THE DIGITAL ELEVATION MODEL (DEM) AND ITS UTILIZATION FOR CONTOUR LINES PLOTTING AND THE BUILDING-UP OF THE 3D REPRESENTATIONS OF THE RELIEF

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ABSTRACT

We present the design of the programs system I_3GSR_2S.The system allows input data randomly or uniformly sampled.For data with random structure,the collocation method is applied where the trend surface is statistically determined and the covariance function may be automatically changed to the purpose of adapting it to the various terrain classes.Uniform structures are processed by program based on the approximation theory with spline functions.

After the computation of the DEM,optional,conventional representation of the relief,or 3D ones may be generated.The method of plotting contour lines make use the digitizing principles in the square schema. 3D representations are carried out by an optimized variant of the Wright method.

Maps with digitally plotted contour lines,as well as exemples of 3D representations are given for exemplification.

RESUME

On présente la conception du système de programmes I_3GSR_2S. Le sytème admet des données initiales échantillonnées de façon uniforme ou aléatoire.Pour les données à structure aléatoire on applique la méthode de la collocation,où la superficie de tendence est statistiquement determinée,tandis que la fonction de covariance peut être automatiquement changée en vue de son adaptation aux différentes classes de terrain.Les structures uniformes sont traitées à partir d'un programme basé sur la théorie de l'approximation à fonctions spline.

Après le calcul du DEM,optionnel peuvent être engendrées des représentations conventionnelles du relief ou 3D.La procédure de traçage des courbes de niveau utilise les principes de numériser en schéma quadratique.Les représentations 3D sont realisées par une variante optimisée de la méthode Wright.

En guise d'exemple on présente des sections de plan à courbes des niveau restituées de façon numérique ainsi que des exemples de représentations 3D.

ZUSAMMENFASSUNG

Es wird das Konzept des Programmierungssystem I_3GSR_2S dargestellt. Das System lässt sich als Muster aleatorisch oder regelmässig genommenen Anfangsdaten zu.Für die Daten mit aleatorischer Struktur wendet man das Kollokationsverfahren an,wobei die Tendenzfläche statistisch bestimmt wird und die Kovarianzfunktion automatisch verändert sein kann,um ihre Anpassung an verschiedene Geländeklassen zu erreichen.Die regelmässigen Strukturen werden mit Hilfe eines Programms verarbeitet dass sich auf die Annäherungsfunktionen mit Spline-Funktionen gründet.

Nach der Berechnung des DEM,optional können konventionelle Darstellungen des Reliefs oder 3D Darstellungen erzeugt werden.

Das Absteckungsverfahren von Höhenlinien verwendet Digitalisierungsprinzipien in quadratischen Schema.Die 3D Darstellungen werden durch eine verbesserte Variante des Wright Methode erzeugt. Zum exemplifizieren werden Karten mit digital zurückgegebenen Höhenlinien und Beispiele 3D Darstellungen dargestellt.

1. INTRODUCTION

Analised from the informative point of view,the digital terrain model (DTM) represents a system that comprises different cathegories of informations (heights,slopes,land values,soil type,hydrological,geological and geotechnical data,etc.) corresponding to the classes of objects and phenomena that forms the surface of the earth.In use,they will be reference data,necessary to the building of different types of special digital models.Out of the multitude of models with special character,the digital elevation model (DEM) presents a particular importance to the activities of topographic mapping as well as those belonging to civil engineering.This includes an ordered multitude of informations concerning the position and the height of certain points on the terrain surface,sampled according to the structural characteristics of the relief and algorithms for approximation and reconstruction of the terrain surface in the new points.Besides the altimetric data,that have an importance of a primary element,DEM requires and comprises some specific features of planimetry e.g. shore lines of lakes and rivers,geomorphologic lines for shape contouring,valley bottoms,break lines,etc.

2. BUILIDING DIGITAL ELEVATION MODEL (DEM) WITHIN THE COMPLEX PROGRAMS I_3GSR_2S

The automatization and the methods of analytical photogrammetry, give new valencies to topographic informations that once acquired according to the principle of checking maximum amount of information,recorded in a unique code and digitally stored,subsequently allow to be endlessy used without any restriction in different applications.It gives the possibility to generate by means an automatic processing and output systems of specific products for different aims,using same data basis resulting in a obvious increase yield and decrease of prices.The implementation of such technologies require the application of digital restitution methods within the building of digital models is the main component.
Following this line,a complex programs for the calculation of DEM using the acquired photogrammetric data was achieved,having as first application procedure the generation of contour lines.
In the present variant I_3GSR_2S(Intercol,Intercub,Interlin,Genmatre, Sol,Rot and Relief-S) the issued modulary complex programs has an advanced universallity degree since admits input data with random and uniform structures.Data sampling are semiautomatically achieved using a system maked of a stereometrograph connected with Independent-102F minicomputer where a composite sampling technique is applied.Arbitrary located points are sampled by operator subsequent to the photointerpretation of the relief contained in the stereomodel or uniformly placed in a square network whose step is determined using transfer function criterion.In both cases the points of the morphologic terrain features which will further complete the structure of the points of current surface description previously recorded are separately measured.

The first stage of data processing consist in their checking and partitioning at calculation units. Then, follows the building of the digital model, that for random distributed data uses the Intercol program conceived on interpolation basis using the collocation method. The range of interpolation (fig.1) within blocks points are determined, is of circular shape and obtained getting R the maximum correlation distance between points equal with $(1/2.5 \ldots 1/4)\lambda$ for even or relatively even terrain, or $(1/4.5 \ldots 1/6.5)\lambda$ for uneven terrain, λ being medium wave length of terrain oscilation in comparison with the general trend surface.

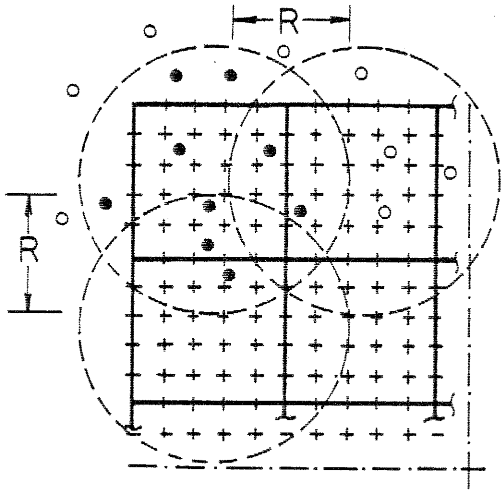

Fig-1
○ – reference points
● – reference points within interpolation range
+ – interpolated points

The choice of trend surface corresponding to every domain is made analysing the standard deviation of differences between the measured heights and their estimated values. These are infered by means all variants resulted after the combination of a basis surface composed of the first six terms of the incomplete bicubic polynom ($A_{22}=A_{23}=A_{32}=A_{33}=0$), with the rest of the terms.

At the beginning, the 12 polynomial coefficients are determined using the method of the least squares:

$$Z=[1\ x\ x^2\ x^3][A][1\ y\ y^2\ y^3]^T \qquad (1)$$

where $A_{22}=A_{23}=A_{32}=A_{33}=0$

$$Ax=L+v\ ;\ (A^TA)x-A^TL=0\ ;\ x=(A^TA)^{-1}A^TL \qquad (2)$$

Then we use the following apllied notations:

$$Z=\underbrace{A_{00}+A_{10}X+A_{01}Y+A_{20}X^2+A_{11}XY+A_{02}Y^2}_{A1}+\underbrace{A_{03}X^3}_{A2}+\underbrace{A_{21}X^2Y}_{A3}+\underbrace{A_{12}XY^2}_{A4}+\underbrace{A_{03}Y^3}_{A5}+\underbrace{A_{31}X^3Y}_{A6}+\underbrace{A_{13}XY^3}_{A7} \qquad (2)$$

According to these, the trend surface variants is obtained by combinning $A_1 \ldots A_7$ elements.

$$\underbrace{A1}\ ;\ \underline{A1}A2A3A4A5A6A7\ ;\ \underline{A1A2}A3A4A5A6A7\ ;\ ----------- \qquad (3)$$

$T_1(x,y)$, $T_2(x,y) \ldots T_m(x,y)$ surfaces will successively result, and owing to them new $\bar{Z}_i$ height values for N reference points from the interpolation range will be calculated, and subsequently the standard deviation of heights differences, σ.

$$\sigma=\left[\frac{1}{N}\sum_{(i=1 \ldots N)}(\bar{Z}_i-Z_i)^2\right]^{\frac{1}{2}}\ ;\quad T_1(X,Y)\longrightarrow\sigma_1\ ;\quad T_2(X,Y)\longrightarrow\sigma_2\ ;------\underset{(m=\text{no. of combinations})}{T_m(X,Y)\longrightarrow\sigma_m} \qquad (4)$$

The surface corresponding to $\sigma_{\min}$ value will be selected as trend surface ($\sigma_{\min} \Rightarrow T_c(X,Y)$. Its coefficients will be redetermined assigned for every equation in the system an weight equal with the reverse value of height difference previously obtained in reference point whose corresponds the equation.

$$(A^TPA)x-A^TPL=0\ ;\qquad x=(A^TPA)^{-1}A^TPL \qquad (5)$$

where $p_i = 1/(\bar{Z}_i-Z_i)$, in the P matrix weight.

Then the z_i height values are calculated as well as the dz_i

differences which participate in the interpolation process using the collocation method

$$Z(i) = T_c(X,Y) + c^T C^{-1} \bar{L} \qquad (6)$$

Since the inverse C^{-1} of covariance matrix and $\bar{L}$ vector remain constant for all interpolated points in a range, the interpolation vector $N=C^{-1}\bar{L}$ is computed so that the interpolation is practically reduced to the product $c^T N$.
The use of this method has the advantage of interpolating a group of points (fig.1) in every range with a diminished time necessary for the computation, since the covariance matrix need to be inverted once.
The programm comprises five covariance functions:

$$F_{C_1} = C(0)^{-kd^2}, \text{ where } C(0)=V, \text{or with filtering } C(0)=(1-10^{-3})V \qquad (7)$$

$$F_{C_2} =(1-(d^2/m^2))^{1/2};\ F_{C_3} = 1/(1 + d^2/m^2);\ F_{C_4}=e^{-2d/D};\ F_{C_5}=1/(1+d/m)$$

Having as decision parameter the comparison between the error of evaluation and the maximum admissible error:

$$m = C(0) - c^T C^{-1} c \leqslant m_{adm} \qquad (8)$$

the covariance **function** can be automatically changed in view of addapting to the terrain surface configuration.
When the range of interpolation is crossed by break lines, this part in subranges that are separately analysed. The computation units process in Intercol program comprises up to 1000 points, and the result of the processing is a model generated as a square network.
The relief modelling starting from the uniformly structurated input comming from the sampling process or from Intercol program is processed using Intercub program. Its algorithm is based upon approximation theory with spline functions used in different variants for the achievement of the digital elevation models(Ref.1,4,5) that combine controlled polynomial approximation with the continuity and smoothness of representation. A completely bicubic polynom is used:

$$Z = F(x,y) = [\ X\ ][\ A\ ][\ Y\ ]^T \qquad (9)$$

the 16 parameters being determinated by imposing four conditions in each of the corner points of a network element which expresses the connection in reference points (Z_i), the continuity of slope (Z'_x and Z'_y) and the smoothness (Z''_{xy}). Considering the origin of the coordinates system in the left corner of every square element and the length of the side of thus is equal to a unit, the [A] matrix elements result as functions only of Z, Z'_x, Z'_y, Z''_{xy} numerical values (fig-2 and(10) formulas).

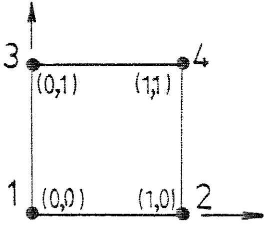

Fig.-2.

$$A_{00}= Z_1;\ A_{01}= Z'_{X1};\ A_{10}= Z'_{Y1};\ A_{11}= Z''_{XY_1};$$
$$A_{20}=\frac{1}{2}[Z'_{X_2}- Z_1]-Z'_{X_1};\ A_{30}=\frac{1}{2}[Z'_{X_2}- Z_1]-Z_2; \qquad (10)$$
$$A_{21}=3[Z'_{X_2}-Z'_{Y_1}]-2Z''_{XY_1}-Z''_{XY_2};$$
$$\vdots$$
$$A_{33}=-----------$$

The consideration of break lines within this program is obtained adding some supplementary conditions concerning the curvature continuity. The processed computation units are dimensioned up to looo points.
After the obtainning of digital model with one of the two procedure above described, the data are stored on magnetic tape or disc in a basis that includes only ($Z_{i,j}$) heights, the step value (h), the (X_o,Y_o) origin and (i,j) dimension. Is not necessary to store the $X_{i,j}$, $Y_{i,j}$ coordinates, since they may be obtained by computation formulas: $X_{ij} = X_o+ih$; $Y_{ij} = Y_o+ jh$.

3. APPLICATIVE WORKING

According to digital model informations, optionaly, conventional relief representations through contour lines at different scales or spatial representations (3D) can be obtained.
The levelling generation proceeding first call Interlin program which computs the maximum and minimum height, the number of contour lines and the points belonging to them on each side of the existing squares of the network, as long as on the break lines conforming to the equidistance introduced by the initial parameter. The new points determined by the linear interpolation are provisionally stored in files labeled according to the values of contour lines.
Consequently will be called the Genmatre program that generate the representation matrix of every contour line. The element of the matrix represents the number of intersections that makes the curve with the sides of the grid squares, their value varying from o to 4. Besides is performed the testing of contour line configuration by comparing elements on every row, column and diagonal of the matrix to detect the areas where the curves have a low determination. The existence of such areas point out the fact that the network of the digital model has a not adequate density thus it doesn't contain a sufficient quantity of information and automaticaly should be thickened with Intercub program, the step of the network becomming h/2. Then the computation sequences from Interlin and Genmatre programs are repeated.
After that the curves fulfil the required conditions imposed of the Genmatre program, the files of temporary stored points are processed with Sol sorting program which are achieved on the principle contour lines digitizing in the square schema. The program is composed by subroutines for most plausible forms of the curves analyzing with respect of the characteristic morphological terrain surface features. The existing contour lines within a computation unity can be closed, the first point being identical with the end point or open, when the end points are set on computation limits unity. Corresponding to that classification, the accurate contour line tract is determined through logical order sorting the points investigating each square using St, Dr, Su and Inf subroutines, not generalising the shape of the curve. The network elements in which the terrain has a saddle form, characterised by the existence of four points belonging to a curve on the square sides are analysed with specific Parabol subroutine, that assimilates the terrain with a hyperbolic paraboloid. The position of the four points in a local coordinated system having its origin in the center of the hyperbole got by intersection of the paraboloid with a horizontal plan equivalent in height with the curve

value and the configuration of the points within neighbouring squares establishing the order for correct points connection.The subroutine make also the distinction between saddle shapes and the cases in which the four points of a square are due to the break line. The files of the points obtained by Sol program makes the curve under the polygon form. The usual contour line aspect is obtained consequently to Rot program that generates additional points on every side by a sliding interpolations method with a 5 degree polynoms. The number of additional points is established according to the charateristics of the used plotter for mapping. The second application procedure makes the generation of spatial representation (3D) by Relief-S program according to a variant of Wright method. The input data are composed of the corresponding heights network to the area that is to be represented and extracted from the general DEM in a [R] matrix, (X_o,Y_o) origin, (h) step and (i,j) the network dimensions. The points are transformed in a special coordinate system for the obtaining 3D view effect:

$$\begin{aligned} X'_{i,j} &= X_o + (i-1)\,T_x + (j-1)\,h \\ Y'_{i,j} &= (i-1)\,T_y + Z_{i,j} - Z_{min} \end{aligned} \tag{11}$$

This is equivalent with the displacement on the horizontal (T_x)- optionally toward the left or the right and on (T_y) vertical, $Y_{i,j}$ coordinate being replaced with the value of $Z_{i,j}$ height. The points along i=1 line that belong to $[R'_{i,j}]$ matrix with transformed data,define the first minimum visibility limit. In comparison with this, computating interval by interval a function of the form:

$$F(\theta) = \left[\theta_{P_{i,j},P_{i+1,j}} - \theta_{P_{i,j},P_{i,j+1}}\right] \tag{12}$$

the visible points of i=2 line are determined. The negative values of the function points out the hidden points and in their place new points are liniary interpolated and they will correct the longitudinal section initially drawn by the point of i=2 line obtaining the following minimal visibility limit. This represent the correct representation line on the horizontal. During the processing its points are consequently located in the line files of representation in crossing direction. The procedure is repeated with the next line of the matrix $[R'_{i,j}]$. Different perspective variants can by obtained by applying some rotation of the data. Fig.3 presented spatial plots - 3D corresponding to a sheet of map at 1:10,000 scale acquired with I_3GSR_2S complex of program and plotted to Aristo automatic drawing installation.

4. CONCLUSIONS

The applications achieved with I_3GSR_2S complex program have proved the possibility getting digitally contour lines which fulfill all the accuracy and mapping quality imposed for levelling representations on great scale maps.As compared to conventional restitution,digital restitution is more accurate in even,relatively even and moderately irregular terrain areas,but less adequate to thoroughly irregular terrain,crossed by a large number of break lines.

Data sampling is a fundamental element in the technique of DEM's and it always conditions the accuracy,fidelity,yield and price.

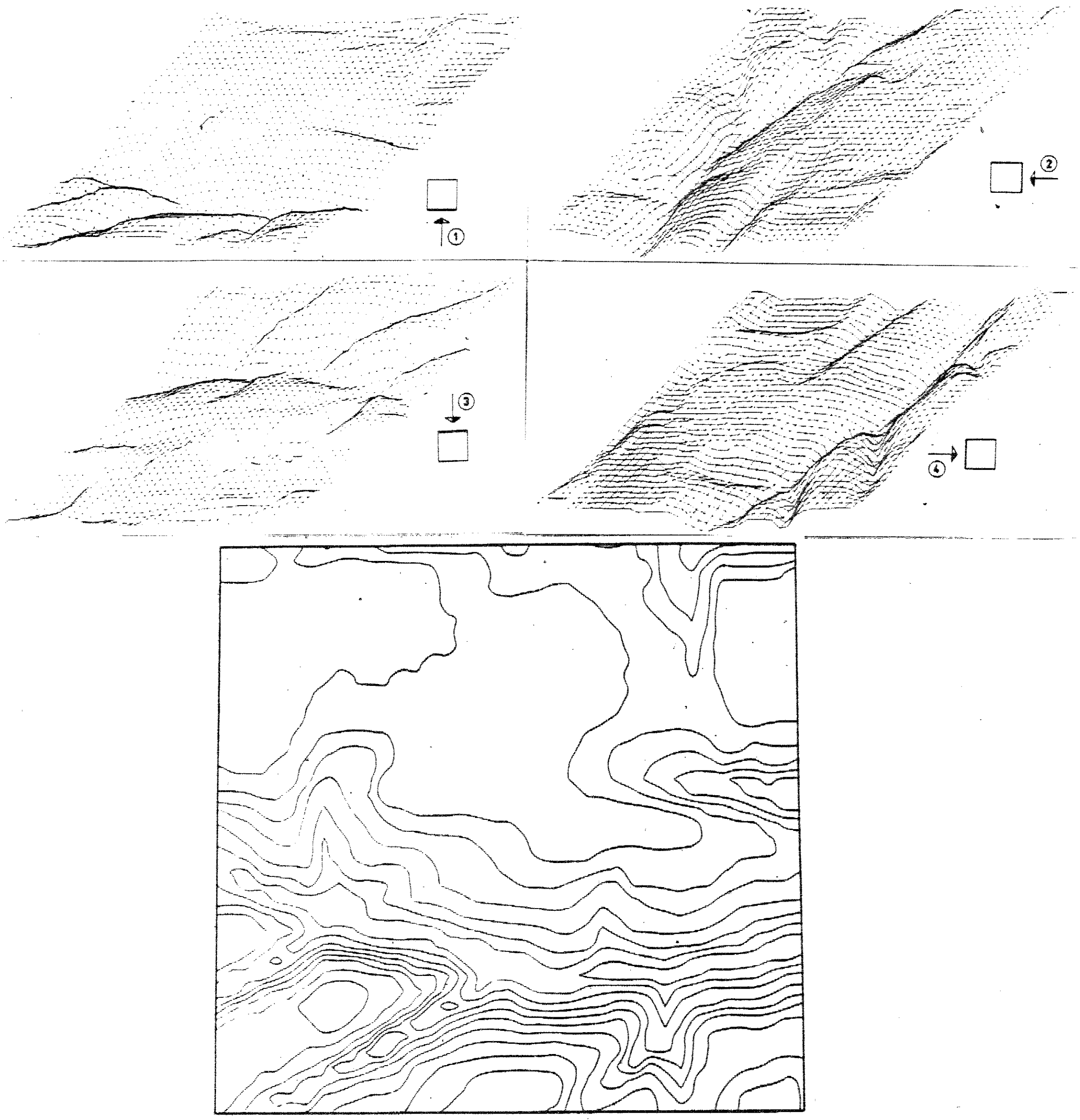

Fig.3 Spatial plots - 3D (top) corresponding to sheet of topographic map at 1:10,000 scale (bottom), acquired with I_3GSR_2S complex of programs.

Thus the use of interfaced devices with computers prove to be the most efficient means of getting the sampling, offering more rapid on line checking, testing and correcting by an interactive process. It also allows tremendous opportunities for the implementation of complex sampling procedures that allow the getting of height information according to the structural characteristics of each terrain class.

Nowadays the digital models impose themselves ever more since they represent a modern data basis, having the capacity of offering informations to a large range of technical and engineering activities.

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