

SPACE RESECTION OF 35mm MODEL AIRCRAFT PHOTOGRAPHY  
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Abstract

The results of a project to determine the survey potential of 35mm non-metric photography, taken from a model aircraft, are assessed. No réseau plate or fiducial marks were incorporated in the camera. The investigation was only concerned with planimetric data. The significance of the effect of fiducial co-ordinate transformations and camera calibration data on accuracy are also discussed.

## INTRODUCTION

This paper reports on a study (Bolt, 1983) that was carried out to determine the accuracies to which model aircraft could be used to survey small areas using non-metric cameras. The cost of conventional aircraft survey may be prohibitive when small companies or small areas are involved. The advent of analytical techniques has allowed the use of non-metric cameras with reasonable degrees of accuracy.

The majority of studies using non-metric cameras have concerned themselves with high accuracies at close range (see Atkinson, 1980). Such techniques have come to be known as self calibration methods. Abdel-Aziz and Karara (1971) and Bopp and Kraus (1978) are among those to have developed such techniques. Fewer studies have considered using non-metric photography from the air at heights below 500m and yet analytical methods make this possible. There has also been greater use of model helicopters (Wester-Ebbinghaus, 1980) than model aircraft.

## MODEL AIRCRAFT PHOTOGRAPHY

In June, 1983 model aircraft photography was taken over the area of a disused gypsum mine near Carlisle by Geoffrey Walton (Consulting Mining and Engineering Geologists, Charlbury) in conjunction with Iris Surveys Ltd.(Stroud). A Konica FS-135mm SLR camera was used, fitted with a nominal 40mm Hexanon AR (f/1.8) lens. The film used was Ilford FP4 (125 ASA).

Visual analysis of the photography suggested that the resolution of the film was likely to pose more of a problem than the effects of aircraft ground speed. This was a finding borne out by Wester-Ebbinghaus (1980). Plan position of the photography was difficult to maintain (Miller, 1980). This was especially so in strong winds. The photography had a high level of redundancy with some areas covered several times. Coverage was also so scattered in nature that a bundle adjustment would be the only reasonable approach for triangulating the photography. This would also allow effective use of redundant photographs.

Ground control was provided after the photography was flown. The flying height was maintained at approximately 400m. At this height a 35mm frame covered 350m by 240m on the ground. This was a small area and the location of sufficient, discrete ground control points was difficult. A consistent amount of clear overlap was virtually impossible to maintain with the model aircraft. Ground control cannot therefore be provided economically within frame overlaps.

## CAMERA CALIBRATION

Self calibration techniques allow the solution of a large number of unknowns including perhaps the x and y co-ordinates of the format (fiducial) corners. This, however, puts high requirements on the amount of ground control necessary and would only be suitable where a few frames were being considered and high accuracy was required.

The aim of this study was to determine to what accuracy ground co-ordinates could be recovered by resection, if the interior orientation elements were solved for by camera calibration.

Interior orientation here would also include any correction to frame co-ordinates for film stretch or lack of flatness. The only remaining unknowns for which ground control must be provided would be the six exterior orientation elements (frame tilts and ground perspective centre).

A method of field calibration similar to that used by Wolf and Loomer (1975) was used, since this was the easiest to implement and the most cost effective. By calibrating the camera in the field the position of the lens node was not as critical as would be the case in a laboratory calibration. The test field was a view over London (Plate 1) as seen from University College London. The approximate view angle of the camera lens was  $45^\circ$ . A total of 20 detail points was observed, using a one second theodolite. Photographs were taken of the test field with the survey camera about four orientations (two diagonals, a vertical and a horizontal), with the lens node over the same position as the central axis of the theodolite.

In camera calibration, it is the point of symmetry falling onto the focal plane that is of real interest. This can be found without any reference to the principal point. Initially one of the centrally positioned test field points was chosen as a reference mark approximating the point of symmetry for the orientation mark concerned. Suitable computer programs were written to reduce the data so that values for film distances and test field angles were known between the central mark and all other points. For each calibration orientation, principal distance values were calculated about the chosen central mark. Since this mark was not the point of symmetry the graph of principal distance against radial distance from the central mark, for any orientation, was unlikely to be symmetrical. By changing the radial position of the central mark by an amount  $R$ , calculated for each orientation, the graph can be made symmetrical. As the radial position of the central mark changed, so the angles between the central axis and rays of all other points subtended at the lens node also changed. This change in angle  $RAD$  can be found as follows:

$$\tan RAD = \frac{R}{f} \quad \therefore RAD = \text{Arctan} \left( \frac{R}{f} \right) \quad (1)$$

where  $f$  = principal distance

Fig.1 shows values for the principal distance plotted against radial distance from the central mark for the two diagonal orientations, after radial distances had been corrected by  $R$  and the angles corrected by  $RAD$ . The central mark now represents the estimated point of symmetry. The graphs in Fig.1 show a high degree of symmetry.

Since no réseau plate was incorporated into the camera format there could be no effective correction for lack of film flatness. However, in order to try and take account of differential film stretch the co-ordinates of the format corners within the camera body were measured. This was done to an accuracy of 0.11mm using a Reflex Metrograph (Scott, 1981). Computer software was written allowing a two dimensional affine or similarity transformation (or none at all) to be computed between camera format corners and negative format corners.

## CALIBRATION RESULTS

The calculated mean principal distance for all orientations was found to be considerably larger (by about 1.4mm) than the manufacturer's given value. As is usually expected, the camera showed decreasing principal distance values with increasing field angle. However, Fig.1 shows that the expected parabolic curve was not particularly well formed for the diagonal orientations. Curves for the horizontal and vertical orientations were worse. The lack of stability was most apparent near the lens axis where measurements were taken over small angles. This destabilisation of what should be a smooth curve was most likely to be due to film deformation. At this stage attempts had only been made to account for this deformation in terms of film stretch. Analysis showed that transformations of negative format corners made no improvement to calibration curve stability. Firstly, the negative format corners were too ill-defined and unstable and, secondly, the format corners of the camera body were not measured to a high enough degree of accuracy. At a photoscale of 1:10 000, an accuracy of 0.11mm represents 1.1m on the ground.

Radial distortion was calculated for the lens as described in Scott (1977). Values were not worked out for the vertical orientation since this had a smaller field angle and was correspondingly unstable. The data were suitably combined to give an overall distortion curve representing distortion values against increasing radial distance from the point of symmetry. These results are shown graphically in Fig.2. A mathematical function was then fitted to the calculated curve. The best fit was achieved with a single co-efficient equation given below.

$$D = K_1 r^3 \quad (2)$$

where

D = radial distortion

$K_1$  = co-efficient and

r = radial distance from point of symmetry

This was in accordance with a finding of Karara and Abdel-Aziz (1974). However, in this case, even equation (2) was a poor fit. The overall fit residual was 20µm. Over most of the camera format this was as much as half of any distortion present. It is doubtful whether correction for radial distortion according to a curve of such poor fit would be significant.

## SPACE RESECTION

The ground control for the areas under study totalled 68 points. These points provided control for a total of 27 photographs. The x and y co-ordinates of the control points in each frame were measured monocularly three times using a comparator. The corners of each negative format were measured at the same time. Having derived the necessary information from camera calibration and the negatives themselves, a space resection was computed solving the six unknowns of exterior orientation for each frame. This required a minimum of four ground control points falling within each frame to allow some degree of redundancy.

The resection was computed using the collinearity formulae:

$$x^T - x_0^T + Dx = -f \begin{bmatrix} r_{11}(X_a - X_s) + r_{21}(Y_a - Y_s) + r_{31}(Z_a - Z_s) \\ r_{13}(X_a - X_s) + r_{23}(Y_a - Y_s) + r_{33}(Z_a - Z_s) \end{bmatrix}$$

$$y^T - y_0^T + Dy = -f \begin{bmatrix} r_{12}(X_a - X_s) + r_{22}(Y_a - Y_s) + r_{32}(Z_a - Z_s) \\ r_{13}(X_a - X_s) + r_{23}(Y_a - Y_s) + r_{33}(Z_a - Z_s) \end{bmatrix}$$

where  $x^T, y^T$  = observed frame co-ordinates after correction for film stretch,  
 $x_0^T, y_0^T$  = co-ordinates of the point of symmetry after correction for film stretch,  
 $Dx, Dy$  = radial distortion corrections for each point in x and y,  
 $f$  = calculated principal distance of the camera,  
 $X_a, Y_a$  = ground control co-ordinates and  
 $X_s, Y_s$  = ground perspective centre.

For the first iteration, approximations for the ground perspective centre were found by taking a mean of the ground co-ordinate values within the frame concerned. The general flight path of the aircraft photography showed a kappa twist of 0.0 - 1.0 rad. Kappa rotation was therefore approximated to 0.5 rad. Trial and error procedures also showed that, to ensure successful convergence, the rotations had to be approximated to within 1.0 rad of the true value or the formulation will become unstable and will fail. Omega and phi rotations were approximated to zero. The solution was iterated until the mean of changes to all unknowns fell below some specified value. Residuals between computed and observed co-ordinates were then calculated at ground scale.

## RESECTION RESULTS

Residuals at plate scale of 100 $\mu$ m or more (1m on the ground) were considered significant and removed from the analysis. Reasons for significant residuals were difficult to isolate. Observational error, photographic resolution and misidentification of ground control points were likely to have been the main causes.

Table I shows a set of results obtained using a two dimensional similarity transformation between camera and negative format corners, and using radial distortion correction. Results showed that planimetric control could be recovered to within 0.3m at ground scale. However, neither transformation for correcting film stretch nor radial distortion correction had a significant effect on the improvement of overall accuracy.

The calibrated principal distance of the camera was 1.4mm greater than the manufacturer's quoted value. Analysis showed that, within the resection, the use of the calibrated value increased planimetric accuracy, on average, by 0.4m at ground scale. This difference was significant but perhaps not as large as might have been expected. Much of the benefit gained by calibrating the principal distance was probably negated by other instabilities within the non-metric system. To achieve an effective increase in overall accuracy, the system must be upgraded as a whole.

Accuracies for ground height determination were estimated from the planimetric results. Two formulae were used, the first of which was a conventional air survey formula:

$$\sigma Z = \frac{Z}{f} \cdot \frac{Z}{b} \cdot \sigma p_x, \quad (4)$$

where  $\sigma Z$  = r.m.s.e. of the height residual expected at ground scale,  
 $Z$  = flying height,  
 $f$  = principal distance,  
 $b$  = photobase value and  
 $\sigma p_x$  = mean r.m.s.e. calculated from the residual fit in  $x$  and  $y$  in the space resection.

Using an approximate value of the photobase of 11mm, the formula estimated that height values could be determined to within 0.85m at ground scale.

A second formula, given below and developed by Abdel-Aziz and Karara (1974) in close range non-metric photography was also used, although  $M_x$  is calculated by a different method.

$$MZ = \frac{Z/f}{B/Z} \cdot \sqrt{2} \cdot M_x \quad (5)$$

$MZ$  = r.m.s.e. of the height residual expected at ground scale,  
 $Z$  = flying height,  
 $f$  = principal distance,  
 $B$  = airbase value and  
 $M_x$  = mean r.m.s.e. calculated from the residual fit in  $x$  and  $y$  in the space resection.

Using the same stereopair as in equation (4), ground height determination was estimated to be to an accuracy of 1.12m. The airbase was calculated to be 106.5m, based on a photobase value of 11mm.

A Kern DSR1 analytical plotter was used to set up the same model as used in equations (4) and (5). Absolute orientation confirmed that the accuracy of planimetric determination was of the order of 0.3m. The accuracy of height determination was calculated to be in the order of 1m. This practical test perhaps provided the most reliable estimate.

## CONCLUSIONS

Results showed that, as would be expected, planimetric accuracy was somewhat higher than height determination. Analysis had also shown that, unless the instabilities within non-metric systems were controlled, the corrective potential of information gained from camera calibration was destroyed. Such instabilities may also cause the calibration results themselves to be unstable and inaccurate. If the comparator used has a digitizer with a re-setting facility, the planimetric accuracies could still be maintained without the use of any co-ordinate transformation between negative format corners and calibration format corners. Establishing the origin in such a way effectively removed the need for shift co-efficients. It was also ensured that the negatives were aligned with the axes of the comparator when observed. This removed the need for rotation co-efficients.

The use of a wide angle lens would increase the ground coverage of each photographic frame and reduce the amount of photography needed to cover a given area. Radial distortion will be larger and accuracies can, therefore, be expected to be lower. However, if instabilities within the non-metric system were further controlled to allow effective correction for radial distortion this need not be the case.

If the accuracies obtained within this study are sufficient for given mapping purposes, then a stereocomparator, digitized in x, y and z, with an origin re-setting facility and with kappa rotation on both plate carriers, is all that would be needed. It may be possible to achieve the required accuracies with no calibration at all.

If greater accuracy is required, the work of other authors suggests that the best system would be one in which a réseau plate is fitted into the focal plane of the camera. This would allow the measurement of film deformation over the whole camera format. A 60mm x 60mm format would provide better photographic resolution along with increased ground coverage. If a réseau plate is used to correct for film stretch and film deformation due to lack of flatness, then field calibration should prove to be an adequately accurate method for correcting radial distortion.

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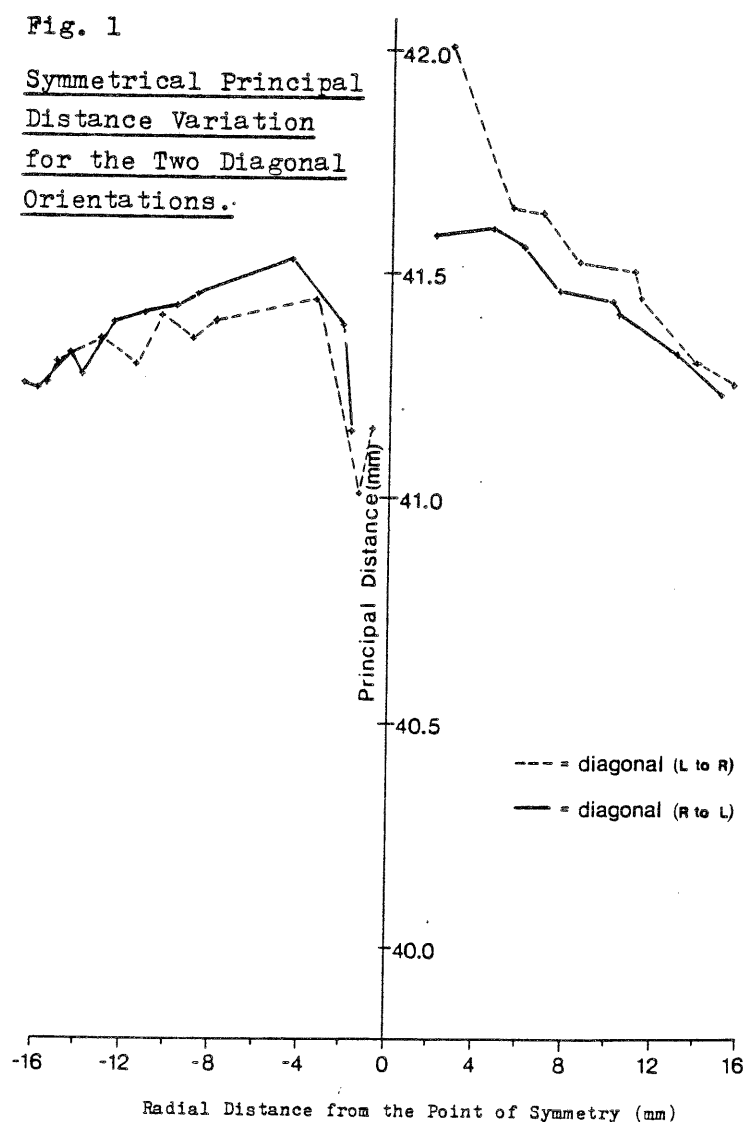
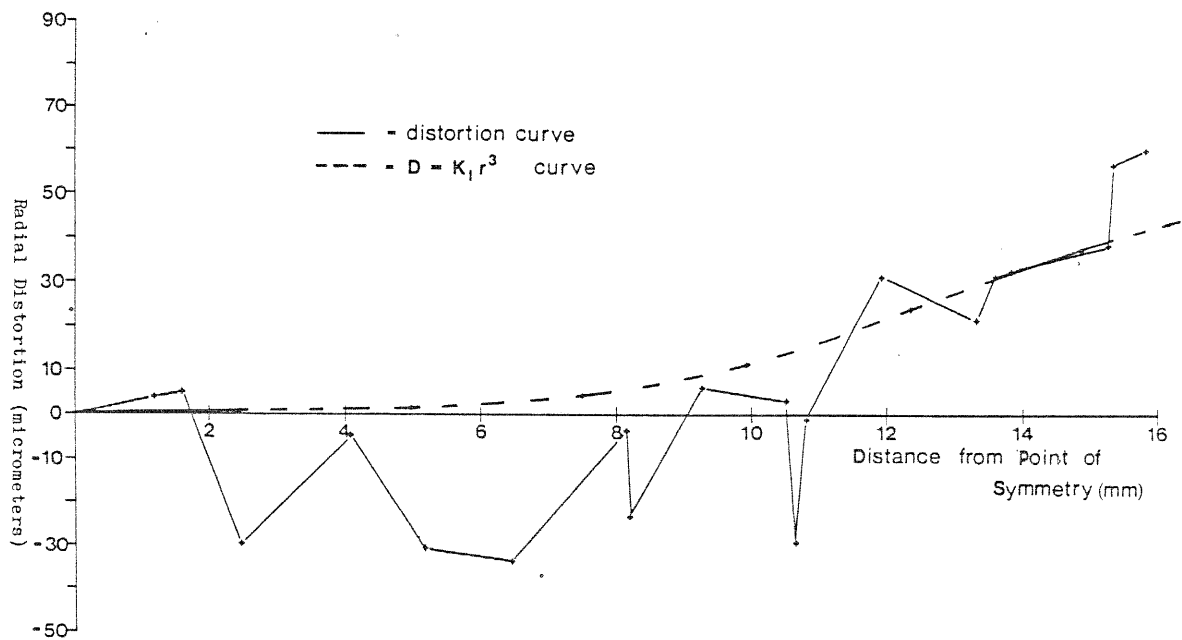
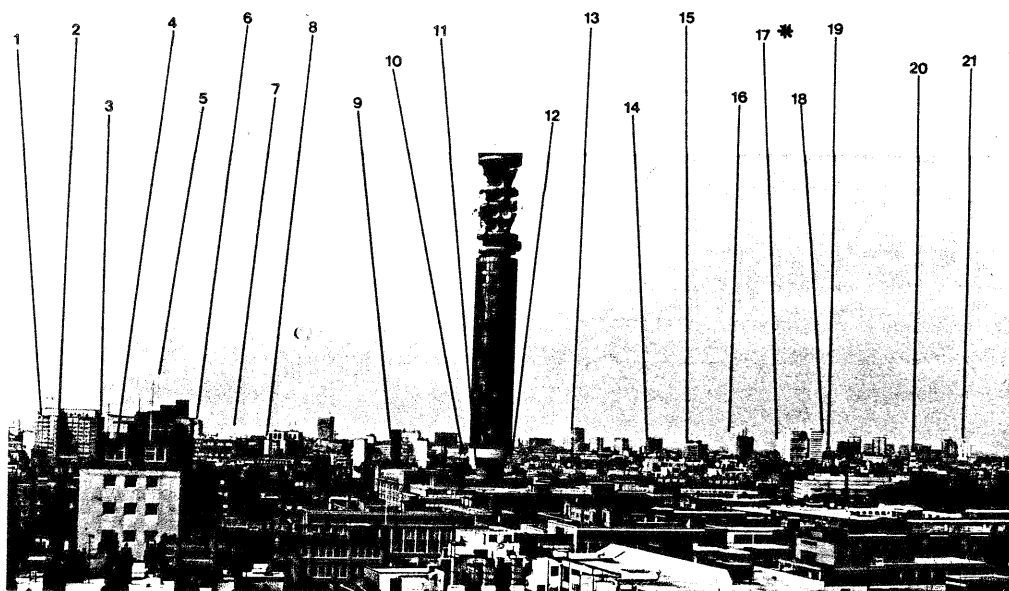




Fig. 2

Camera Lens Radial Distortion.



\* Point 17 was mis -observed and removed leaving 20 points.

Table. I Resection Results Using a 2D-Similarity Transformation with Radial Distortion Correction.

Frame No.	$\omega$	Rotations (Radians) $\phi$	$k$	Flying Ht. (m)	Average Ground Accuracy (m)	R.M.S.E. (m)
1.	0.099	0.217	0.536	367.87	0.112	0.143
2.	-0.186	0.257	0.413	437.59	0.270	0.341
3.	-0.018	-0.025	0.364	464.83	0.139	0.194
4.	0.102	-0.132	0.406	380.68	0.225	0.306
5.	-0.054	0.093	0.235	438.37	0.157	0.207
6.	0.113	-0.063	0.544	411.57	0.235	0.276
7.	-0.086	-0.065	0.602	396.61	0.208	0.290
8.	0.179	-0.089	0.633	395.51	0.247	0.299
9.	0.037	0.083	0.719	388.28	0.189	0.249
10.	-0.023	-0.157	0.530	444.68	0.239	0.282
11.	-0.000	0.039	0.767	395.68	0.209	0.259
12.	0.066	0.035	0.861	392.58	0.262	0.323
13.	0.095	0.106	0.990	401.26	0.133	0.150
14.	-0.052	-0.235	1.193	399.75	0.173	0.200
15.	-0.071	0.019	1.074	367.20	0.409	0.448
16.	-0.055	-0.035	4.614	377.87	0.167	0.183
17.	-0.282	0.091	0.951	402.40	0.356	0.458
18.	-0.254	-0.033	1.297	420.75	0.256	0.321
19.	-0.015	-0.021	1.280	415.84	0.206	0.262
20.	-0.057	0.000	1.236	420.24	0.381	0.458
21.	-0.032	-0.030	0.756	369.16	0.207	0.252
22.	-0.171	0.116	0.964	404.66	0.323	0.379
23.	0.005	-0.011	0.757	368.78	0.096	0.112
24.	-0.157	0.187	0.696	439.66	0.206	0.252
25.	-0.112	0.119	0.622	471.41	0.173	0.202
26.	-0.076	-0.006	0.879	447.64	0.299	0.332
27.	-0.099	0.096	0.877	456.91	0.000*	0.000*

\* residuals computed with only three control points and therefore no redundancy.