TRIDIMENSIONAL REPRESENTATION BASED ON THE DIGITAL TERRAIN MODEL. ITS DEVELOPMENT AND APPLICATION

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ABSTRACT: A programme set to develope tridimensional representation is given. The surface of the considered terrain zones is initially recognized by digital terrain models derived as a square network, using collocation method or bicubic spline functions. Representation is carried out in three phases: digital model point expressions in homogeneous coordinates, optional transformation in central or oblique perspective projections, and surface visible part determination. A test based on the comparison of the orientation angle is used in determination of visibility phase. Representations are used in contouring chek, some project location studies, and open-cast mining check as well. The above mentioned applications are exemplified by sheets made using ARISTOMAT plotter.

Introduction

Today, photogrammetric and remote sensing methods are intensively used to obtain information regarding the surfaces of the Earth and other planets, at the same time, they provide a large variety of products in which digital data are ever increasing employed. So, the digital elevation model is one of the modern products, which importance grows concurrently with the technological automation development. After it has been made and stored as an information entity on a data storage media, always, the digital elevation model can be compared with the latent relief image, which after being processed automatically facilitates to solve problems within various fields of activity based on the results obtained. Among its main applications, we can mention: ortho and stereophotomaps, digital levelling plotting, morphometric parameter computations, and thematic topographical map (map for slope classes, curvatures, sun exposure of slopes, relief energy, structural lines), satellite recording uses, 3D representation generation, engineering design implementations, terrain correlation guidance systems or information geosystems. Further, we shall analyse 3D representations knowing,nowadays, many uses owing its possibility to be an efficient way for studying, designing, understanding and prognosis as regards the many activities related to the terrain.

Digital Model Generation To Be Used in 3D Representations

The orthogonal projection is usually used to carry out relief representations resulting graphically in contour lines traced on topographical maps. When representations describing the relief configuration in space are needed, we must previously define the segments of the represented relief by a digital model, as a prerequisite condition. In this approach, INTERCOL and INTERCUB programmes, components of I_3SRPR_3 processing system aiming at digital levelling plotting, are used to obtain digital models as inputs within 3D representations.

The ground information necessary to develop digital model for a relief segment are derived photogrammetrically. Both non-uniform data structures containing points, which the human operator selected after analysing relief in each measured stereomodel by photointerpretation and uniform structures representing square networks, which sampling interval is established by the transfer function, are collected. In both cases, morphological lines and points are measured separately and after registration their data are added to those used in current description.

Non-uniform structure processings are made by INTERCOL programme based on interpolation principle, using colocation method:

$$Z(x,y) = T(x,y) + c^T C^{-1} L \qquad (1)$$

The interpolation range we have used is a circular one. Its ray is established considering the maximum correlation distance ranging from $(1/2.5 \ldots\ldots 1/4)\lambda_m$ to $(1/4.5 \ldots\ldots 1/6.5)\lambda_m$ for flat or near flat terrains and rough terrains, respectively. To determine parameter λ_m values (the average wavelength of the terrain oscillations as against an average horizontal reference plane) 4 or 5 profiles large enough and densely sampled, are used; after a preliminary analysis of the modelled terrain, these profiles are disposed on a direction rendering variation range as complete as possible.

The programme algorithm makes the tendency surface selection using a statistical procedure based on the analysis of the variance among (Z_i) measured reference elevations and their values established successively, using interpolation surfaces derived from the bicubic incomplete polynomial. It includes exponential Gauss and Hirvonen functions to compute covariances, having the possibility to be automatically exchanged by comparing interpolated height errors with the maximum admissible error ($m = (C(0) - c^T C^{-1} c) \leq m_a$) and it determines point blocks within interpolation ranges. Computation units processed can comprise 2500 reference points, and the processing result is as a square network.

When input data are uniformly structured, the digital elevation model is made calling INTERCUB programme. It uses the complete bicubic polynomial, as a modelling spline function:

$$Z(x,y) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \cdot \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ y \\ y^2 \\ y^3 \end{bmatrix} \qquad (2)$$

Polynomial coefficients are established by imposing four connexion conditions in each corner point belonging to a surface element, which we have analysed. The conditions used assures: the exact connexion of (Z) height values, tangent plane continuity along x,y (Z'_x, Z'_y) coordinate axis directions, and (Z''_{xy}) surface smoothing. Here, computation units can contain as far as 5,000 points, and the digital model is output as a square network, as if INTERCOL programme had been used.

3D Representation Generations and Applications

To obtain 3D representations corresponding to a relief segment as a digital model - like, processing methods used in programme development employ homogeneous coordinate technique. Homogeneous coordinate operation is an efficient way to describe curves and surfaces and, as it is well known, it was developed within the projective geometry framework as a practical means in theorem verifications.

Problems related to object representations situated in a n-dimensional (nD) space have always a correspondent in ((n+1)D) homogeneous space and in many cases they are easily solved when they are transposed in this space. The results obtained can be introduce in (nD) space by projection, after they have undergone the solving stage. Such a strategy used to develop perspective representations, in which homogeneous coordinate is the main element, results in two basic advantages:
- the possibility to treat uniform transformation operations becomes a reality;
- all transformations fitting a representation can be grouped in one transformation matrix-operator, using a matrix product, having the same effect as their sequential application (Newman, Sproul 1979), (Dubayah, Dozier 1986).

Considering the homogeneous representation, $[x\ y\ z]$ position vector corresponding to a point within (3D) space is represented homogeneously in (4D) space by $[x\ y\ z\ m]$ vector having an extra element represented by $m \neq 0$ scalar, named the scale factor. $[x\ y\ z\ m]$ vector transformation into (4D) space is given by relation (Dubayah, Dozier 1986);

$$[X\ Y\ Z\ M] = [x\ y\ z\ m] \cdot T \qquad (3)$$

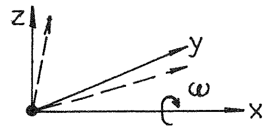
where $[X\ Y\ Z\ M]$ are the transformed homogeneous coordinates and T is a 4x4 transformation matrix. The transformed uniform coordinates are $[\bar{x}\ \bar{y}\ \bar{z}\ 1] = [X/M\ Y/M\ Z/M\ M/M]$ and if M = 1 (in affine transformation) $\bar{x} = X$; $\bar{y} = Y$; $\bar{z} = Z$

In their matrix shape, the geometric linear transformation operations, when $[x\ y\ z\ m]$; (m=1) vector is envisaged, to be transposed in various representation types (parallel, perspective central, a.s.o.) can contain :

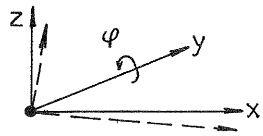
a) translations

$$[\bar{x}\ \ \bar{y}\ \ \bar{z}\ \ 1] = [x\ \ y\ \ z\ \ 1] \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}}^{T_t} \qquad (4)$$

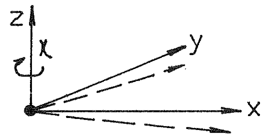
(Tx, Ty, Tz = displacements along x,y,z axis directions).
b) rotations defined after the rotation sense has been specified in connection with rotation axis positions and directions.



$$\begin{bmatrix}\bar{x} & \bar{y} & \bar{z} & 1\end{bmatrix} = \begin{bmatrix}x & y & z & 1\end{bmatrix} \overbrace{\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\omega & -\sin\omega & 0\\ 0 & \sin\omega & \cos\omega & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}^{R\omega} \qquad (5)$$



$$\begin{bmatrix}\bar{x} & \bar{y} & \bar{z} & 1\end{bmatrix} = \begin{bmatrix}x & y & z & 1\end{bmatrix} \overbrace{\begin{bmatrix}\cos\varphi & 0 & \sin\varphi & 0\\ 0 & 1 & 0 & 0\\ -\sin\varphi & 0 & \cos\varphi & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}^{R\varphi} \qquad (6)$$



$$\begin{bmatrix}\bar{x} & \bar{y} & \bar{z} & 1\end{bmatrix} = \begin{bmatrix}x & y & z & 1\end{bmatrix} \overbrace{\begin{bmatrix}\cos\chi & -\sin\chi & 0 & 0\\ \sin\chi & \cos\chi & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}^{R\chi} \qquad (7)$$

c) scaling, using elements situated on the transformation matrix diagonal.

$$\begin{bmatrix}\bar{x} & \bar{y} & \bar{z} & 1\end{bmatrix} = \begin{bmatrix}x & y & z & 1\end{bmatrix} \overbrace{\begin{bmatrix}m_x & 0 & 0 & 0\\ 0 & m_y & 0 & 0\\ 0 & 0 & m_z & 0\\ 0 & 0 & 0 & m_g\end{bmatrix}}^{Tm} \qquad (8)$$

(m_x, m_y, m_z = local scale factors, m_g = global scale factor)

Considering observations on advantages of homogeneous coordinate uses to develop perspective representations, transformations from relations (4 - 8) applied in the representation process are simplified, using a general matrix $T_G = R\omega \cdot R\varphi \cdot R\chi \cdot T_t \cdot T_m$.

A set of three programmes (RELIEF - C,D,S) generating representations within the relief space have been implemented, based on the concepts previously described.

RELIEF-C programme represents the relief in the central perspective as a frontal view, using the projection point specified only by X_p and Z_p coordinates. According to this representation variant at the beginnning, O(XYZ) reference coordinate system corresponding to the digital model is planimetrically shifted in the represented relief segment centre during a preliminary computation stage (Figure 1). Then, T_G general transformation matrix is obtained carrying out the following transformations:
- digital model points are so translated that the projection point to be in the origin:

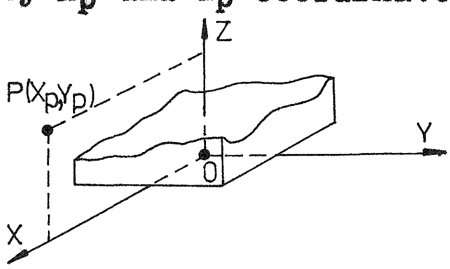
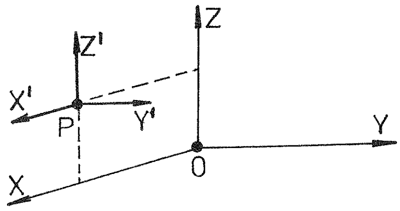


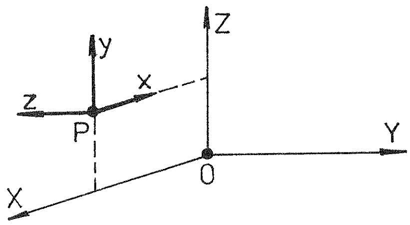
Figure 1

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -T_{x_p} & 0 & -T_{z_p} & 1 \end{bmatrix} \Longrightarrow$$



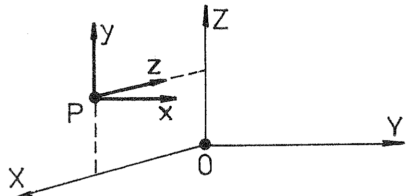
- the viewing system considered is a left Cartesian one (x y z); z axis corresponds to the viewing axis, x axis is right oriented and y axis is up oriented; in such a case, axis redistributions are applied (Z elevations of the digital model are oriented towards y direction):

$$T_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$

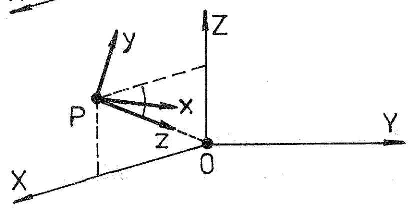


- to orient z axis on the viewing direction pointing O(XYZ) reference coordinate system origin, a 90^o rotation is made around y axis followed by a second ($\cos\omega = X_p/\sqrt{X_p^2 + Y_p^2}$; $\sin\omega = Z_p/\sqrt{X_p^2 + Y_p^2}$) rotation around x axis.

$$R_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$



$$R_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega & 0 \\ 0 & \sin\omega & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$



The general transformation matrix resulted from T_1, T_2, R_3, R_4, transformations has the following shape:

$$T_G = \begin{bmatrix} 0 & -\sin\omega & -\cos\omega & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega & 0 \\ 0 & T_{x_p}\sin\omega - T_{z_p}\cos\omega & T_{z_p}\sin\omega + T_{x_p}\cos\omega & 1 \end{bmatrix}$$

($\bar{x}_i$, $\bar{y}_i$, $\bar{z}_i$, 1) transformed coordinates are projected on (xvy) viewing plane, which corresponding plane is ZOY in the reference coordinate system, after the digital model points are transformed using T_G matrix.

$$[x_i \quad y_i \quad 0 \quad 1] = [\bar{x}_i \quad \bar{y}_i \quad \bar{z}_i \quad 1] \begin{bmatrix} 1/\bar{z}_i & 0 & 0 & 0 \\ 0 & 1/\bar{z}_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lines within the model network are sequentially processed, and $1/\bar{z}_i$ ratio is updated corresponding to each line, in order to carry out their computations.

The hidden zones within the represented relief segment are established using successive profile comparing method, when the points projected on the viewing plane having a $[R]_{ij}$ matrix shape are at our disposal. Data of the first time, belonging to R_{ij} define (P_1) profile geometrically making the first horizon or the first minimum visibility line; the $[R]_{ij}$ matrix analysis is carried out from South to North. The points disposed on the second R_{ij} line, which succession gives the new (P_2) profile as against (P_1) profile are analysed, using the procedure given in the Figure b2 diagram. (P_1) profile is considered devided into $(I_1, I_2 \ldots . I_n)$ intervals marked by its segment ends.

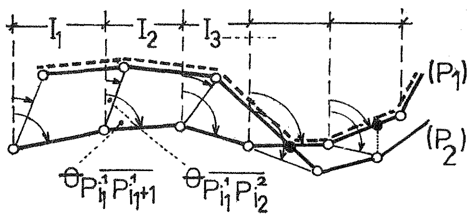


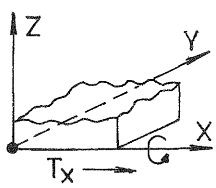
Figure 2

The successive positions of (P_2) points within each interval are compared with the limit position given by (P_1) profile segment defining the interval to establish visibility. A function,which values represent the orientation angle differences, such as:

$$F(\theta) = (\theta_{\overline{P^1_{i_1}P^1_{i_1+1}}} - \theta_{\overline{P^1_{i_1}P^2_{i_2}}}) \qquad (9)$$

is used in the above mentioned comparison: The function negative values show (P_2) profile hidden points replaced by new points given by segment intersections or being interpolated linearly; together with (P_1) points, they modify its initial location line. Thus, the next minimum visibility limit indicated in Figure 2 by broken lines is obtained. After analysing a profile pair, this computation sequence is repeated until $[R]_{ij}$ matrix is finished. The procedure results mentioned above are introduced into FO and FV files structured as a matrix corresponding to the profiles traced horizontally and vertically, using a plotter. FO file is generated line by line and FV file is obtained by simultaneous row generations.

The oblique perspective representations are made, using RELIEF-D and S programmes. The representation in the oblique perspective projection deviated towards to right is implemented by RELIEF-D programme processing the digital model in a left Cartesian system (Figure 3). Two transformation combinations result in a T_G transformation matrix, i.e. a translation in x axis direction followed by a rotation around the same axis.



$$\overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (I-1)T_x & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega & \sin\omega & 0 \\ 0 & -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}^{T_G}$$

Figure 3

(in which: h = digital model square network interval; $T_x = h(\sin\alpha / \cos\alpha)$; $\alpha = 30^o \ldots \ldots 45^o$; I=1,N;N= number of lines within the network.) Digital model network is transformed line by line. Line translation size increases depending on I index values progressively. After transformation, coordinates resulted are projected on ZOX ($\bar{x} = X$; $\bar{y} = 0$; $\bar{z} = Z$) plane. Visibility test and outputs necessary in plottings are made, using the above mentioned procedure. The second oblique perspective representation, deviated towards the left is implemented by RELIEF-S programme processing the digital model. The same procedure, transformations, as well as, computations an used as when RELIEF-D programme is employed, but with a right Cartesian system. Programmes are written in FORTRAN language, and ARISTO plotter is used to obtain outputs.

Considering the high 3D representation capacity to describe the relief configuration at the beginning, they have been applied in levelling plotting checkings, using I_3SRPR_3 programme system.

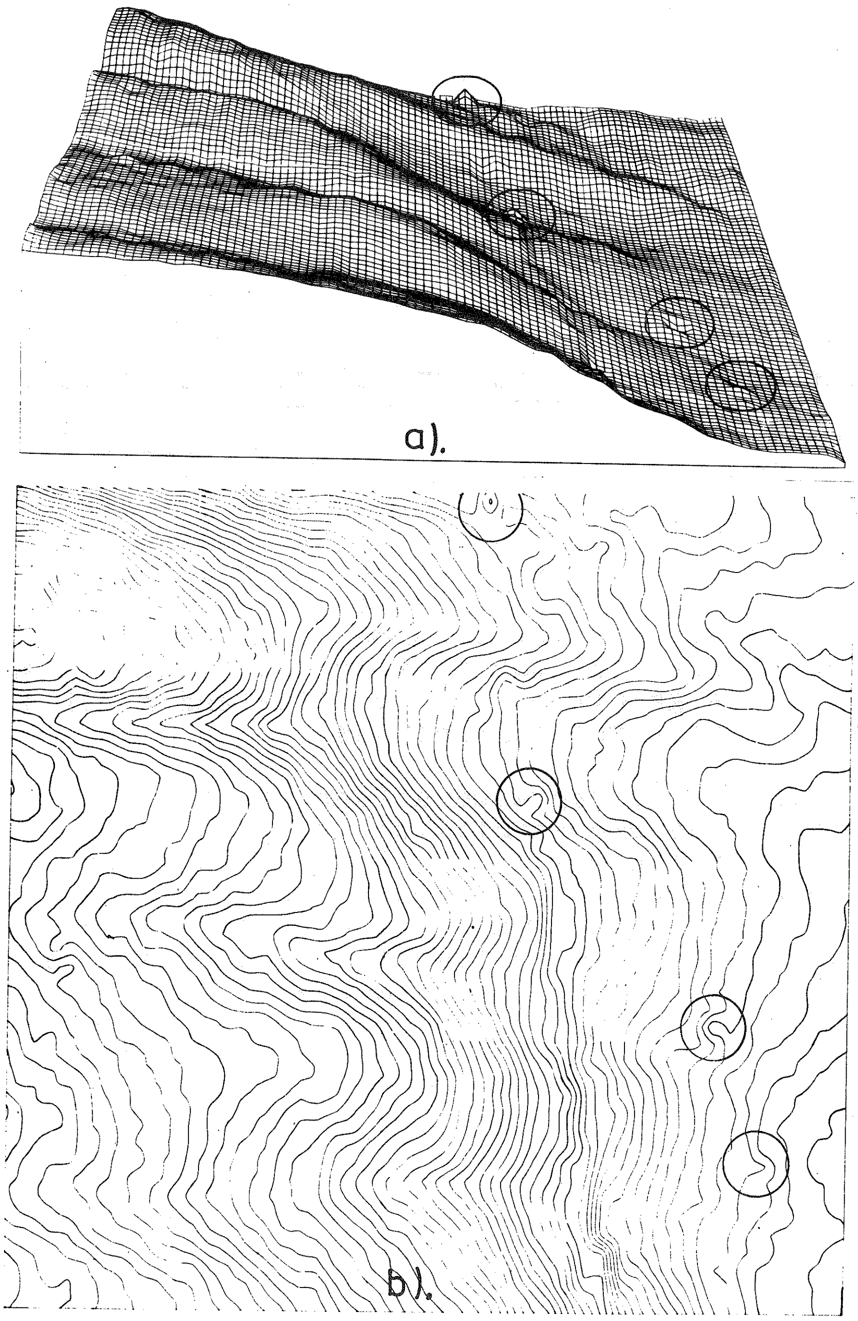
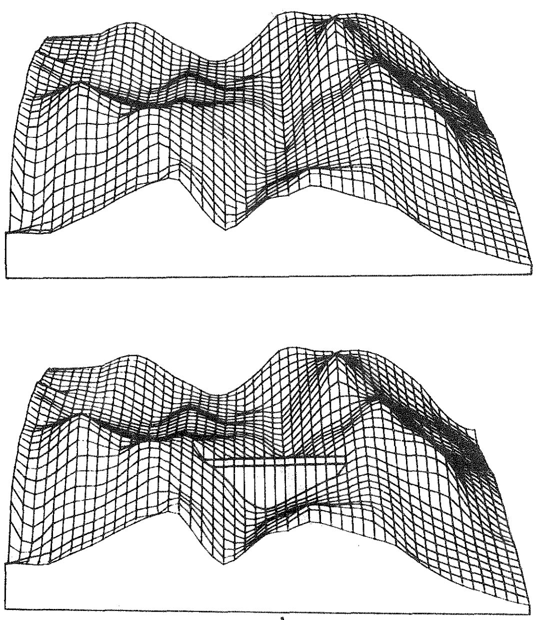
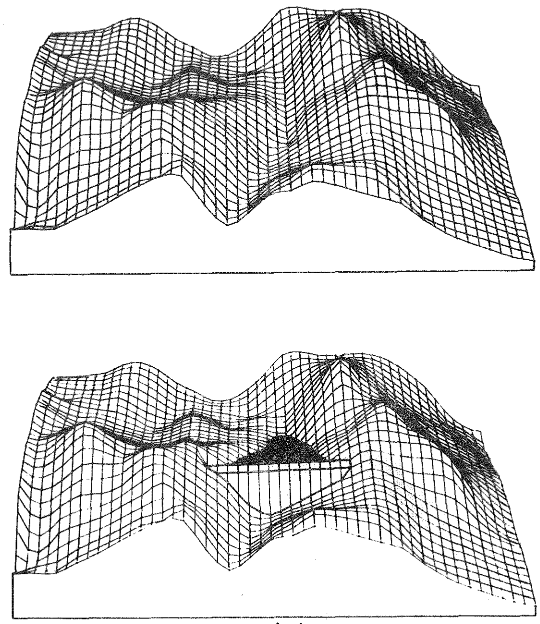


Figure 4 3D central perspective representation compared with an contour lines overlay generated by I_3SRPR_3 programme system.

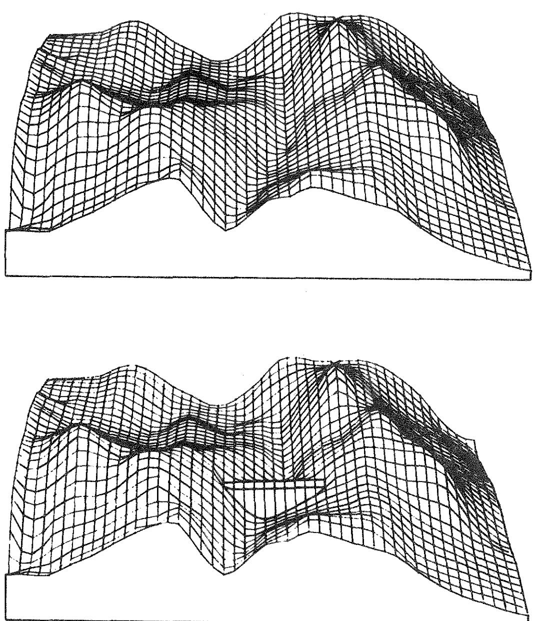
Comparing the overlay derived by digital plotting with the perspective representation (Figures 4 a,b), we can establish the correctness of curves traced depending on relief shapes to be found on the ground. At the same time, the verosimility of the computation procedures implemented by programmes establishing contour line traces and shapes can be obtained.



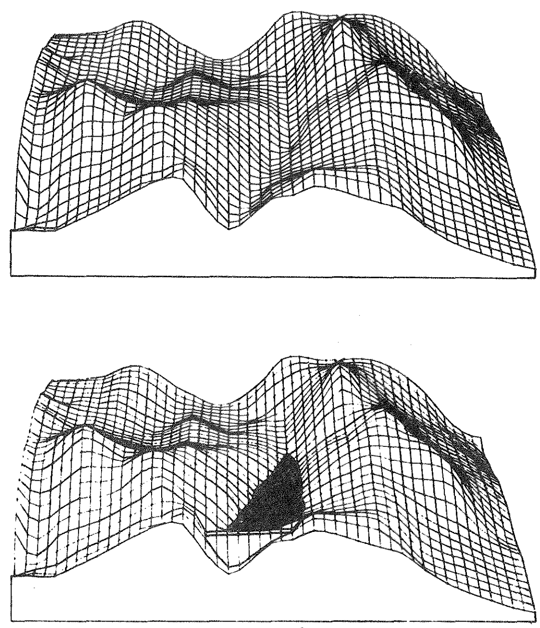
a).



b).



c).



d).

Figure 5 Investigate dam locations.

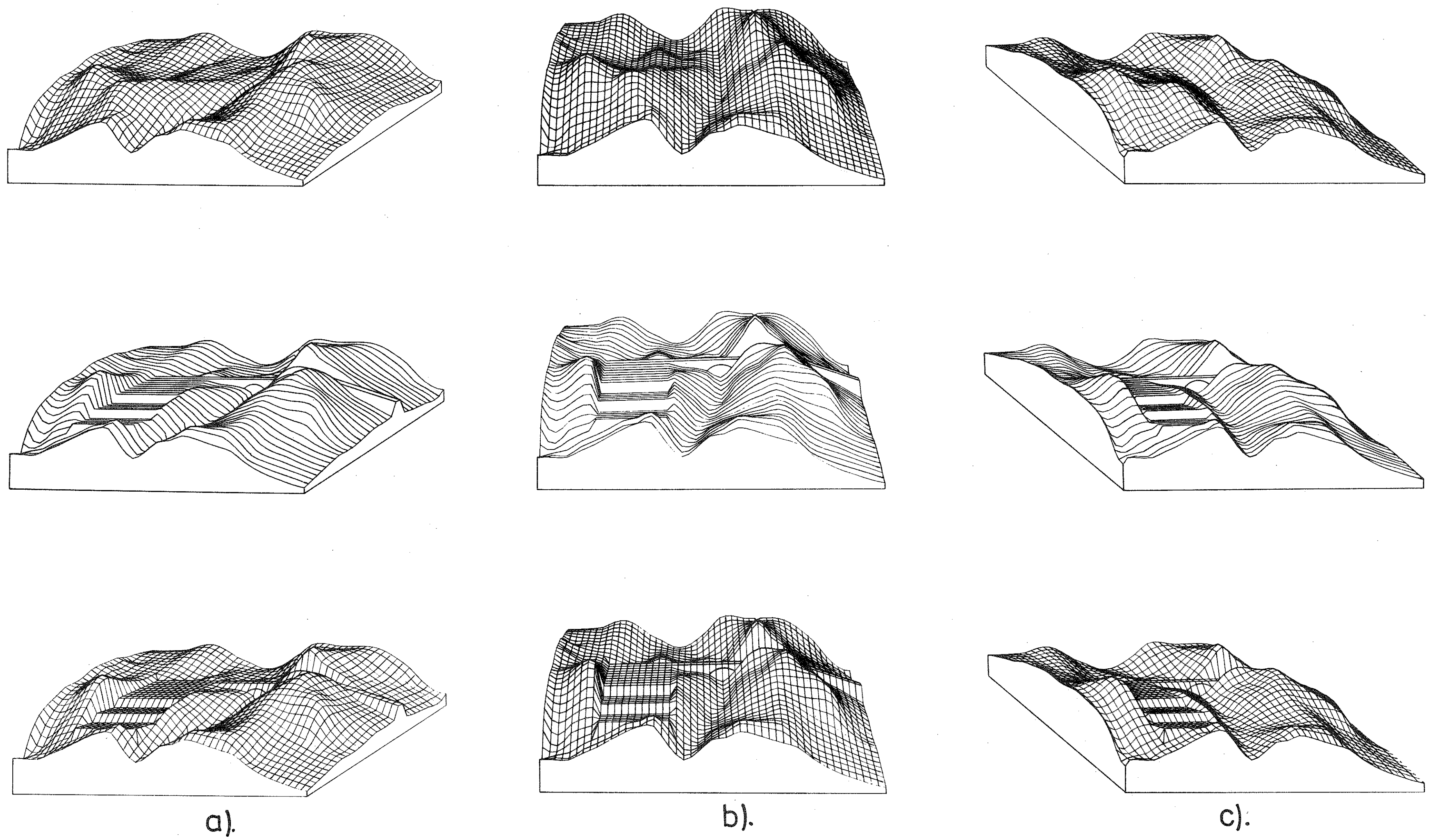


Figure 6 Open-pit mining visualisation.

Both the representation fidelity and comprehensibility of the programmes simulating microforms within digital model network are well evaluated,introducing some erroneous elevation values considering the contour interval values. Subroutines implementing designed elevation values as inputs for the digital model have been introduced in the 3D representation programmes, in order to investigate dam locations. After the designed elevations have been placed in planimetric positions corresponding to various designing versions, representations showing the designed dam and,eventually, the respective storage lake are generated (Figures 5 a,b,c,d). The same subroutines are used in open-pit mining visualisation (Figures 6 a,b,c).

Conclusions

3D representations have proved to be a very useful checking tool. Used in digital levelling plotting, they are a real help in evaluating both the traced curve qualities and computation programme comprehensibility. They can be also used to estimate overlay qualities derived by various methods in practical use. Just one more mention,they are a very efficient way to visualise engineering designs, to detect the eventual designing errors, and to study the impact among designed buildings and the environs.In the near future,we have an intention to make 3D representation tests, in our desire to evaluate generalisation procedures and DEM data collecting checking, as well.

RÉSUMÉ: On présente une série de programmes pour construire des représentations tridimentionnelles du terrain. La superficie des terrains représentés est définie initialement par des modèles numerique de relief générés sous la forme de quadrillage, en utilisant la méthode de collocation ou les fonctions "spline" bicubiques. La construction des représentations se réalise en trois étapes:l'expression des points du modèle numerique en coordonnées homogènes, par option la transformation en projection perspective centrale ou oblique et la détermination des parties visibles de la superficie. Dans l'étape de détermination de la visibilité on applique un test basé sur la comparaison de l'angle d'orientation. Les représentations sont utilisées pour la vérification du filage des courbes de niveau pour l'étude de l'emplacement des constructions hydrotechniques et la visualisation des projets des exploitations minières à ciel ouvert. Les applications mentionnées sont exemplifiées par des planches réalisées au système de dessin automate ARISTOMAT.

REFERENCES

Dubayah,R.O., Dozier, J. 1986 Orthographic Terrain Views Using Data Derived from Digital Elevation Models. P.E.R.S., vol.52, 4, 509-518.

Newman,W.M., Sproul, R.F. 1979 Principles of Interactive Computer Graphics, New York, McGraw-Hill.