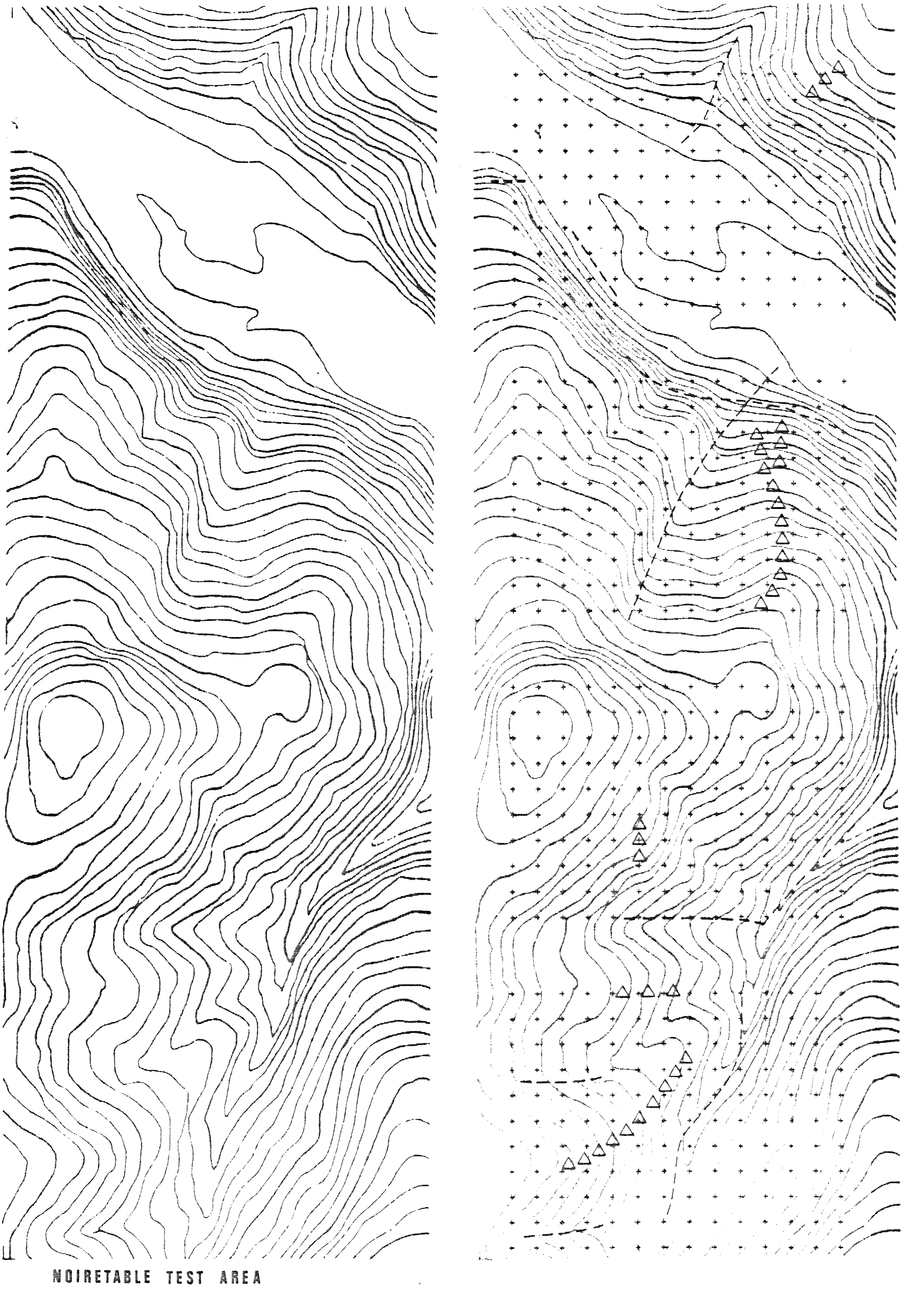
The research on digital elevation models is the object of a commission (B) of OEEPE. The main purpose is to verify the suitability of digital elevation models to derive, by simple computer procedures, relevant features of the terrain like impluvia and crests and to optimize a DEM data bank.
The test area examined in this paper is a hilly zone of central France, proposed by the french study group of commission B, and especially mapped by the IGN staff. A grid of measured elevations (average spacing 40 m) has been used for computations and a very detailed contour line map has been drawn from more dense terrain data to allow verification. Out of this zone, a strip of 0.640x2 km, containing 800 grid points, has been extracted and processed.

2. Mathematical approach

In a previous experience a stepwise collocation approach has been used to predict the elevation in the some manner as for a signal in a stochastic process. Where this method failed (relatively high noise with respect to the signal) it has been realized that standardized noises were significant and showed relevant terrain features. Let us remember that noise is the difference between observed values and values interpolated by collocation (signal). It seemed reasonable that further information could be gained by examining the first and second finite differences Δ and Δ^2 (slopes and curvatures) of standardized noises affecting the elevations. In fact, the collocation method smooths the terrain irregularities so that the analysis of noises makes evident a systematic behaviour corresponding to protrusions and hollows, and the analysis of the finite differences of noises shows where the surface changes its trend.



The computation of finite differences is easy.



NOIRETABLE TEST AREA

If n_{ij} is the noise at P_{ij} of the grid

$$\Delta n_{i,j/i} = (n_{i+1,j} - n_{i-1,j})/2l$$
$$\Delta n_{i,j/j} = (n_{i,j+1} - n_{i,j-1})/2l$$
$$\Delta^2 n_{i,j/ii} = (n_{i+1,j} - 2n_{i,j} + n_{i,j-1})/l^2$$
$$\Delta^2 n_{i,j/jj} = (n_{i,j+1} - 2n_{i,j} + n_{i,j-1})/l^2$$
$$\Delta^2 n_{i,j/ij} = \Delta^2 n_{i,j/ji} = (n_{i+1,j+1} - n_{i+1,j-1} - n_{i-1,j+1} + n_{i-1,j-1})/l^2$$

where l is the grid spacing.
Clearly the noise slope Δn is the gradient of elevation noise (its components are $\Delta n_{i,j/i}$ and $\Delta n_{i,j/j}$) and the noise curvature is the Hessian tensor of the some elevation noise.
The maximum noise slope is the norm of the gradient; that is

$$|\Delta n| = \sqrt{(\Delta n_{i,j/i})^2 + (\Delta n_{i,j/j})^2} \quad ;$$

the maximum noise curvature is the maximum eigenvalue of the Hessian matrix.

$$\max(\Delta^2 n) = \max\left[\Delta^2 n_{i,j/ii} + \Delta^2 n_{i,j/jj} \pm \sqrt{(\Delta^2 n_{i,j/ii} - \Delta^2 n_{i,j/jj}) + 4\Delta^2 n_{i,j/ij}}\right]/2$$

3.Analysis of the results

To easily perform the comparison of the true terrain with the noise field, n, Δ n, max(Δ^2n) have been mapped in fig. 1,2,3,4. The first map (fig.1) shows the discrepancies (noises) between interpolated and true values of elevation. Positive errors mean that the interpolated surface is lower than the true one; the contrary holds for negative errors.
To avoid computer difficulties the examined strip has been divided into four parts, each containing about 200 grid points. Of course for the border points of each part no finite differences have been computed.
Contour lines have an interval of 10 meters.
Sinking and generally concave features (ditches, deep river beds, valleys etc.) are to be searched where large negative values nestle. At a first sight no difference exsists between a deep valley and any abrupt concave change of slope, because the statistical method developes an interpolated surface which could have a very marked prevailing slope. A first draw of valleys and similar features is possible by interpolating evident linear trends of negative values (vectorization). Tops, crests and generally convex features are to be searched in positive zones. A much more detailed information on terrain break lines can be gained from the examination of the mapped values of noise slopes. In fig.2 error vectors are shown; in fig.3 the norm of the vector with the sign of its largest component is drawn; this device allows a regionalization of the map with region borders which are significant and actually correspond to the most relevant linear features. So, for instance, the riverbed in the deep valley in the lower part of the map can be easily drawn as the dividing line between positive and negative regions. This line falls entirely in a negative region of elevation noise values (fig.1).
As could be easily inferred the most detailed description of terrain irregular patterns is contained in the map of noise curvatures (fig.4) and the reason of this lies in the general properties of second order derivatives (obtained in this case through finite differences applied to the elevation error, or noise) to evidence local changes of the shape of a surface.

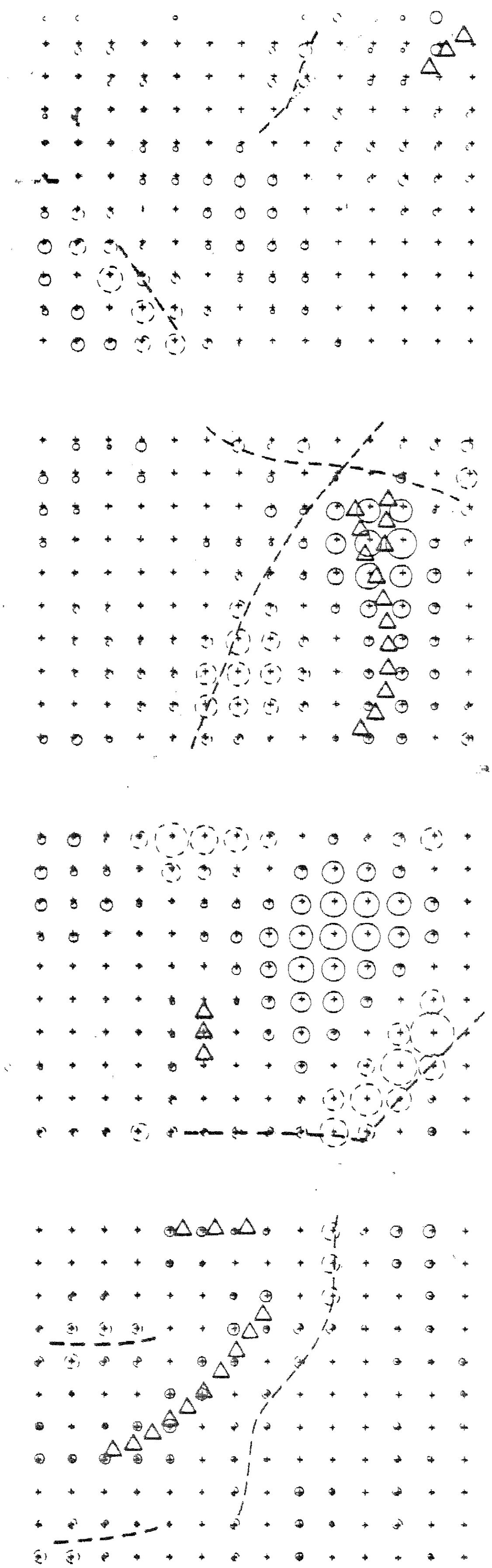


FIG. 1

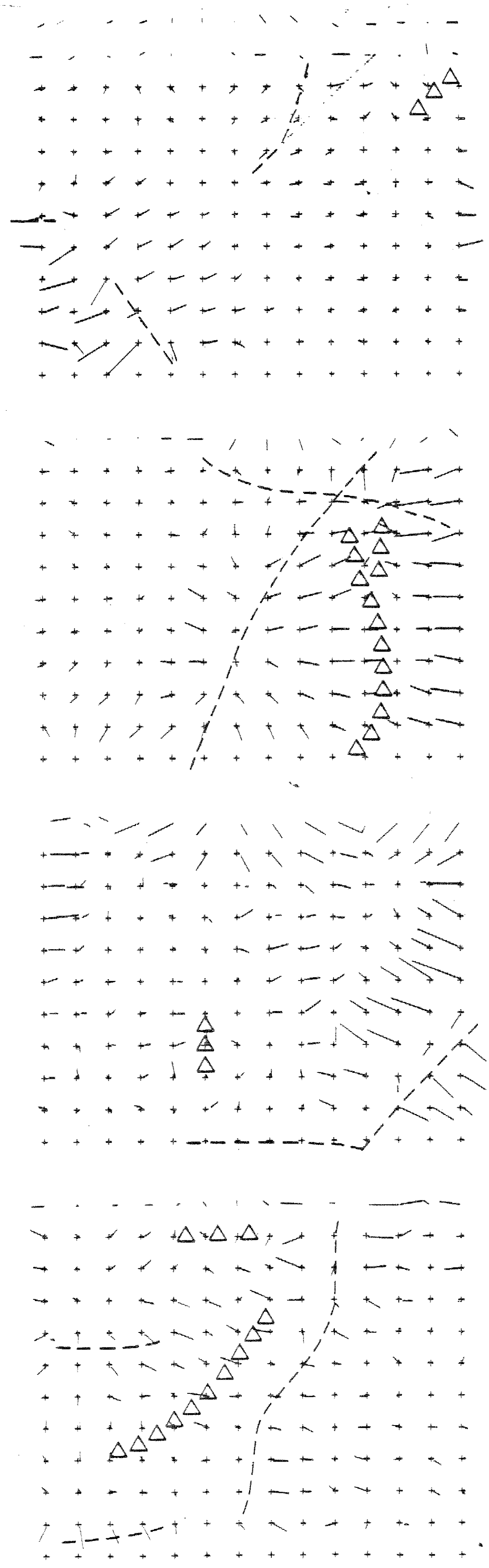


FIG. 2

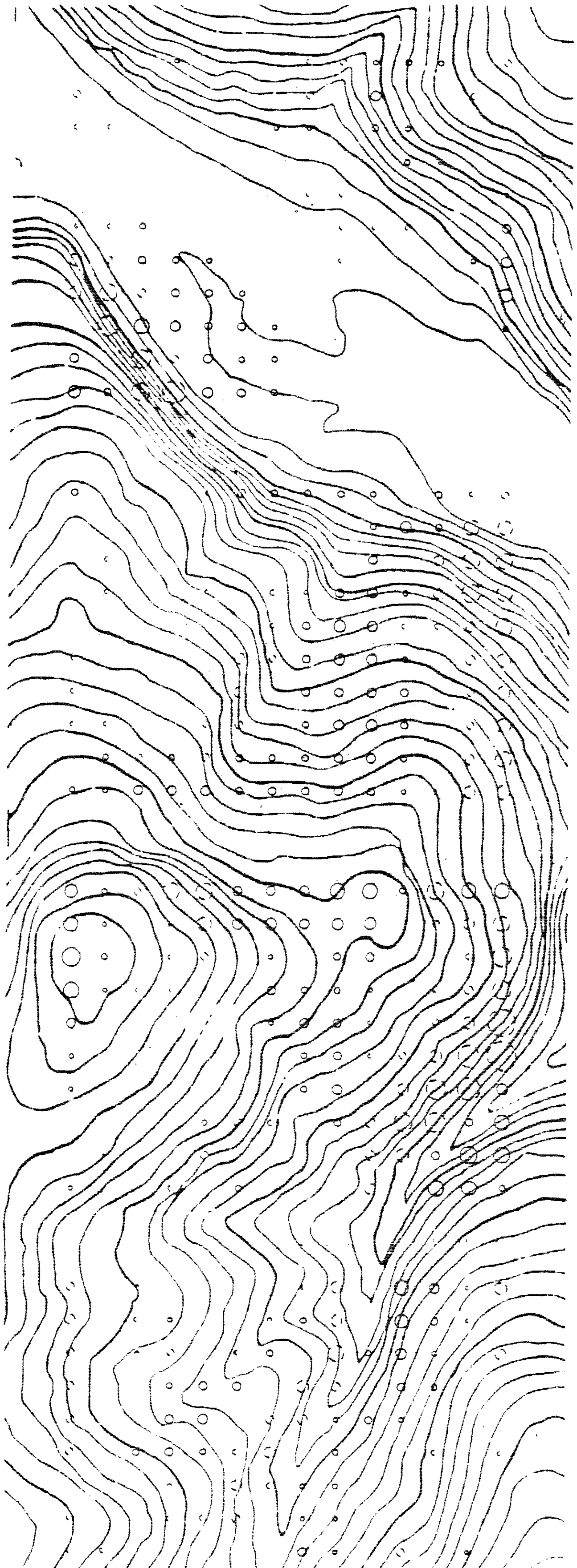


FIG. 3

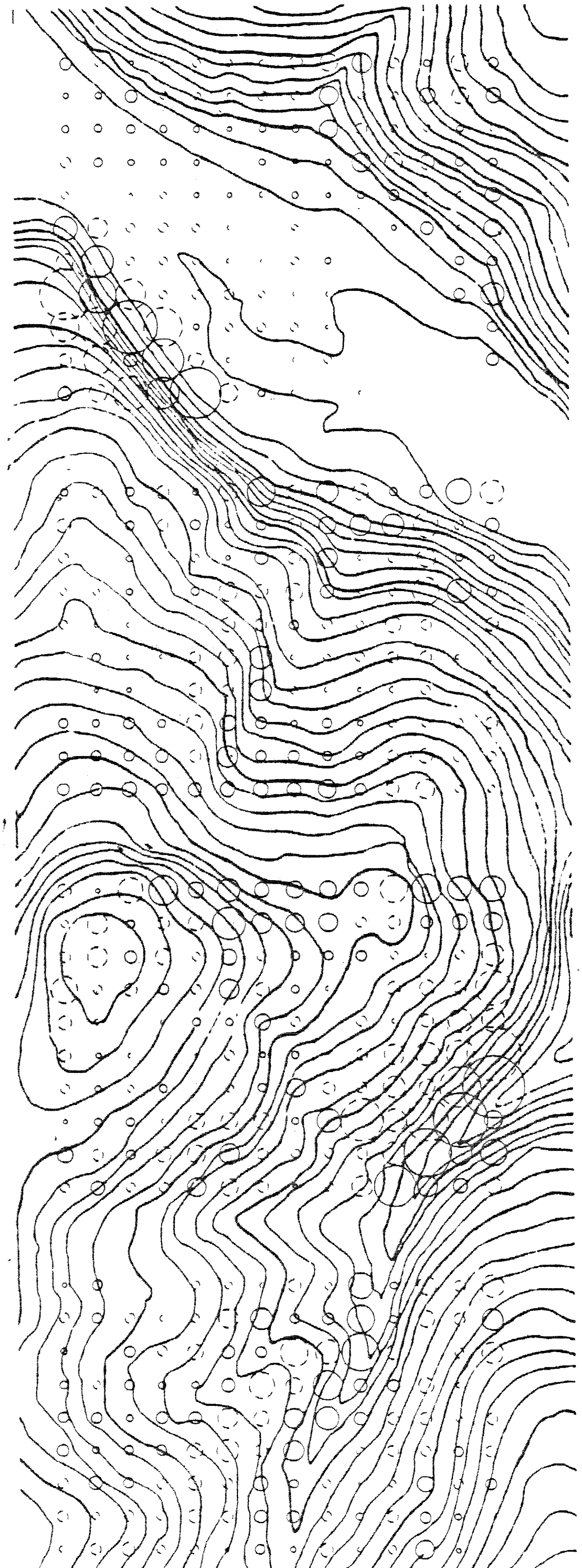


FIG. 4

Positive errors clearly show concave features and locate all the impluvia (valleys, river beds) in the examined strip. Negative errors correspond to crest lines.

4. Conclusions

The automatic tracking of break lines is very important because it allows a partitioning of the surface into statistically homogeneous areas. Without changing the density of the grid points , but simply associating them in an optimum way, a much more accurate DEM is obtained. This means that the statistical analysis (i.e. a new estimation of terrain covariance and a new run of stepwise collocation) can be repeated on each homogeneous area to check the validity of such integrated DEM.
Not only elevations , but also their first and second derivatives (slopes and curvatures) can be predicted by means of collocation; this would give a much finer description of the terrain and a valuable aid in identifying runoff patterns, saddle shaped features and other relevant landforms.
The actually employed approach to this problem implies the use of a Taylor Karman structure, whose mathematical setup is illustrated in the Appendix; its physical and statistical meaning lies essentially in the hypotesis of an isotropic structure of the covariance function of the elevation.
Taylor Karman values of slopes and curvatures have been predicted and compared with true values (derived from elevations). The results (not reported here for the sake of brevity) show a different behaviour from those obtained via finite differences; indeed the derivatives give an information of very local character, while the finite differences smooth the processes. So this last will be used to detect break lines and to partition the terrain in homogeneous areas, and the Taylor Karman values will be considered as good interpolated values.

Appendix

Let p_P be the slope vector , that is the gradient of elevations in P :

$$p_P = \nabla_P q_P = \begin{vmatrix} \frac{\partial q}{\partial x} \\ \frac{\partial q}{\partial y} \end{vmatrix}_P$$

and C the curvature Hessian matrix :

$$C_P = \nabla_P \left(\nabla_P^T q_P \right) = \begin{vmatrix} \frac{\partial^2 q}{\partial x^2} & \frac{\partial^2 q}{\partial x \partial y} \\ \frac{\partial^2 q}{\partial x \partial y} & \frac{\partial^2 q}{\partial y^2} \end{vmatrix}_P$$

The autocovariance function of elevations is estimable, under the hypothesis that it is homogeneous and isotropic :

$$C_{q_P q_Q} = \phi(r_{PQ})$$

The crosscovariance function between slopes and elevations is derived using the Taylor Karman structure :

$$C_{p_p q_Q} = E(\nabla_p q_p\ q_Q) = \nabla_p C(q_p q_Q) = \nabla_p \phi(r_{PQ}) = \phi'(r_{PQ})\frac{\underline{r}_{PQ}}{r_{PQ}}$$

The crosscovariance function between curvatures and elevations is obtained in a similar way :

$$C_{p_p q_Q} = E\ \nabla_p(\nabla_p^T q_p) q_Q = \nabla_p\ \nabla_p^T E(q_p q_Q) = \nabla_p \nabla_p^T C_{q_p q_Q} =$$

$$= \nabla_p \nabla_p^T \phi(r_{PQ}) = \left[\phi''(r_{PQ}) - \phi'(r_{PQ})\frac{1}{r_{PQ}}\right] \frac{\underline{r}_{PQ}\,\underline{r}_{PQ}}{r_{PQ}^2} + \phi'(r_{PQ})\frac{1}{r_{PQ}}\ I$$

where I is the identity matrix.
In the case of the DEM examples analyzed in the paper the above formulas give :

$C_{q_p q_Q} = aJ_0\,(br_{PQ})$ ---which is the best fit function of empirical values of terrain covariance

$$C_{p_p q_Q} = -abJ_1\,(br_{PQ})\ \frac{1}{r_{PQ}}$$

$$C_{c_p q_Q} = ab\ J_2\,(br_{PQ})\frac{\underline{r}_{PQ}\ \underline{r}_{PQ}}{r_{PQ}^2} - abJ_1\,(br_{PQ})\frac{1}{r_{PQ}}\ I$$

where J_0, J_1, J_2 are Bessel functions of 0,1,2 order.

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