DETERMINING TOPOGRAPHIC STRUCTURE LINES BY DIGITAL ELEVATION DATA

Hüseyin Gazi BAŞ, Dr.
Istanbul Technical University,Engineering Faculty of Sakarya
TURKEY
Com.N. 3

INTRODUCTION

In the last ten years the digital terrain models have been produced by means of the computers which have been used faster in compilation of the maps and the automated mapping can be performed planimetrically and with contours based on these models. The development of automated mapping methods is closely connected with the creation of a structural model which used the structure lines. As it is known, valley lines and ridge lines form the framework of the terrain forms in topography. The valleys from these characteristics, which are also known as structure lines in topography, represent down level of lines and the ridges represent high level of lines. Topographic features such as crets of ridges, drainage divides, flatnesses etc. exist between these lines. Since structural lines depict the boundaries of existing changes in topographic surface, they have an important role in the presentation of morphometric and morphologic characteristics of relief (Rudyy, 1985).

In this study, by making use of the fact that the structure lines depict boundaries of actual changes in topographic surface, a mathematical approach is introduced for determining topographic structure lines by means of digital elevation data, and a sample application of aforementioned approach is presented.

MATHEMATICAL APPROACH

The function expressed by (1) represents a topographic surface.

$$z=f(x,y) \qquad (1)$$

When z is equal to a constant in above function, a space curve, which is the geometrical position of points elevations of which have the same value of z on the surface, is obtained. The shape of the above mentioned curve is not destroyed when it projected on the plane Ox,y and at the same time the contours are obtained. Structure lines are the geometrical positions of the extrem points of topographic surface. Therefore study of the extrems of function is essential to discover these points. The tangent plane at the point x_o, y_o must be parellel to the horizantal plane to have the maximum or minimum of the function there.It means that $f_x(x_o,y_o)$ and $f_y(x_o,y_o)$, which are the first order partial derivates of the function, are equal to zero. This is an essential condition. $f_x(x_o,y_o)$ gives the slope of the tangent of the curve which is obtained by cutting the surface with a plane parellel to xz plane while $f_y(x_o,y_o)$ gives the slope of the tangent of the curve which is obtained by cutting the surface with a plane parellel to the yz plane. The fact that these slope are equal to zero means that tha tangent plane on the mentioned point is parellel to the horizantal plane. There are also additional conditions, which contain the higher order partial derivates for investigation of surface, but the equality of the first order partial derivates to zero is suffecient to find the critical points, which indicate the real changes in relief.

By making use of the relation between the derivates and differences, the partial derivates of function given with equation (1) along the the x and y axes can be found. For this purpose above function is expended according to Taylor by taking difference interval as h for direction of each axis. Then the partial derivates are obtained by using the differences between to consecutive points of surface.This case is studied for a function of one variable by using the following notation.

$$f_o=f(x_o),\quad f_1=f(x_o+h),\quad f_{-1}=f(x_o-h),\quad f_o'=f_x \text{ and } f_o''=f_{xx}$$

If function is expended according to Taylor at step interval h, the following expression is obtained (Rao, 1982).

$$f_1=f(x_o+h)=f_o+\frac{h}{1!}f_o'+\frac{h^2}{2!}f_o''+\ldots \tag{2}$$

and similarly

$$f_{-1}=f(x_o-h)=f_o-\frac{h}{1!}f_o'+\frac{h^2}{2!}f_o''-\ldots \tag{3}$$

by subtracting equation (3) from equation (2), one is obtained

$$f_1-f_{-1}=2hf_o' \tag{4}$$

Then f_o' which represents f_x is expressed by following equation

$$f_o'=\frac{1}{2h}(f_1-f_{-1}) \tag{5}$$

As it previously expressed, if the detailed investigation of surface is desired, the higher order partial derivates are needed. The following equation is obtained as second order derivate according to x by following the same way,

$$f_{xx}=\frac{1}{h^2}(f_1-2f_o+f_{-1}) \tag{6}$$

and the others are also found by following the same way.

If the above mathematical approach is applied to a function of two variables given by equation (1) according to the arrengement of points shown in Figure 1, the following equations are obtained for the partial derivates of point 0 (zero). The first order partial derivates are:

$$\left(\frac{\partial f}{\partial x}\right)_o=f_x=\frac{1}{2h}(f_1-f_3) \tag{7}$$

and

$$\left(\frac{\partial f}{\partial y}\right)_o=f_y=\frac{1}{2h}(f_5-f_7) \tag{8}$$

The second order partial derivates are:

$$f_{xx}=\frac{1}{h^2}(f_1-2f_o+f_3) \tag{9}$$

and

$$f_{yy}=\frac{1}{h^2}(f_5-2f_o+f_7) \tag{10}$$

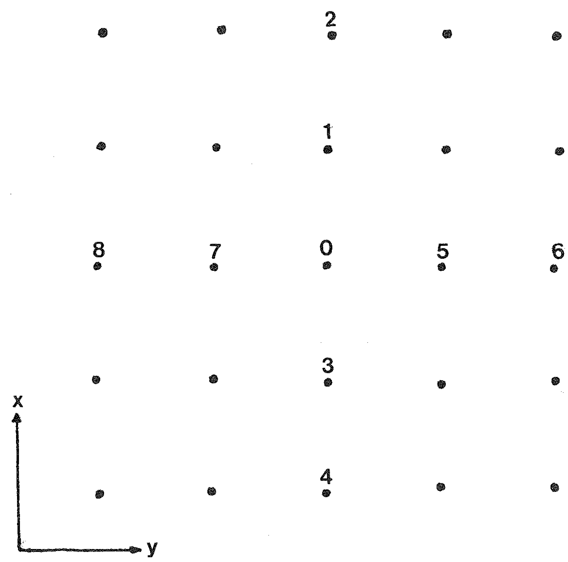

Figure 1. Arrangment of points partial derivates

PRACTICAL APPLICATION

The digital elevation data, which represent a topographic surface and ensure determining structure lines in an analytical manner, may be obtained by using one or more of the following several sources (Doyle, 1978; Gossard, 1978; Sıyam, 1981; Segu, 1985).

a. Direct measurements on photogrammetric stereomodels,
b. ground surveys,
c. existing topographic maps,
d. existing orthophotomaps superimposed with contours
e. and some other sources.

In this study it has been preferred to digitize an existing topographic map. Storing the digital elevation data in the computers arbitrarly causes both the occupation of the memory of computer too long and may arise a problem at the represantation of the surface being studied. Therefore in the firs stage of processing the surface which will be studied is divided into a rectangular grid and then each intersection of a rectangular grid is loaded into the memory of computer with its elevation. Thus, the elevations of points H(i,j) that represent surface have a matrix form, where i and j show the intersection of rectangular grid along the x- and y- axes respectively; H represents spot height. The following mathematical expressions which are obtained by applying the equations (7) and (8) are used to determine the first order partial derivates, along the x- and y- axes, at related point of surface.

$$FX(i,j)=(H(i+1,j)-H(i-1,j))/2dx \qquad (11)$$

$$FY(i,j)=(H(i,j+1)-H(i,j-1))/2dy \qquad (12)$$

where dx and dy are equal intervals between the grid lines along the x- and y- axes respectively.

Critical points forming structure lines which belong to surface shown in Figure 2 have been determined by using the above equations (11), (12) with the aim of practical application. As seen from the Figure 2,

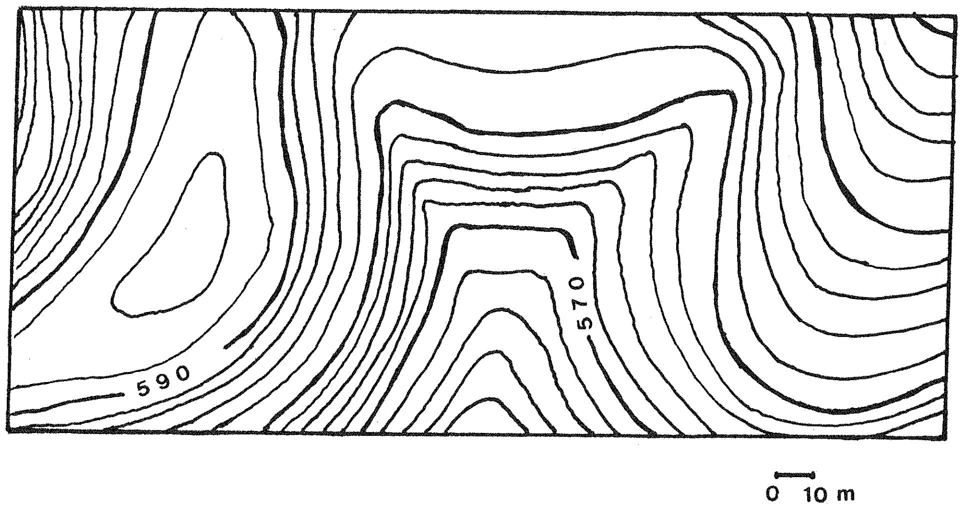

Figure 2. Topographic map.

such a surface does not include the essential digital data for identifying structure lines. Therefore firstly a square grid with a line interval of 10 m. was set up on surface, then the elevation of each intersection of square grid was determined by linear interpolation.

The choice of the interval between the points is important when the square grid is set up. It has been observed from results of several experiments, which have been done with different scale of map and different line interval, that these intervals between the points have an important effect on the accuracy of structure lines. The research works that have been done about the accuracy of digital elevation models have also stated that the density of the points which represent surface is an important parameter which effects the accuracy of digital model (Accerman, 1978; Wong and Siyam, 1983).

The first order partial derivates of all points of surface have been computed by using the data ensured from digitasing an existing topographic map. Then the structure lines of surface shown in Figure 2 have been obtained by joining the points where the partial derivates were zero. The related surface and the structure lines obtained by analytical way are shown in Figure 3.

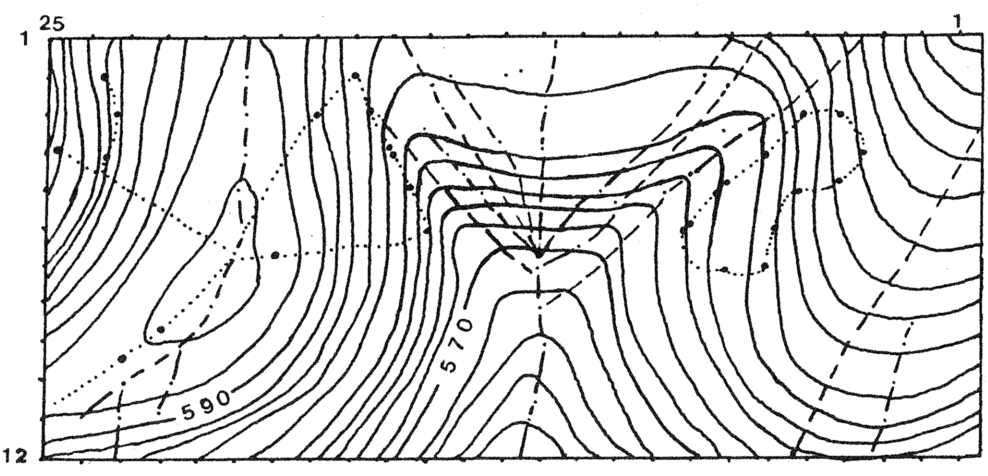

Figure 3. Topographic structure lines determined from digital elevation data.
------- : structure lines obtained by using equation (11).
.... : structure lines obtained by using equation (12).

The partial derivates of surface points along y- axis are presented in Table 1. As seen from the table, if derivates at any two consecutive points change their sign from positive to negative or vice-versa it means there is a zero-point between these two points. In this case position of such a zero-point may be easily determined by linear interpolation. Structure lines can be plotted together with computations if the computer has a plotting equipment.

Table 1. The first order partial derivates along the y-axis

I	J→ 2	3	4	5	6	7	8	9	10
2	-.1	-.14	-.13	-.14	-.15	-.16	-.21	-.20	-.21
3	-.09	-.06	-.06	-.09	-.1	-.13	-.16	-.18	-.19
4	-.09	0	.09	-.05	-.19	-.1	-.11	-.13	-.19
5	-.22	0	.05	0	-.03	-.05	-.08	-.1	-.15
6	-.3	-.13	0	.03	.01	0	-.01	-.06	-.16
7	-.2	-.19	-.11	.02	.03	0	-.05	-.11	-.16
8	-.13	-.16	-.26	-.14	.02	-.02	-.05	-.09	-.15
9	-.11	-.11	-.25	-.23	-.13	-.11	-.05	-.1	-.15
10	-.61	-.1	-.2	-.4	-.35	-.1	-.04	-.09	-.11
11	-.1	-.1	-.2	-.39	-.4	-.24	-.14	-.11	-.11
12	-.61	-.11	-.21	-.4	-.39	-.24	-.18	-.19	-.18
13	-.12	-.13	-.23	-.39	-.38	-.22	-.1	-.1	-.17
14	-.05	-.15	-.34	-.38	-.22	-.03	-.09	-.14	-.14
15	-.14	-.16	-.15	-.05	0	-.05	-.1	-.15	-.26
16	-.1	-.05	.01	.05	.03	-.05	-.1	-.18	-.26
17	.01	.04	.05	.06	.03	-.09	-.18	-.2	-.2
18	.05	0	.03	.05	.04	-.04	-.11	-.24	-.31
19	.06	.03	.03	.04	.03	-.02	-.1	-.19	-.3
20	.05	.05	.05	.08	.05	-.03	-.08	-.15	-.26
21	.05	.05	.06	.03	.07	.13	.03	-.04	-.23
22	.06	.08	.04	.08	.13	.11	.09	-.04	-.12
23	.05	.03	.03	.1	.18	.2	.15	.05	-.05
24	-.04	-.05	-.03	.15	.28	.28	.3	.2	0
25	-.05	.01	.08	.14	.18	.34	.45	.35	.16

It is quoten in Rudyy's study (1985) that significantly more structure lines are identified by analytical method than structure lines plotted as a results of visual analysis of a stereomodel. But when the partial derivate equations given by Rudyy's study have been used, computer-time extended by 30-35 percent more when the partial derivate equations (11) and (12) given in this study have been used. However it has been seen as a result of computations and plottings that the accuracy of both methods was at the same level. The structure lines determined by Rudyy's equations are shown in Figure 4.

CONCLUSION

A mathematical approach has been presented to determine the topographic structure lines using the digital elevation data. The structure lines can easily be obtained by means of this approach. A similar analytical method is given in Rudyy's study. Althought the results obtained by using both methods are equivalent as cartographic, the approach given in this study is faster 30-35 percent than aferomentioned analytical method. If the approach given here is developed, it can be used to determine analytically the area of the drainage basin and capacity of reservoir.

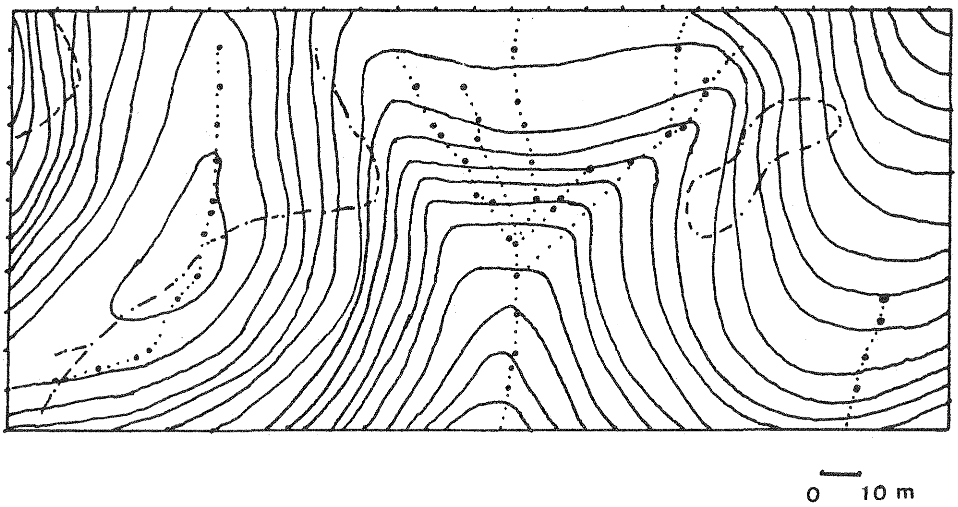

Figure 4. Structure lines obtained by using derivate equations given in Rudyy's study (1985).

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