INVESTIGATION OF THE ACCURACY OF A BLOCK ADJUSTMENT ACCORDING TO THE BUNDLE METHOD

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Commission III Presented Paper

Abstract

For the purpose of evaluating the accuracy of a block adjustment according to the bundle method, three blocks have been investigated, each based on different images and control points.

The investigations show that there is no satisfactory agreement between the results from actual data (Block 1) and simulated data (Block 3). This poses the problem to what extent existing geodetic data may be used for the evaulation of photogrammetric accuracy.

Sommaire

Afin de déterminer la précision d'une compensation par blocs selon la méthode des faisceaux, trois blocs avec différents types de signalisation des points et de film ont été analysés.

La comparaison des blocs 1 et 3 nous montre que les résultats effectifs (Bloc 1) et ceux des données simulées (Bloc 3) ne concordent pas de façon satisfaisante.

Ceci pose la problème suivant: comment peut-on à partir des informations à disposition et avec les moyens actuels analyser les capacités de telle compensation?

Zusammenfassung

Zur Beurteilung der Genauigkeit der Blockausgleichung nach der Bündelmethode wurden drei Blöcke mit verschiedenartigem Bild- und Objektpunktmaterial untersucht.

Die Ergebnisse der Untersuchungen haben gezeigt, dass der Vergleich zwischen den tatsächlichen Resultaten (Block 1) und den Resultaten mit simulierten Daten (Block 3) keine zufriedenstellende Uebereinstimmung gibt.

Damit stellt sich das Problem, wie man mit heutigen Mitteln die Leistungsfähigkeit aus den zur Verfügung gestellten Stützwerten beurteilen kann.

1 The general problem of a block adjustment

In 1972 a program for the block adjustment according to the bundle method was published by the Federal Institute of Technology in Zurich [10]. Several block adjustments were made with this program at the Institute of Geodesy and Photogrammetry [7], [9], [11]. The following investigations of the accuracy of an aerotriangulation were computed with a modified program of the bundle method on the CDC computer at the Federal Institute of Technology. The accuracies attained with this program were described in a previously published article [8].

The theoretic investigations of the bundle method are explained in [3], [6], [16]. The question to be answered here is: How does the method chosen to evaluate the accuracy of practical examples agree with the mathematical model $\mathbf{F}_{(\mathbf{y})} = \mathbf{0}$ (see [13]).

The computer solution of numeric photogrammetry is based on the method of least squares. This is theoretically justified only if the a piori errors of the introduced values have a Gaussian distribution. The ensueing result is the best estimate for the unknowns.

If these errors do not show a normal distribution, the obtained results for the unknowns and their relative accuracy should be regarded with caution because they are based on an incorrectly applied mathematical model.

It is known that at least the image coordinates are affected by a series of random and systematic error components. There are different methods to determine the influence of these errors on a block triangulation. Using block 'Oberschwaben' [1], several problems were investigated by applying the block adjustment method.

2 Research material, coordinate measurements and transformation of comparator coordinates

A part of block 'Oberschwaben', OEEPE block Zürich, was used for a practical investigation. The block is almost quadratic and consists of 7 strips with 16 photographs each. The size is 35 x 35 km^2.

Specifications: 112 wide-angle photographs taken with an aerial camera Zeiss RMK A, calibrated focal length f = 153 mm, image scale = 1 : 28'000, longitudinal overlap 60 % and lateral overlap 20 %. Glass diapositives with marked control and tie points were used for the measurements.

The glass diapositives of the aerial photographs (2nd and 3rd generation) were measured by the author using a WILD stereocomparator STK1 at the Institute of Geodesy and Photogrammetry of the Federal Institute of Technology at Zurich. The measured coordinates were then transformed into the image coordinate system with an affine transformation. The reduction of the image coordinates to the principal point takes into account the radially symmetric components of the lens distortion and refraction. Table 1 shows the mean error values in µm calculated from double measurements of the different strips and of the complete block. The standard deviation for all of the measured image coordinates of the block is

$$m_p = \pm \frac{\sqrt{\sum_1^n m_{pxy}^2}}{n} \ \mu m$$

whereby m_{pxy} = standard deviation in the individual strips and n = the number of strips.

Strip No.	Number of points	Number of models	Standard deviation in mμ computed from differences
2	477	15	3.3
4	496	12	2.6
6	526	15	3.1
8	496	15	2.5
10	520	15	2.4
12	526	15	2.5
14	518	15	2.5
Total	3559	105	$m_p = 2.7$

Table 1 Standard deviation of the image coordinates computed from differences

3 Investigation procedure

In order to investigate the accuracy of the block adjustment according to the bundle method, the material was treated under three different assumptions. Figure 1 shows how the input image coordinates and object coordinates were chosen. All of the control point coordinates, given in a Gauss-Krüger, respectively UTM-projection, were transformed into a spatial Cartesian coordinate system.

The image coordinates were assumed to be equally accurate and uncorrelated and were given the weight 1. The control point coordinates were considered to be observations according to [12] with different weights (see Tabel 2). All of the given tie points with unknown geodetic coordinates were introduced into the adjustment with the weight zero.

4 General remarks concerning the adjustment

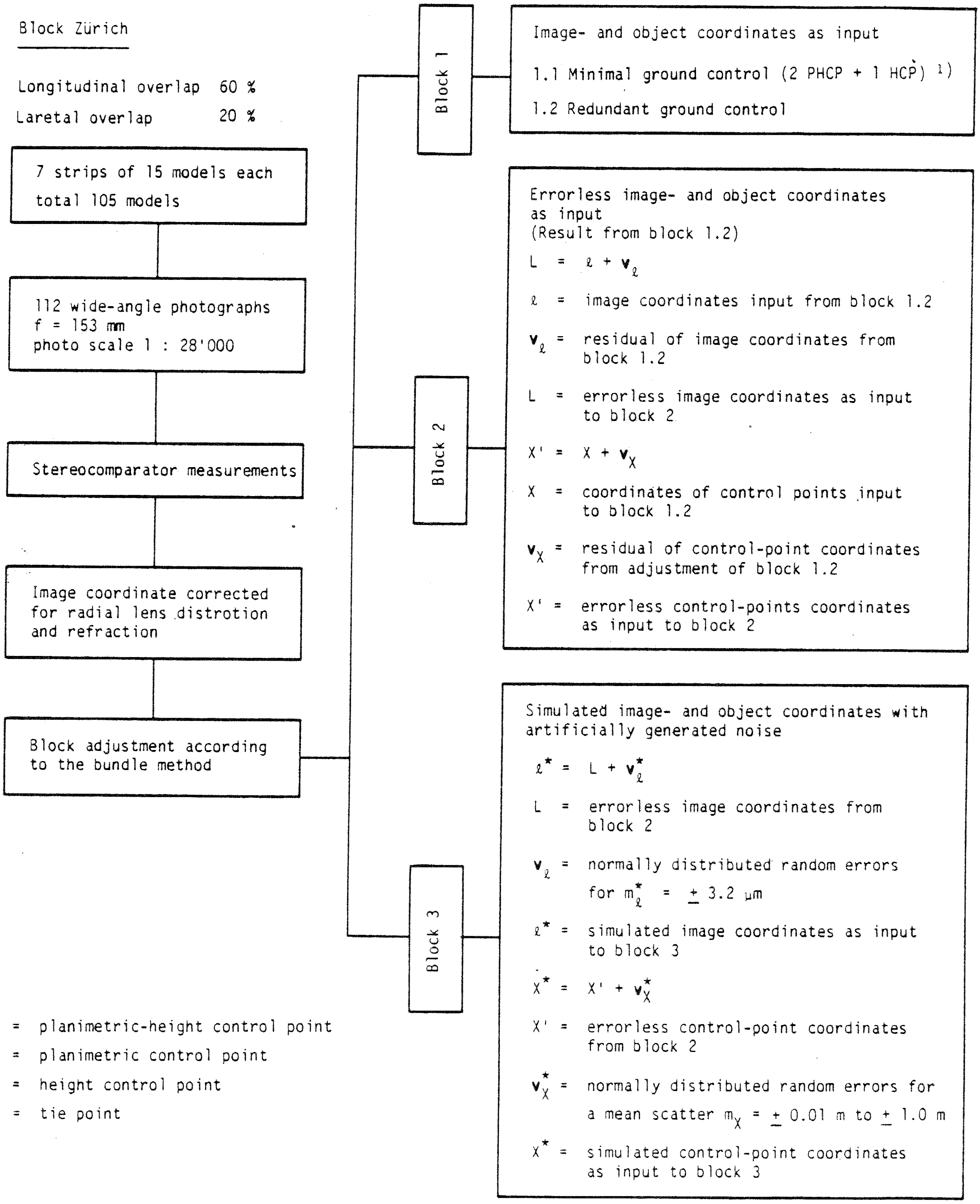
To investigate the accuracy of block Zürich, the block triangulation was computed according ot the model of the colinearity condition (14), under the condition that the sum of the squares of the weighted residuals of the observations, together with the sum of the squares of the weighted residuals of the introduced control values ($\mathbf{v}_\ell^T\mathbf{P}_\ell\mathbf{v}_\ell + \mathbf{v}_x^T\mathbf{P}_x\mathbf{v}_x$) would be a minimum [15]. The residuals of the measured image coordinates are represented by $\mathbf{v}_\ell$ and the residuals of the given control points by $\mathbf{v}_x$. The introduced weights are of course normalized (see Table 2).

5 Block adjustment

5.1 Block 1

The following cases were investigated (see Table 2):

1. Block 1.1: adjustment with minimal control, at least 2 planimetric and height control points (2 PHCP) and 1 height control point (1 HCP). This adjustment is interesting because the resulting $\mathbf{v}_\ell$ values are based on a minimal defect elimination which corresponds to an adjustment with conditions [13] where only the noise of the image coordinates has an influence.



1)
PHCP = planimetric-height control point
PCP = planimetric control point
HCP = height control point
TP = tie point

Figure 1

Input and processing of the data used for the investigation

Block	Version	Number of control points	a priori standard error (m_X) of the control points in meters	Weight	
				image coord. P_ℓ	control points P_X
1.1	1	2 PHCP + 1 HCP	0.01	1	1024.0 E-10
1.2	2	240 PHCP + 35 PCP	0.01	1	1024.0 E-10
	3		0.10		10.24 E-10
	4		0.13		6.25 E-10
	5		0.20		2.56 E-10
	6		0.30		1.067 E-10
	7		0.40		0.625 E-10
	8		0.60		0.284 E-10
	9		1.0		0.1024 E-10

Table 2 Overview of block adjustments Block 1

2. Block 1.2: a further adjustment of the image coordinates and object coordinates was then carried out with all of the given geodetic control points (240 planimetric and height control points and 35 planimetric control points). The weight of the given control points was reduced incrementally with respect to the a priori accuracy of the control points ($\pm$ 0.01 m to $\pm$ 1.0 m, see Table 2).

These results are interesting for the evaluation of the systematic error components of the image information and object information in a block triangulation.

5.1.1 Adjustment of block 1.1 with minimal control

Version 1

To solve the normal equation system in this special case, 7 coordinates of suitable control points were kept fixed (minimal constraint). The residuals of the control point coordinates $\mathbf{v}_X$ become zero and are independent of $\mathbf{P}_X$, even though th $\mathbf{P}_X$-values affect the $\mathbf{Q}_{XX}$ matrix. The sum of the squares of the weighted residuals ($\mathbf{v}_\ell^T\mathbf{P}_\ell\mathbf{v}_\ell + \mathbf{v}_X^T\mathbf{P}_X\mathbf{v}_X$) becomes ($\dot{\mathbf{v}}_\ell^T\mathbf{P}_\ell\dot{\mathbf{v}}_\ell$), whereby the $\dot{\mathbf{v}}_\ell$ values correspond to residuals from an adjustment with conditions.

The resulting mean estimate of the variance factor is given by the following equation:

$$m_0 = \left(\frac{\dot{\mathbf{v}}_\ell^T \mathbf{P}_\ell \dot{\mathbf{v}}_\ell}{n - u} \right)^{1/2}$$

n and u are defined as follows:

$n = n_\ell + n_s$	=	all parameters with a weight > zero in the mathematical model ($\mathbf{F}_{(\mathbf{y})} = \mathbf{0}$)
n_ℓ	=	number of image coordinates
n_s	=	number of introduced control values
r	=	rank deficiency of the system $\mathbf{F}_{(\mathbf{y})} = \mathbf{0}$, in this case $r = 7$, resulting in $n_s = r = 7$
u	=	number of parameters in the y-vector corresponding to the number of orientation parameters in 112 images (112 x 6) + 3 x number of object points (760 x 3)
u	=	112 x 6 + 760 x 3 2952
$n = n_\ell + 7$	=	2764 x 2 + 6 x 2 + 12 x 2 + 7 = 5571
Therefore:		(n - u) = 2619

As already mentioned, the resulting residuals correspond to a conditional adjustment, i.e. they show only the discrepancy within the photogrammetric triangulation based on the coplanarity condition of homologous rays. The estimate of the variance factor (see example in Table 3 and Table 6) is $m_o = 3.2$ µm, in this case $= m_\ell$.

Version	Type of points	Image space: $\mathbf{v}^T_{\ell_x} \mathbf{P}_{\ell_x} \mathbf{v}_{\ell_x}$ / $\mathbf{v}^T_{\ell_y} \mathbf{P}_{\ell_y} \mathbf{v}_{\ell_y}$	Image space: $\mathbf{v}_\ell^T \mathbf{P}_\ell \mathbf{v}_\ell$	Object space: $\mathbf{v}_X^T \mathbf{P}_X \mathbf{v}_X$ / $\mathbf{v}_Y^T \mathbf{P}_Y \mathbf{v}_Y$ / $\mathbf{v}_Z^T \mathbf{P}_Z \mathbf{v}_Z$	Object space: $\mathbf{v}_X^T \mathbf{P}_X \mathbf{v}_X$	$\dot{\mathbf{v}}_\ell^T \mathbf{P}_\ell \dot{\mathbf{v}}_\ell$	n - u	m_ℓ µm
1	NP	.92268 E-08 .16766 E-07	2.61567 E-08	.84195 E-17	0	2.61567 E-08	2619	3.2
	PCP	.42979 E-10 .32796 E-10		.46338 E-17				
	PHCP	.42299 E-10 .45827 E-10		.12493 E-21				

Tabel 3 Minimal ground control 2 PHCP + 1 HCP ($m_x = 0.01$ m, $P_x = 1024.0$ E-10) Block 1.1

It is known that in such a solution, a part of the basic noise, due to the flexibility of the coplanarity condition, will affect the triangulation geometry (for example model distortion). This portion is normally small and shows up in vertical errors of the object and in somewhat distorted orientation parameters. Therefore 3.2 µm represents a lower limit. As expected, this value is larger than the accuracy of the image coordinate measurements itself, which averaged to ± 2.7 µm from double measurements (see Table 1). Therefore, the 3.2 µm reflects an increase of noise as compared to the actual measurements.

This increase is due to various errors such as film deformation, incomplete compensation of distortion, refraction anomalies, etc. These errors show the gap between reality and the mathematical model. Provided that they have systematic components, these will, as mentioned above, be partially compensated by translation parameters and rotation values of the exterior orientation (Xo, Yo, Zo, ω, ϕ, κ) and by object deformation.

The estimate of the variance factor $m_\ell = \pm 3.2$ µm seems to be within expectations and was therefore introduced in all further investigations with $\mathbf{P}_\ell = 1$.

5.1.2 Adjustment of block 1.2 with redundant control

As already mentioned, 240 planimetric and height control points (PHCP) and 35 planimetric control points (PCP), spread evenly across the test area, were used. They gave enough redundant information for defect elimination. The adjustment was carried out under the condition

$$(\mathbf{v}_\ell^T \mathbf{P}_\ell \mathbf{v}_\ell + \mathbf{v}_x^T \mathbf{P}_x \mathbf{v}_x) = \text{minimum}$$

The sum of the squares of the weighted residuals of the image coordinates plus the sum of the squares of the weighted residuals of the control points are minimized. Normalized weights must be introduced. Before the adjustment, the weights $\mathbf{P}_\ell$ and $\mathbf{P}_x$ were calculated for the assumed accuracy of the image coordinates and control point coordinates m_ℓ and m_x (see Table 2). For a single observation, the weight is calculated according to the standard deviation of the observation. Therefore, the equation for control point coordinates X_i is

$$\mathbf{P}_{xi} = \frac{K}{m^2_{xi}}$$

whereby

K is introduced as the square of the mean estimate of the variance factor m of the block adjustment 1.1 with minimal control (2 PHCP + 1 HCP) (see Table 2)

m_x = mean of a priori error of the geodetic control point coordinates

The value of m_o is calculated as follows:

$$m_o = \left(\frac{\mathbf{v}^T\mathbf{P}\mathbf{v}}{n - u}\right)^{1/2}$$

whereby $(\mathbf{v}^T\mathbf{P}\mathbf{v}) = (\mathbf{v}_\ell^T \mathbf{P}_\ell \mathbf{v}_\ell + \mathbf{v}_x^T \mathbf{P}_x \mathbf{v}_x)$

n = (2 x image points) + (number of control point coordinates)

u = (6 x number of aerial photographs) + (3 x number of total points).

Type of points	Number of object points	Number of object points coord.
TP	485	1455
PCP	35	105
PHCP	240	720
Total	760	2280

Tabel 4 Number of object points

Type of parameters	Number of observations n	Number of unknowns u	n - u
Orientation elements		112 x 6 = 672	- 672
TP	1915 x 2 = 3830 (n_ℓ)	3 x 485 = 1455	2375
PCP	108 x 2 = 216 (n_ℓ) 35 x 2 70 (n_s) 286	3 x 35 = 105	181
PHCP	759 x 2 = 1518 (n_ℓ) 240 x 3 = 720 (n_s) 2238	3 x 240 = 720	1518
Total	6354	2952	3402

Tabel 5 Overview of observations and unknowns

The quality of the total measurement is expressed by m_o. The standard deviation of the control point coordinates is calculated with

$$m_x = \frac{m}{\sqrt{\mathbf{P}_X}}$$

The accuracy assumptions of the given control points were deteriorated incrementally in Tests 2 to 9. This led to reduced weight assumptions for the $\mathbf{P}_X$ values and to increased m_x values. Therefore, the m_o values are reduced incrementally as seen in Table 6.

Version	Number of control points	$\mathbf{P}_x$	m_X a priori m	$\mathbf{v}_\ell^T\mathbf{P}_\ell\mathbf{v}_\ell$	$\mathbf{v}_X^T\mathbf{P}_X\mathbf{v}_X$	$\dot{\mathbf{v}}_\ell^T\mathbf{P}_\ell\dot{\mathbf{v}}_\ell$	$\mathbf{v}^T\mathbf{P}\mathbf{v}$	n - u	m_ℓ µm	m_o µm	m_X a posteriori m
1	2 PHCP + 1 HCP	1024.0 E-10		2.61567 E-08	0	2.615670 E-08	2.615670 E-08	2619	3.2	3.2	-
2	240 PHCP + 35 HCP	1024.0 E-10	0.01	10.18623 E-08	0.10912 E-08		10.29735 E-08	3402		5.5	0.02
3		10.24 E-10	0.10	5.82007 E-08	1.28736 E-08		7.10743 E-08			4.6	0.14
4		6.25 E-10	0.13	5.17111 E-08	1.30656 E-08		6.47767 E-08			4.4	0.17
5		2.56 E-10	0.20	4.14565 E-08	1.19575 E-08		5.34140 E-08			4.0	0.25
6		1.067 E-10	0.30	3.49063 E-08	0.91580 E-08		4.40643 E-08			3.6	0.35
7		0.625 E-10	0.40	3.27088 E-08	0.70266 E-08		3.97354 E-08			3.4	0.43
8		0.284 E-10	0.60	3.10000 E-08	0.43185 E-08		3.53188 E-08			3.2	0.60
9		0.1024 E-10	1.0	2.97969 E-08	0.22909 E-08		3.20878 E-08			3.1	0.96

Table 6 Overview of the results of the different test versions Block 1

In Version 2 the a posteriori standard deviation of 0.02 m for the control values is unrealistically small and results in an unfavorable value of $m_o = \pm 5.5$ µm. The m_o-value of ± 3.2 µm for version 8 corresponds well with that of version 1 (minimal ground control). However, under normal circumstances the correspondant standard deviation of $m_X = \pm 0.60$ m is rather unfavorable for the accuracy of the control points.

The aim of the adjustments of blocks 2 and 3 was on the one hand to account for correctly applied software and on the other hand to prove the assumptions of the adjustment philosophy when using ideal input material.

5.2 Block 2

The data of this investigation is taken from the adjustment results of block 1.2. The corrections of the original image coordinates and object coordinates in block 1.2 can easily be obtained by applying the **v**-values of the previous block adjustment. Since each image point and obeject point has a definite residual, simple addition of the residuals will give the improved image coordinates and object coordinates. The adjusted image coordinates are obtained by the following equation (see Figure 1).

$$L = \ell + \mathbf{v}_\ell$$

For the adjusted object coordinates (see Figure 1)

$$X' = X + v_X$$

According to the least squares method, the corrected values, together with the resulting unknowns should fulfill the underlying mathematical model without constraint. In order to test this tehoretic assumption, the bundle adjustment was repeated with the corrected image coordinates and control point coordinates.

After the block adjustment the mean estimate of the variance factor according to the equation

$$m_o = \left(\frac{\mathbf{v}^T \mathbf{P} \mathbf{v}}{n - u} \right)^{1/2}$$

should be equal to zero since errorless input information theoretically yields no **v**-values.

Version	Type of points	Image space: $\mathbf{v}_{L_x}^T \mathbf{P}_{L_x} \mathbf{v}_{L_x}$ / $\mathbf{v}_{L_y}^T \mathbf{P}_{L_y} \mathbf{v}_{L_y}$	Image space: $\mathbf{v}_L^T \mathbf{P}_L \mathbf{v}_L$	Object space: $\mathbf{v}_X^T \mathbf{P}_X \mathbf{v}_X$ / $\mathbf{v}_Y^T \mathbf{P}_Y \mathbf{v}_Y$ / $\mathbf{v}_Z^T \mathbf{P}_Z \mathbf{v}_Z$	Object space: $\mathbf{v}_X^T \mathbf{P}_X \mathbf{v}_X$	$\mathbf{v}^T \mathbf{P} \mathbf{v}$	n-u	m_o µm
1	TP	1.2467 E-12 1.5613 E-12	15.4026 E-12	0.38067 E-12	0.74682 E-12	16.14942 E-12	3402	0.07
	PCP	0.8080 E-12 0.5768 E-12		0.31940 E-12				
	PHCP	6.1590 E-12 5.0508 E-12		0.04675 E-12				

Table 7 Redundant ground control (m_x = 0.01 m, P_x = 1024.0 E-10) Block 2

The mean estimate of the variance factor m_o after the block adjustment was m_o = 0.07 µm (see Table 7). This discrepancy to zero can be explained by rounding errors since double precision was not applied in the computation process. Therefore, the adjustment software can be regarded as sufficiently errorless.

5.3 Block 3

5.3.1 Simulated image coordinates and object coordinates

To check the influence of random errors in the image coordinates (L) and control point coordinates on the result of the applied block adjustment, the errorless control point coordinates (X') of block 2 are first superimposed with normally distributed random values (Gaussian values). As a result, the photographic image coordinates and the control point coordinates become two-dimensional, respectively three-dimensional random variables with appropriate weights.

The normally distributed random values were computed with the program RANG from the computation center at the Federal Institute of Technology. They were generated independently for the image coordinates x and y and the control point coordinates X', Y' and Z'. The simulated image coordinates were taken from the following equation (see Figure 1):

$$\ell^{\star} = L + \mathbf{v}_{\ell}^{\star}$$

The $\mathbf{v}_{\ell}^{\star}$ values were computed with the random generator for the mean scatter m_{ℓ} = $\pm$ 3.2 µm (see Table 8). The simulated control point coordinates were taken from the equation (see Figure 1)

$$X^{\star} = X' + \mathbf{v}^{\star}$$

The control point coordinates of block 3 (240 planimetric and height control points and 35 planimetric control points) were generated in four steps (a - d), each with a different standard deviation (see Table 8).

Testversion	Errorless image coord. from block 2	Mean scatter for m_{ℓ}	Simulated image coord. block 3	Errorless control points coord. from block 2	Mean scatter m_X m	Simulated control points coord. as input to block 3 $X^{\star} = X + v_X^{\star}$
a	L	3.2 µm	$\ell^{\star} = L + v_{\ell}^{\star}$	X	0.01	$X_a^{\star} = X + v_X^{\star}$
b					0.10	$X_b^{\star} = X + v_X^{\star}$
c					0.30	$X_c^{\star} = X + v_X^{\star}$
d					1.0	$X_d^{\star} = X + v_X^{\star}$

Table 8 Generated random numbers for simulation of image and control point coordinates Block 3

The effectiveness of the applied random generator was tested independently for $\mathbf{v}_X^*$, $\mathbf{v}_Y^*$ and $\mathbf{v}_Z^*$ by calculating the standard deviation (see Table 9). A comparison of the introduced standard deviation and the calculated standard deviation from the random numbers in Table 9 shows small but negligible differences. Therefore, the generator used for the random numbers can be considered as being sufficiently correct.

Testversion	Errorless control points from block 2 X'	Simulated control points coord. from block 3 X*	Mean scatter $m_{X,Y,Z}$ m	Standard deviation computed from random numbers		
				m_X m	m_Y m	m_Z m
a	X'	X_a^*	0.01	0.01	0.01	0.01
b		X_b^*	0.10	0.10	0.10	0.10
c		X_c^*	0.30	0.31	0.28	0.29
d		X_d^*	1.00	1.04	0.98	0.96

Table 9 Testversions with simulated random numbers Block 3

5.3.2 Computation

Five different block adjustments were carried out for the investigation of the accuracy of simulated image coordinates and control point coordinates (see Table 10).

Block	Version	Number of control points	Standard error (m_X) of control points in meters	Weight	
				Image coord. P_ℓ	Control points P_X
3.1	1.3	2 PHCP + 1 HCP	0.01	1 3.2 μm	1024.0 E-10
3.2	2.3a	240 PHCP + 35 HCP	0.01	1 3.2 μm	1024.0 E-10
	3.3b		0.10		10.24 E-10
	4.3c		0.30		1.067 E-10
	5.3d		1.0		0.1024 E-10

Table 10 Overview of block adjustments Block 3

The standard deviation of the image coordinates was assumed to be 3.2 μm for all tests and introduced with the weight $\mathbf{P}_\ell = 1$. The image coordinates were assumed to be equally accurate and uncorrelated. The control point coordinates were considered to be observations with different weights under the assumption of uncorrelated errors.

5.3.3 Results of the block adjustment

The most important results for the evaluation of the attained accuracy are represented in Table 11. In Version 1.3 there are at least 7 control point coordinates (2 PHCP + 1 HCP) available for defect elimination. As expected, Table 11 shows that the sum of the squares of the weighted residuals of the control point coordinates ($\mathbf{v}_x^T\mathbf{P}_x\mathbf{v}_x$) is zero.

With a normally distributed error of the image coordinates and object point coordinates, the estimate of the variance factor m_0 after the adjustment in Version a - d with full constraint is ± 3.2 µm (see Table 11).

Version	Number of control points	$\mathbf{P}_x$	m_x a priori m	$\mathbf{v}_\ell^T\mathbf{P}_\ell\mathbf{v}_\ell$	$\mathbf{v}_x^T\mathbf{P}_x\mathbf{v}_x$	$\dot{\mathbf{v}}_\ell^T\mathbf{P}_\ell\dot{\mathbf{v}}_\ell$	$\mathbf{v}^T\mathbf{P}\,\mathbf{v}$	n - u	m_ℓ µm	m_0 µm	m_x a posteriori m
1.3	2 PHCP + 1 HCP	1024.0 E-10		2.63268 E-08	0	2.63268 E-08	2.63268 E-08	2619	3.2	3.2	-
2.3a	240 PHCP + 35 HCP	1024.0 E-10	0.01	3.46324 E-08	0.01912 E-08		3.48236 E-08	3402		3.2	0.01
3.3b		10.24 E-10	0.10	2.98830 E-08	0.49088 E-08		3.47918 E-08			3.2	0.10
4.3c		1.067 E-10	0.30	2.73422 E-08	0.67668 E-08		3.41090 E-08			3.2	0.31
5.3d		0.1024 E-10	1.0	2.65799 E-08	0.78304 E-08		3.44103 E-08			3.2	0.99

Table 11 Overview of the results of the different test versions Block 3

6 Interpretation and analysis of the results

Table 6 shows the most important statistical data of the adjustment of block 1 for the evaluation of the attained accuracy. As a first stept the adjustment of block 1.1 (Version 1) was carried out under minimal constraint. The resulting mean estimate of the variance factor $m_0 = m_\ell = \pm\ 3.2$ µm is a measure of accuracy reflecting only the noise of the measured image coordinates. It corresponds to the practical expectations.

In Version 2 all of the control point coordinates in block 1.2 were given a very small standard deviation (see Table 2), respectively a very large weight, resulting in a very small ($\mathbf{v}_x^T\mathbf{P}_x\mathbf{v}_x$) component. With modern computers the theoretical value m_x = zero, respectively $\mathbf{P}_x = \infty$ can be calculated with sufficient accuracy.

After the block adjustment, a mean estimate of the variance factor m_0 = ± 5.5 µm and a standard deviation of ± 0.02 m for the control point coordinates resulted for all of the 240 planimetric and height control points, the 35 planimetric control points and the 485 tie points (see Table 6). The significant differences between the a posteriori mean estimate of the variance factor $m_\ell = \pm\ 5.5$ µm and the corresponding mean estimate of the variance factor $m_\ell = \pm\ 3.2$ µm obtained in block 1.1 (Version 1, see Table 3) is primarily caused by the constraint between the ground control points and the purely photogrammetric triangulation. Practically spoken, photogrammetry is built on the assumption of errorless geodesy. The result shows that in this example the geodetic control information was overrated with respect to its accuracy.

In a further step, the weight of the given ground control points was incrementally reduced in Versions 3 to 9. The results can be seen in Table 6.

A few remarks should be made concerning the origin and accuracy of the geodetic control information. Specifications about the fourth order geodetic triangulation net from which the control points in block Oberschwaben were taken, can be found in [5].

The given heights are probably the results of leveling, whereby the local geoid undulations were not taken into account. Despite a careful study of the geodetic data, the accuracy of the geodetic horizontal coordinates could not be established. They seem to have faults which probably cause local systematic errors (as much as a decimeter). Such conditions, known for example also in Switzerland [2], must often be tolerated because of organizational and specifically economic reasons. A statistically correct interpretation of the adjustment results will be complicated, if not made altogether impossible by such conditions.

The accuracy of the heights can probably best be described with the term "standard deviation", under the assumption that the geoid undulations, which exceed the accuracy of leveling, show a normally distributed standard deviation with respect to their local mean value.

It is much more difficult to make a quantitative statement. As far as the topographic structure of the Oberschwaben area is concerned, a m_Z value of $\pm$ 5 to $\pm$ 10 cm would seem appropriate. Assuming a normally distributed noise of $\pm$ 10 cm for the horizontal coordinates X and Y, we get the input values for Version 3. Therefore, in Table 6 the mean estimate of the variance factor m_0, respectively the standard deviation of the image coordinates m_0 is $\pm$ 4.6 μm and the somewhat larger a posteriori standard deviation of the coordinates m_X is $\pm$ 0.14 m. With respect to m_ℓ, these results are not completely satisfactory. Therefore, further adjustments were made in Version 4 to 9 with an incrementally increased standard deviation for the control points.

Version 4 is only slightly different from Version 3. The standard deviation of the coordinates m_X a priori was increased from 0.10 to 0.13 m The result is predictable in as much as the mean estimate of the variance factor is reduced by only 0.2 μm to $\pm$ 4.4 μm and the a posteriori obtained m_X value is increased by 4 cm to $\pm$ 17 cm. The corresponding v_ℓ values of the more than 5500 residuals show about 35 residuals with a value between two to three times 4.4 μm. Only 2 residuals exceed these values.

A further point of interest is Version 6, in which a mean estimate of the variance factor m_0 = $\pm$ 3.6 m was obtained under the assumption of a standard coordinate deviation of $\pm$ 0.30 m (see Table 6). The residuals of the image coordinates of this test are comparable to the result of Version 1 and for practical purposes satisfactory. The computed residuals for the ground control points however, are large and in addition, systematically arranged. The problem lies in the determination of the systematic errors of the terrestrial control points.

For photogrammetric accuracy, this can lead to a mixture of geodetic and photogrammetric errors. The remaining systematic errors in the object space which cannot be mathematically isolated are almost normally distributed by applying the, in this case theoretically unjustified Gaussian adjustment principle. Furthermore, the problem with constraint leads to a symmetric arrangement. In orhter words, the obtained coordinate residuals demonstrate the arrangement of systematic vortex formations which are often found in similar cases, but which do not allow any conclusions to be made regarding the originally present systematic errors.

It remains to be emphasized that, because of todays' limited possibilities in dealing with systematic errors, experiments to demonstrate the efficiency of photogrammetric triangulation are only reasonable if the geodetic control elements are carefully examined to assure that they correspond to the classic concept of normally distributed errors.

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