PHOTOMECHANICAL METHOD OF NADIR RECTIFYING OF EXCESSIVELY TILTED AERIAL PHOTOGRAPHS

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1. Introduction

Aerial photographs, characterized by tilt angle-nadir distance greater than in normal case, can not be elaborated with the use of commonly utilized analog stereoplotters. In order to make it possible, flight mission should be repeated, or existing excessively tilted photographs should be rectified. Considering lack of analytical stereoplotters, it is advisable to make preliminary nadir rectifying at a scale of 1:1, obtaining exactly vertical photographs, and next to make elaboration using analog stereoplotter. The proposed method assumes use of 4 fiducial marks as points for rectifying. The suitable distribution of these points at photo corners assures accuracy of rectifying, appropriate for stereoscopic elaboration with the use of stereoplotters.

2. Theoretical principles of the method.

Two assumptions were put as a basis of method:

a/ projective geometry theorem, concerning requirement of projectivity of planes, expressed by general formula:

$$X_o = \frac{Ax' + By' + C}{Dx' + Ey' + 1}$$

$$Y_o = \frac{Fx' + Gy' + H}{Dx' + Ey' + 1} \qquad /1/$$

where X_o, Y_o - coordinates of point on the exactly vertical photo;
x', y' - coordinates of corresponding point on the tilted photo;
A, B...H - coefficients of rectifying,

b/ properties of photo isocentre:

- angles with vertices in photo isocentre, existing beetwen corresponding points on vertical and tilted photo, are equal to each other;
- scale along horizontal line, perpendicular to the line of steepest slope, passing through isocentre on the tilted photo is equal

$$\frac{1}{m} = \frac{c_k}{H_o} , \qquad /2/$$

where m- scale denominator, c_k - camera constant, H_o - flight height.

For soling that problem the modified formulas /1/ were applied, enabling practical use in case, if only some elements of photo orientation are known:

$$\bar{x}_o = c_k \frac{\bar{x}}{c_k - \bar{x}\sin v}$$

$$\bar{y}_o = c_k \frac{\bar{y}}{c_k - \bar{x}\sin v} \qquad /3/$$

where : $\bar{x}_o$, $\bar{y}_o$ - coordinates of fiducial marks on exactly vertical photo ($\bar{II}_o$) in coordinate system defined by origin in isocentre (I_o) and streight line vv as $\bar{x}_o$-axis;

$\bar{x}$, $\bar{y}$ - coordinates of fiducial marks on tilted photo ($\bar{II}$) in coordinate system defined by origin in isocentre I' and straight line vv' as $\bar{x}$ - axis;

c_k - principal distance of aerial camera /camera constant/;
ν - angle of tilt /nadir distance/.

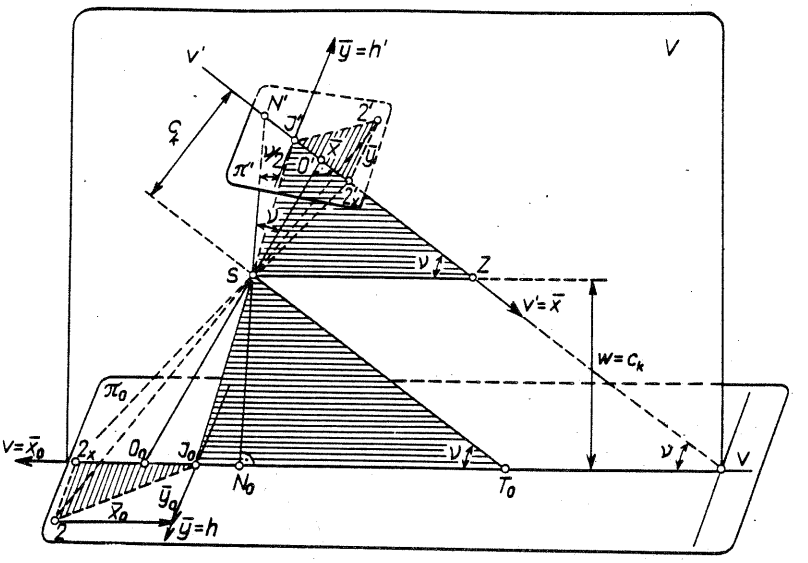

Figure 1.

Formulas /3/ are derived from geometrical relationships, existing in similar triangles $\Delta (SI_oT_o) \sim \Delta (SI'T)$ and $\Delta (2S2_x) \sim \Delta (2'S2_x')$, shown in Figure 1 :

$$\frac{T_oI_o+\bar{x}_o}{T_oS} = \frac{SZ}{Zf'-\bar{x}}$$

$$\frac{\bar{y}}{\bar{y}} = \frac{ST_o}{ZI'-\bar{x}}$$

where : $T_oI_o = T_oS = \frac{c_k}{\sin\nu} = ZS = ZI'$

Moreover as it can be seen from Fig.1 tilted photo π' is projected through projection centre S into exactly photo π_o in uniform scale 1:1, determined by scale denominator $m = H_o : c_k$

3. Practical implementation of method conception.

Commonly known method of photo rectifying with the use of ordinary photomechanical rectifiers of C.Zeiss SEG I or Wild E4 type, was applied to realize the theoretical assumptions. Adjustment points are formed by 4 fiducial marks, isocentre I' marked on the photo and corresponding points of base for rectifying, having coordinates calculated according to formulas /3/. The following data must be known in order to calculate these coordinates :

- coordinates of fiducial marks x', y', measured on precise stereocomparator, for instance C.Zeiss Stecometer ;
- angle elements of relative orientation : tilt of photo ν and direction angle of straight line of steepest slope $v'v'$ (necessary to transform coordinate system (O', x', y') into system (I', $\bar{x}$, $\bar{y}$) ;
- aerial camera constant c_k, calculated on the basis of calibration data.

Computations are performed using computers with the help of specially prepared program.

Angles ν, α, necessary for computations are also determined, using analytical method. Results of aerotriangulation performed for condensation of photogrammetric framework, are used for that purpose. On the basis of adjusted terrain coordinates (X,Y,H) of aerotriangulation points, coordinates of projection centre S are calculated using method of double-point interpretation in space, and next elements ν, α are computed.

3.1. Accuracy analysis of the method.

Accuracy for particular stages of the work was estimated, considering usefulness of nadir rectified photographs for stereoscopic elaboration on the stereoplotter.

Analysis of accuracy of nadir rectifying was done through simulation of tilt of square grid, engraved on flat glass plate, 230x230 mm in size. For examination the author selected 25 points- intersection of grid lines, in 50 mm intervals, and isocentre I', marked on particular grids, according to coordinates, calculated for simulated tilts : $v = 2^g, 4^g, 6^g, 8^g, 10^g, 12^g, 14^g$.

The examination was done as follows :

- on the basis of known coordinates of intersections of non-rectified grids, coordinates of the same points were calculated using formulas /3/, after its simulated rectifying into [illegible] grids tilted according to given above angles v ;
- bases for rectifying were prepared - 4 fiducial marks and isocentre were marked, according to image coordinates $\bar{x}_o, \bar{y}_o$, calculated for each grid with the use of formulas /3/ ;
- precise adjustment of original grids placed in rectifier carrier with the base placed on rectifier screen was done, using Wild instrument. This adjustment can be done with accuracy ± 0.05 mm ;
- after photo processing coordinates of 25 points of each rectified grid were measured using Stecometer ;
- after suitable transformation of image coordinates measured on rectified grids, they were compared with corresponding theoretical values resulting from formulas /3/ ;
- accuracy assessment for nadir rectifying of square grids is presented in table 1.

Table 1.

Tilt v g	Rectifying I			Rectifying II			Number
	m_x	m_y	m_p	m_x	m_y	m_p	of points
2	0.016	0.033	0.037	-	-	-	25
4	0.015	0.013	0.020	-	-	-	"
6	0.044	0.055	0.070	0.041	0.033	0.054	"
8	0.073	0.076	0.105	0.053	0.061	0.081	"
10	0.161	0.160	0.227	0.159	0.164	0.229	"
12	-	-	-	-	-	-	
14	-	-	-	-	-	-	

The following conclussions can be drawn from the given above values of mean errors m_x, m_y, m_p (in mm at photoscale) :

- for angles of tilt up to 6^g rectifying can be done with accuracy $m_p = \pm 0.05$ mm at photoscale. In table 1 results of repeated independent rectifying are given;
- as angle of tilt increases, decrease of accuracy of rectifying to $m_p = \pm 0.23$ mm is observed. This is caused by difficulties in precise (± 0.05 mm) adjustment of fiducial marks in rectifier carrier with corresponding points placed on much inclined screen. This phenomenon can be overcome through two-stage rectifying of the same photo. At first stage photo tilted with angle of tilt for instance $v=10^g$ is rectified to photo with angle of tilt $v=5^g$, i.e. with convenient screen tilt. At next stage second exposure is done after proper insertion of rectified once photo into rectifier carrier with unchanged screen

tilt. Preliminary examinations reveal, that as a result of two-stage rectifying exactly vertical photographs are acquired, characterized by accuracy sufficient for its stereoscopic elaboration on analog stereoplotter.

3.2. Practical application.

Briefly described examinations of practical results were preceded by theoretical accuracy analysis consuming :

- determination of tilt angle (for instance for photographs with 10^g tilt required accuracy of determination of angle ν is $m_\nu = 3^c \div 6^c$ according to assumed accuracy of photo adjustment in rectifier);
- introduction of photo incentricity in rectifier;
- measurement of coordinates of fiducial marks and preparation of base for rectifying.

On the basis of conducted studies "Technology of photomechanical method of nadir rectifying of excessively tilted photographs to exactly vertical photos with geometrical conditions maintained"was elaborated at the Institute of Geodesy and Cartography. Program PN (nadir rectifying) for calculation of points for rectifying with the use of computer written by member of research group M.Sc. J.Ziobro, is the important element of that technology. It was implemented operationally at the State Geodetic-Cartographic Enterprise and is successfully applied according to demands. Program PN was included into system of program for analytical aerotriangulation used in that Enterprise.

If in the block of photos under elaboration excessively tilted photographs were found ($\nu > 4^g$) they are chosen for nadir rectifying. In this way photographs which couldn't be elaborated on analog stereoplotter, considering its angle parameters, need not be repeated.

As an example of the results of computations for nadir rectifying with the use of PN program, the data from computer printout are presented:

Photo 4066

c_k = 152.80

QM (ω) = -787.68 FI (φ) = 125.18

NI (ν) = 797.51 KA (α) = 1299.39

Coordinates of fiducial marks for marking points I,O,N on non-rectifying photo :

No	x	y
1	606.01	393.99
2	606.01	606.01
3	393.99	606.01
4	393.99	393.99
0	500.00	500.00
I	498.06	509.38
N	496.10	518.84

Coordinates rectified for base preparation

No	x	y
1	618.26	431.35
2	592.52	651.59
3	364.53	599.37
4	424.80	394.29
I	500.00	500.00

During elaboration of nadir rectified photographs on the stereoplotter, they should be inserted in the carriers in such a way that nadir point N' (marked before on the original photo) should match principal point of photo carrier.

Conclusions.

On the basis of conducted research works and ptactical results the following conclusions can be done :

a/ photomechanical method of nadir rectifying of photographs can be effectively applied, if :
- after photogrammetric flight performed in unfavourable conditions among correct photographs aerial photos excessively tilted (over 3^g) will be found;
- for stereoscopic elaboration of such photos from necessity conventional analog stereoplotters widely used in practice, are utilized.

As rapid technical-scientific progress has been lately made in the field of technique of photo acquisition and construction of modern photogrammetric instruments, the presented method should be treated as supplementary method, enabling to extend the range of use of existing photogrammetric instrumentation.

b/ using photomechanical rectifiers photographs can be nadir rectified at a scale 1:1 without utilizing rectifier counters with the accuracy sufficient for stereoscopic photo elaboration on the analog stereoplotters.

Implementation of the presented method in photogrammetric production was possible, as analytical methods of aerotriangulation adjustment are applied in production practice, hence the results of aerotriangulation enable to calculate precisely elements of absolute orientation of photographs.