CONTROL OF A COMPARATOR BY PROPAGATION OF VARIANCE AND COVARIANCE

José Catalán
Instituto Geográfico Nacional
España
Commission III

Abstract

The photocoordinates measurements, as it is well known are made - by a comparator (stereo or mono). Images of the fiducial marks - appear on each photograph. They are defined and marked better than images of pass or control points. We can say that the images of - the fiducial marks, as they appear in two sucessive photos, are - located in the same relative positions. By comparing the lenght between fiducial marks in all sucessive pair of photos of the block, we can stimate the uncertainty of a single coordinate measurement by application of propagation law of variances and covariances. - So, it is possible a coarse control of the comparator by using as reference the uncertainty of a single coordinate measurement obtained from the usual measurements on the fiducial marks, as they are obtained in an aerotriangulation block. A numerical example is also given.

Introduction

The control of a comparator referred to in this paper is done by - means of the evaluation of the uncertainty of a single photo-coordinate measurement, by using the photo-coordinates as they are obtained from the usual measuring process by a comparator. So, it - will be possible a coarse control of the comparator by using as - reference the numerical value obtained.
A possible method of evaluation could be defined, as follows :1) To chose a set of "fixed" points along the set of photos, for - example the four fiducial marks. 2) To express the photo-coordinates of the fiducial marks referred to the photograph system which origin is the principal point. 3) To compute both of the mean and the variance of the photocoordinates, using the numerical value - of the variance as reference.
Unfortunately, we can not assume that the images of the four - - fiducial marks were fixed points on the whole block. But we can - say that the images of the four fiducial marks, as they appear in two sucessive photographs, are located in the same relative positions, because it is possible to assume that the shrinkage film - is the same in two sucessive photographs. So, the actual distance between a pair fiducial marks is the same on the left and the - right photographs. If we compute each difference of distances - obtained from each pair of left and right photographs, we would obtain a set of deviations (so many deviations as models in the block for each pair of fiducial marks we use). By standardizing this deviations we can realise that they are normally distributed, N (0,1) for the whole block. The propagation law of variances - and covariances allow us to link the variance of the standardised deviations to the variance of a single photo-coordinate for the -

whole set of measurements made by an operator on the whole block.

Statistical background

Let $(x_i\ y_i)$ be the coordinates of the four fiducial marks measured in the left photograph and $(x_i^*\ y_i^*)$ those in the right photograph. We considerer $x_i\ y_i\ x_i^*\ y_i^*$ as the values of the random variables - $x\ y\ x^*\ y^*$. With the usual notation,

$$X^T = (x_1\ y_1\ \dots\ x_4\ y_4\ x_1^*\ y_1^*\ \dots\ x_4^*\ y_4^*)$$

the expected value of any coordinate is $E(x_i) = \bar{x}_i$; the deviations are $\Delta x_i = x_i - E(x_i)$ and then the deviations vector is

$$e^T = (\Delta x_1 \Delta y_1 \dots \Delta x_4 \Delta y_4 \Delta x_1^* \Delta y_1^* \dots \Delta x_4^* \Delta y_4^*)$$

By definition, the variance of X is

$$\underset{16x16}{V(X)} = E\{e.e^T\} = E\left\{ \begin{vmatrix} (\Delta x_1)^2 & \Delta x_1 \Delta x_2 \dots \Delta x_1 \Delta y_4^* \\ \Delta x_1 \Delta y_1 & (\Delta y_1)^2 \dots \Delta y_1 \Delta y_4^* \\ \vdots & \vdots \\ \Delta y_4 \Delta x_1 & \Delta y_4^* \Delta y_1 \dots (\Delta y_4^*)^2 \end{vmatrix} \right\}$$

If the measurements are all mutually independent, then - - $E(\Delta x_i\ \Delta y_j) = 0$, and if they are measured in the same metrological conditions $E(\Delta x_1)^2 = E(\Delta y_1)^2 \dots = E(\Delta y_4^*) = \sigma^2$ and then

$$\underset{16x16}{V(X)} = \sigma^2 I \qquad (1)$$

Propagation law

We wish to calculate the numeral value of σ^2 by using the law of propagation of variances and covariances. For this purpose we are going to built up a function G the variance of which is, 1) easy to compute, 2) connected with V(X).
The function is $G^T = (G_a\ G_b)$, where $G_a = l_a - l_a^*$ and $G_b = l_b - l_b^*$.
The l_a and l_a^* are the distances between the fiducials marks number 1 and 2 in the left and right photographs, respectively. The same for l_b and l_b^* between de fiducials marks number 3 and 4. Then

$$G_a(x_i\ y_i\ x_i^*\ y_i^*) = l_a - l_a^* =$$
$$= \left[(x_2-x_1)^2 + (y_2-y_1)^2\right]^{1/2} - \left[(x_2^*-x_1^*)^2 + (y_2^*-y_1^*)^2\right]^{1/2}$$

Expanding in a Taylor series about $\bar{X}$ and keeping only the first - term,

$$G_a = G_a(\bar{x}_i\ \bar{y}_i\ \bar{x}_i^*\ \bar{y}_i^*) + \left(\frac{\partial G_a}{\partial x_i}\right)_{\bar{X}} \Delta x_i + \left(\frac{\partial G_a}{\partial y_i}\right)_{\bar{X}} \Delta y_i +$$
$$+ \left(\frac{\partial G_a}{\partial x_i^*}\right)_{\bar{X}} \Delta x_i^* + \left(\frac{\partial G_a}{\partial y_i^*}\right)_{\bar{X}} \Delta y_i^* \qquad (i=1,2)$$

and the same for G_b (i = 3,4). In matrix notation

$$\begin{bmatrix} G_a \\ G_b \end{bmatrix} = \begin{bmatrix} \bar{l}_a - \bar{l}_a^* \\ \bar{l}_b - \bar{l}_b^* \end{bmatrix} + \begin{bmatrix} \frac{\partial G_a}{\partial x_1} & \frac{\partial G_a}{\partial y_1} & \cdots & \frac{\partial G_a}{\partial y_4^*} \\ \frac{\partial G_b}{\partial x_1} & \frac{\partial G_b}{\partial y_1} & \cdots & \frac{\partial G_b}{\partial y_4^*} \end{bmatrix}_{\bar{X}} \cdot \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \vdots \\ \Delta y_4^* \end{bmatrix}$$

or

$$G = E(G) + M \, . \, e$$

By definition

$$V(G) = E\left\{(G-E(G)).(G-E(G))^T\right\} =$$

$$= E\left\{(M.e).(M.e)^T\right\} =$$

$$= M.E\left\{e.e^T\right\} M^T$$

where $E\left\{e.e^T\right\} = V(X)$, and in acordance with (1)

$$V(G) = \sigma^2 \; M.M^T$$

after computation $M \; M^T = 4 \; I$, and then

$$\underset{2x2}{V(G)} = 4 \; \sigma^2 \; I \qquad (2)$$

Standardized deviations

For each model we can compute two deviations: $d_a = l_a - l_a^*$ and $d_b = l_b - l_b^*$, whose common variance is $4\sigma^2$ (in accordance with(2)), where σ^2 is the variance difined in (1), or as usual, the variance of unit weight for the measurements of photo-coordinates of fiducial marks.
The standardized deviations are $(d_a - \bar{d}_a)/4\sigma^2$ and $(d_b - \bar{d}_b)/4\sigma^2$. We can assume that the two sets of deviations are normally distributed and also that $\bar{d}_a = \bar{d}_b = 0$. So, the two sets of standardized residuals, $d_a/4\sigma^2$ and $d_b/4\sigma^2$ are the observed values of the same random variable, d, normally distributed N(0,1).
The numerical values obtained in the next section confirm the assumption.

Numerical example

First at all we compute the deviations da and d_b for the whole block. Second we compute the mean, m_d, and the variance σ_d^2, for the whole d_a and d_b.
If the measurements are free of gross errors the mean $m_d \simeq 0$ and $\sigma_d \simeq 2\sigma$, that is the standard deviation of unit weight for the measurements of photocoordinates of fiducial marks is $\sigma_d/2$.

In order to prevent the effects of gross errors we can consider that the randon variable t= $[(n-2d)/(n-1-d^2)]^{1/2}$,(n is twice the number of models) is a Student one with n-2 degrees of freedom. The outliers should be those values beyond the values of - Student's distribution for a spicified level of significance; for example a=o.ool. In this case and for photogrammetric - blocks (n> 120) the test value for rejection is 3.29, that is the Students distribution becames a normal one.
The method has been applied to a 4 strip, 105 models block, - photoscale 1:30000. For the functions G_a and G_b 105x2 deviations have been computed. The mean and the standard deviation are m_d=0.00032 mm, s_d=0.008 mm. All the values appear multiplied by 500. From the distribution graph we can see that the deviations are normally distributed.
From s_d, the standard deviation of unit weigth for the measurements of photocoordinates of fiducial marks is $s_d/2$=0.004 mm.

Table I

mod.	d_a(x500)	d_b(x500)	mod.	d_a(x500)	d_b(x500)	mod.	d_a(x500)	d_b(x500)
1	4.99573	2.85645	2	-3.33252	-3.24402	3	2.33765	1.19324
4	-3.29590	-3.22571	5	0.10986	6.10657	6	-2.85950	-0.86365
7	-0.09766	0.90027	8	-2.41394	-3.34473	9	-0.66834	2.22168
10	-1.02539	-4.92859	11	-0.86365	-2.88086	12	-2.94190	-1.98059
13	5.91431	2.89307	14	0.93994	0.94299	15	2.60010	-2.43836
16	3.20435	4.16260	17	-4.76379	-0.74463	18	-4.90112	1.12305
19	5.01099	1.97144	20	-4.29688	-0.30823	21	5.16968	2.15149
22	-6.98853	0.09155	23	1.07727	-0.90332	24	-1.04065	-4.08325
25	3.21350	-2.79541	26	-15.40222	-4.19922	27	7.97424	-1.20850
28	-2.63062	-0.62866	29	-0.14954	-0.12207	30	2.08130	2.15149
31	-1.08948	-3.13416	32	2.79541	1.83716	33	-3.74146	-1.72730
***								
34	0.85144	-0.08240	35	-2.87170	-1.91040	36	0.42419	0.31128
37	-3.36914	-5.38330	38	0.83008	1.78833	39	-3.89099	-2.77405
40	0.81787	-0.18311	41	0.17395	1.16882	42	0.04883	0.97656
43	-0.13123	-2.16064	44	4.93164	-1.06506	45	1.16882	1.23596
46	4.86450	-0.08850	47	-2.06604	-4.11987	48	1.16882	4.21753
49	-5.63049	-2.64587	50	0.34180	1.27869	51	-3.63159	-1.65100
52	5.78003	-1.30005	53	-2.53601	-1.43433	54	2.97546	-2.05689
55	1.06506	-2.01416	56	-0.01221	4.04053	57	-1.85242	-2.82593
58	2.79236	3.73840	59	-1.08032	-2.05689	60	0.00916	5.09644
61	-0.01526	-4.02527	62	1.11694	-0.87891	63	-2.74658	-3.69873
64	3.46069	2.51465	65	-2.87476	-1.83716	66	7.04956	2.01416
67	-2.97546	-4.00086	68	1.10779	-0.90027	69	-2.99072	-1.04065
70	2.74963	0.74158	71	3.21655	0.15259	72	2.68555	6.67114
73	-4.73328	-3.68652	74	4.06799	2.05078	75	-0.10986	1.83716
76	1.50147	2.52686	77	-7.69653	-9.57947	78	3.43933	4.38538
79	-1.01929	3.99170	80	-1.39160	-2.38953	81	2.45972	2.42615
82	-1.76086	-3.74451	83	4.33350	1.20544	84	-3.60107	-3.58276
85	3.15552	2.19727	86	1.06201	-4.91638	87	0.95825	0.95825
88	-0.95825	-0.90942	89	0.91553	2.85034	90	-4.89197	-2.83203
91	1.98975	3.01514	92	2.83814	1.73035	93	5.05371	3.09448
94	-1.87378	-2.93274	95	2.78931	1.80054	96	-4.73938	-3.76282
97	3.97644	2.03247	98	-2.31018	-3.26538	99	2.82288	1.79138
100	-4.61731	-2.57263	101	3.24097	2.21558	102	-4.22974	0.84229
***								
103	0.68359	1.62354	104	1.31531	-1.68457	105	1.62659	2.58179

Table II

```
210                                            n
1 1 2 17 54 66 48 18 3 0 0                     fi
-0.159959972 4.01896000                        md, sd (x500)
      MM.
<-0.0090|
 -0.0080|
 -0.0060|*
 -0.0040|************
 -0.0020|*****************************************
  0.0   |**************************************************
  0.0020|*************************************
  0.0040|*************
  0.0060|**
  0.0080|
> 0.0090|
```

References

/1/ ALLISY A. : "Some statistical methods used in metrology" in Metrology and Fundamentals Constant. Ed. F. Milone, P.Giacomo. North Holland, 1980

/2/ BJERHAMMAR A. : Theory of errors and generalized matrix inverses. Elsevier, 1973