AN ALTERNATIVE MATHEMATICAL MODEL TO THE COLLINEARITY EQUATION USING STRAIGHT FEATURES

A. M. G. Tommaselli
Department of Cartography
IPEA/UNESP - C.P. 957
19100 - Presidente Prudente-SP.
Brasil

J. B. Lugnani
Department of Geosciences
CPGCG/UFPr - C.P.19010
81540 - Curitiba-PR.
Brasil

ISPRS Commission III

ABSTRACT

The aim of this paper is to suggest a mathematical model in order to establish a funtional relationship between straight features in object and image space, without necessity of point to point correpondence.

This mathematical model is based on the equivalence between definition parameters from the planes determined by each straight feature - either in the image and in the object space - and the Perspective Center (P.C.).

It is presented the development of the methodology aiming the application of this mathematical model to the resection and analytical stereomodel formation problems.

The obtained practical results, using simulate data, are presented and discussed, showing that the "equivalent planes mathematical model" works succesfully.

Content

1. INTRODUCTION

A great problem in mapping is control points implantation and maintenance. These points are located in block or model region, in order to produce a suitable geometrical configuration. In some cases it is impossible to select natural points in these regions and targetting of ground control is a high-cost alternative. Besides, these points are short-lived, although they lead to better accuracy in the photogrammetric process.

In this context, a great innovation has been the use of digital entites such as straight features, as control (Masry (1980) , Lugnani (1980)). In cities, spatial features, mainly straight features, are plentiful and their utilization will allow the establishment of permanent ground control.

Collinearity equation is the most useful model in conventional photogrammetry, but it needs point to point correspondence. Developed method (Lugnani (1980), Souza(1982)) using spatial features use collinearity equation combined with constraints in an iterative process wich interpolates points in object-space, belonging to spatial feature, correspondent to observed point in image space.

2. MATHEMATICAL MODEL

The aim of this paper is to suggest a mathematical model, in order to establish a functional relationship between straight features in object and image space, without the need of point to point correspondence.

Let E_1 a straight feature in object space. This spatial entity may be defined by two points whose coordinates are known and considered as a straight line and, therefore, represented by a parametric equation:

$$\begin{aligned} X &= X1 + l.t \\ Y &= Y1 + m.t \\ Z &= Z1 + n.t \end{aligned} \qquad \text{where: } \vec{r} = \begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} X2 - X1 \\ Y2 - Y1 \\ Z2 - Z1 \end{bmatrix} \qquad (2.1)$$

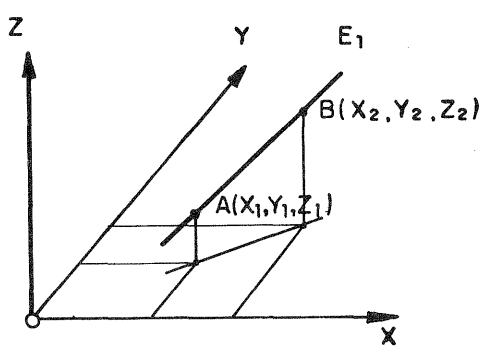

Figure 2.1. Straight Feature in Space.

A straight feature has its image as a straight line if we assume a photograph in which systematic errors were eliminated.

The feature in object and image space establish a plane, passing through the perspective center, whose general equation is:

$$A.X + B.Y + C.Z + D = 0 \qquad (2.2)$$

In figure 2.2. the "non-necessity" of point to point correspondence is ilustrated.

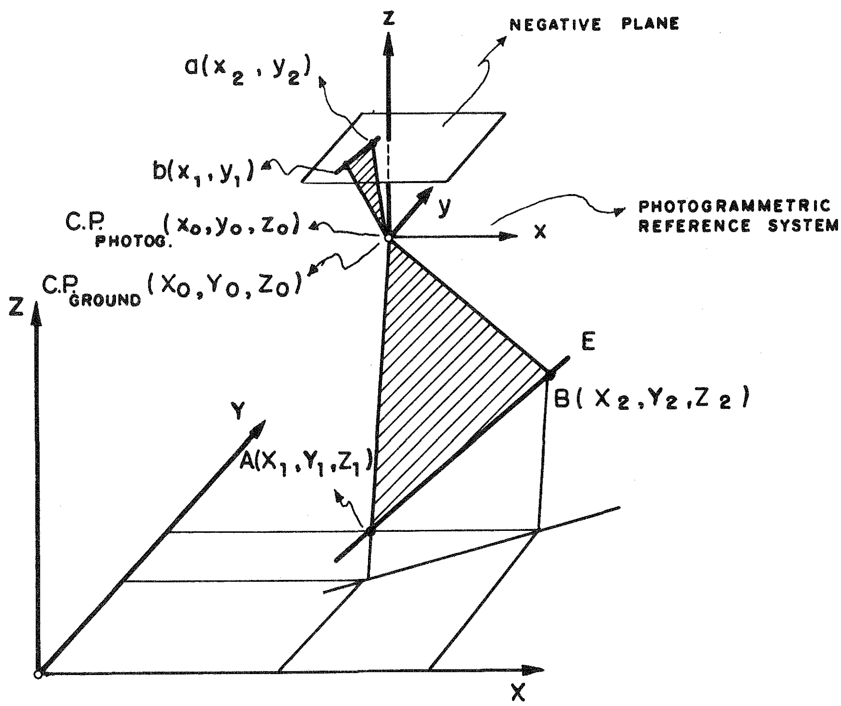

Figure 2.2. The plane established by image and object straight feature.

The plane equation may be obtained from coordinates of three "non-collinear" points by computation of the determinant:

$$\begin{vmatrix} x - xo & y - yo & z - zo \\ x1 - xo & y1 - yo & z1 - zo \\ x2 - xo & y2 - yo & z2 - zo \end{vmatrix} = 0 \qquad (2.3)$$

The plane equation established by the feature and the perspective center in the image space will be obtained in function of two points photocoordinates belonging to this feature and coordinates of the perspective center in the photogrammetric system.

The perspective center is the origin of the photogrammetric system, so:

$$\begin{bmatrix} xo \\ yo \\ zo \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2.4)$$

Replacing in (2.3) we get:

$$\begin{vmatrix} x & y & z \\ x1 & y1 & f \\ x2 & y2 & f \end{vmatrix} = 0 \qquad (2.5)$$

Where x1,y1, x2, y2 are photocoordinates of two points belonging to the feature, and f is the camera constant (z coordinate of the two points).

Then:

$$(f.y1 - f.y2)x + (f.x2-f.x1)y + (x1.y2-x2.y1) = 0 \qquad (2.6)$$

In order to simplify the mathematical model, let:

$$\begin{aligned} A &= f.y1 - f.y2 \\ B &= f.x2 - f.x1 \\ C &= x1.y2- x2.y1 \end{aligned} \qquad (2.7)$$

These quantities are denominated "pseudo-observations", obtained by grouping the photocoordinates and the camera constant, according to (2.7).

Now (2.6) becomes:

$$Ax + By + Cz = 0 \qquad (2.8)$$

Using the three-dimensional similarity transformation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \lambda'.M. \begin{bmatrix} X - Xo \\ Y - Yo \\ Z - Zo \end{bmatrix} \qquad (2.9)$$

Where M = M(k).M(ϕ)M(w) and Xo, Yo, Zo are the ground coordinates of the perspective center.

Appling (2.9) into (2.8) and after some algebric manipulations, we get:

$$\underbrace{\lambda'(A.m11+Bm21+Cm31)}_{A1}X + \underbrace{\lambda'(Am12+Bm22+Cm32)}_{A2}Y +$$

$$+\underbrace{\lambda'(Am13+Bm23+Cm33)}_{A3}Z - \lambda'[(Am11+Bm21+Cm31)Xo +$$

$$+ \underbrace{(Am12+Bm22+Cm32)Yo + (Am13+Bm23+Cm33)Zo]}_{A4} = 0 \qquad (2.10)$$

Now, let us establish the plane equation by elements of object space.

$$\begin{vmatrix} X - Xo & Y - Yo & Z - Zo \\ X1 - Xo & Y1 - Yo & Z1 - Zo \\ X2 - Xo & Y2 - Yo & Z2 - Zo \end{vmatrix} = 0 \qquad (2.11)$$

After some algebric manipulation and appling (2.1) we get:

$$\underbrace{[n(Y1-Yo) - m(Z1-Zo)]}_{A2}X + \underbrace{[l(Z1-Zo) - n(X1-Xo)]}_{B2}Y +$$

$$+ \underbrace{[m(X1-Xo) - l(Y1-Yo)]}_{C2}Z + [(mZ1-nY1)Xo +$$

$$+ (nX1-lZ1)Yo + (lY1-mX1)Zo] = 0 \qquad (2.12)$$

The equations (2.10) and (2.12) describes the same plane in diferent spaces. Thus, the coeficients will be multiples.

Appling this concept we get four equations:

$$A.m11+B.m21+C.m31- \lambda.n(Y1-Yo)+ \lambda.m(Z1-Zo) = 0$$

$$A.m12+B.m22+C.m32- \lambda.l(Z1-Zo)+ \lambda.n(X1-Xo) = 0$$

$$A.m13+B.m23+C.m33- \lambda.m(X1-Xo)+ \lambda.l(Y1-Yo) = 0$$

$$-(A.m11+B.m21+C.m31)Xo-(A.m12+B.m22+C.m32)Yo -$$

$$(A.m13+B.m23+C.m33)Zo- \lambda(m.Z1-n.Y1)Xo- \lambda(n.X1-l.Z1)Yo- \lambda.$$

$$(l.Y1-m.X1)Zo = 0 \qquad (2.13)$$

However, the fourth equation is a linear combination of the others. Therefore we eliminate this last equation of the model

$$\begin{aligned} &A.m11+B.m21+C.m31- \lambda.n(Y1-Yo)+ \lambda.m(Z1-Zo) = 0 \\ &A.m12+B.m22+C.m32- \lambda.l(Z1-Zo)+ \lambda.n(X1-Xo) = 0 \\ &A.m13+B.m23+C.m33- \lambda.m(X1-Xo)+ \lambda.l(Y1-Yo) = 0 \end{aligned} \qquad (2.14)$$

Now, let F an anti-symmetric matrix:

$$F = \begin{bmatrix} 0 & n & -m \\ -n & 0 & l \\ m & -l & 0 \end{bmatrix} \qquad (2.15)$$

Equations (2.14) may be written, using matricial notation:

$$\{[\ A\ B\ C\ ].M^{T}\} + \lambda.F. \begin{bmatrix} X1 - Xo \\ Y1 - Yo \\ Z1 - Zo \end{bmatrix} = 0 \qquad (2.16)$$

After some matricial manipulations we get the final form of the named "equivalent planes" mathematical model.

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = -\lambda.M.F. \begin{bmatrix} X1 - Xo \\ Y1 - Yo \\ Z1 - Zo \end{bmatrix} \quad (2.17)$$

3. APPLICATION OF THE "EQUIVALENT-PLANES MODEL" IN PHOTOGRAMMETRIC PROBLEMS.

3.1. Space Resection

Space resection is the most basic problem in photogrammetry and requires the determination of six parameters of exterior orientation (κ, ϕ, ω, Xo, Yo, Zo).

The mathematical model introduced in this paper is explicit, or:

$$La = F(Xa) \quad (3.1)$$

If we have more pseudo-observations than parameters we can apply the least squares method in order to get a unique solution for the unknowns.

This well known solution is given, by:

$$Xa = X0+X \quad \text{were} \quad X = -(A^T PA)^{-1} (A^T PL) \quad (3.2)$$

$$A = \left.\frac{\partial F}{\partial Xa}\right|_{X0}$$

X0 = vector of aproximated parameters
L = L0 - Lb L0 = F(X0)
Lb = vector of pseudo-observations

The vector of ajusted parameters is:

$$Xa^T = [\kappa, \phi, \omega, Xo, Yo, Zo, \lambda, X1, Y1, Z1, l, m, n \ldots, \lambda^i, X_1^i, Y_1^i, Z_1^i, l^i, m^i, n^i]$$

The design matrix (A) is given by the partial derivatives of the mathematical model with respect to the parameters.

$$\frac{\partial}{\partial k} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = (-\lambda) \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} M.F. \begin{bmatrix} X1 - Xo \\ Y1 - Yo \\ Z1 - Zo \end{bmatrix}$$

$$\frac{\partial}{\partial \phi} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = (-\lambda) \begin{bmatrix} 0 & 0 & -\cos k \\ 0 & 0 & \text{sen} k \\ \cos k & -\text{sen} k & 0 \end{bmatrix} .M.F. \begin{bmatrix} X1 - Xo \\ Y1 - Yo \\ Z1 - Zo \end{bmatrix}$$

$$\frac{\partial}{\partial w}\begin{bmatrix}A\\B\\C\end{bmatrix} = (-\lambda).M.\begin{bmatrix}0&0&0\\0&0&1\\0&-1&0\end{bmatrix}.F.\begin{bmatrix}X1-Xo\\Y1-Yo\\Z1-Zo\end{bmatrix}$$

$$\frac{\partial}{\partial Xo}\begin{bmatrix}A\\B\\C\end{bmatrix} = (-\lambda)\ M.\begin{bmatrix}0\\n\\-m\end{bmatrix} \qquad \frac{\partial}{\partial Y1}\begin{bmatrix}A\\B\\C\end{bmatrix} = \lambda.M.\begin{bmatrix}-n\\0\\1\end{bmatrix}$$

$$\frac{\partial}{\partial Yo}\begin{bmatrix}A\\B\\C\end{bmatrix} = (-\lambda)\ M\begin{bmatrix}-n\\0\\1\end{bmatrix} \qquad \frac{\partial}{\partial Z1}\begin{bmatrix}A\\B\\C\end{bmatrix} = \lambda.M.\begin{bmatrix}m\\-1\\0\end{bmatrix}$$

$$\frac{\partial}{\partial Zo}\begin{bmatrix}A\\B\\C\end{bmatrix} = (-\lambda)\ M\begin{bmatrix}m\\-1\\0\end{bmatrix} \qquad \frac{\partial}{\partial l}\begin{bmatrix}A\\B\\C\end{bmatrix} = -\lambda.M.\begin{bmatrix}0\\Z1-Zo\\-Y1+Yo\end{bmatrix}$$

$$\frac{\partial}{\partial \lambda}\begin{bmatrix}A\\B\\C\end{bmatrix} = -M.F.\begin{bmatrix}X1-Xo\\Y1-Yo\\Z1-Yo\end{bmatrix} \qquad \frac{\partial}{\partial m}\begin{bmatrix}A\\B\\C\end{bmatrix}\ -\lambda.M\begin{bmatrix}-Z1-Zo\\0\\X1-Xo\end{bmatrix}$$

$$\frac{\partial}{\partial X1}\begin{bmatrix}A\\B\\C\end{bmatrix} = \lambda\ .\ M\begin{bmatrix}0\\n\\-m\end{bmatrix} \qquad \frac{\partial}{\partial n}\begin{bmatrix}A\\B\\C\end{bmatrix} = -\lambda.M\begin{bmatrix}Y1-Yo\\-X1+Xo\\0\end{bmatrix}$$

The elements of the weight matrix (P) will be defined using propagation of covariances. We assume that the photocoordinates have the some variance and no correlation.

So, the variance-covariance matrix of the "pseudo- observations" will be:

$$\Sigma Y = \begin{bmatrix}f^2.\sigma x^2 + f^2.\sigma x^2 & 0 & -f.x2.\sigma x^2-f.xl.\sigma x^2\\ 0 & f^2.\sigma x^2+f^2.\sigma x^2 & -f.y2.\sigma x^2-f.yl.\sigma x^2\\ -f.x2.\sigma x^2-f.xl.\sigma x^2 & -f.y2.\sigma x^2-f.yl.\sigma x^2 & (y2^2+x2^2+y1^2+x1^2)\sigma x^2\end{bmatrix} \qquad (3.3)$$

Thus, the weight matrix will be:

$$P = \sigma_o^2.\ (\ \Sigma Y)^{-1} \qquad (3.4)$$

Where σ_o^2 is the variance factor.

The estimation of aproximated parameters may be perfomed by conventional methods, except to parameter λ, which represents a scale(proportionateness) factor. This parameter can be evaluated by means of:

$$\lambda^\circ = - \frac{(x1.y2 - x2y1)}{[m^\circ(x1^\circ - X^\circ_o) - 1^\circ(Y1 - Y^\circ_o)]} \quad (3.5)$$

This parameter (λ) needs to be evaluated for each feature in a single photo.

3.2. Combining "Equivalent-Planes" with collinearity mathematical Model

In urban areas there is great amount of straight features and single points. Thus the method will be more efficient if it utilizes both features and single points. For this, is necessary to introduce more equations in the original mathematical model, corresponding to the collinearity equation:

$$La = F(Xa)$$
$$L'a = G(X'a) \quad (3.6)$$

The vector of ajusted parameters becomes:

$$Xa''^T = [\kappa, \phi, \omega, Xo, Yo, Zo, \lambda, X1, Y1, Z1, l, m, n, \ldots n^i, X, Y, Z, \ldots X^n, Y^n, Z^n]$$

Similarly:

$$Lb''^T = [A', B', C', A'', B'', C'', \ldots A^i, B^i, C^i, x^1, y^1, \ldots x^m, y^m]$$

In order to obtain the design matrix A, we introduce the partial derivatives of the adicional mathematical model (collinearity) with respect to the parameters of exterior orientation and the ground coordinates of the single points.

3.3. Weighted Constraints:

In this paper we use weighted constraints to introduce ground control information in normal matrix and therefore, eliminate rank deficience. The final solution, using constraints is:

$$X = -(A^T PA + C^T PcC)^{-1}(A^T PL + C^T PcLc) \quad (3.7)$$

where: C, Pc, Lc are matricial elements corresponding to constraints. By this method we can establish fixed values for a group of parameters, e.g; parameters of straight features or ground coordinates of single points.

In problems of space resection we usually know the ground coordinates of a single points and equation of straight features. The unknowns will be the six parameters of exterior orientation and the scale factor for each feature. This factor is an aditional unknown because the definition parameters of the planes in object and image space are multiples.

3.4. Determining "Straight Features" by Photogrammetric Intersection of Planes

The greatest goal of the model presented in this paper is the possibility of determination of straight features equations by photogrammetric intersection, as showed in figure 2.4.

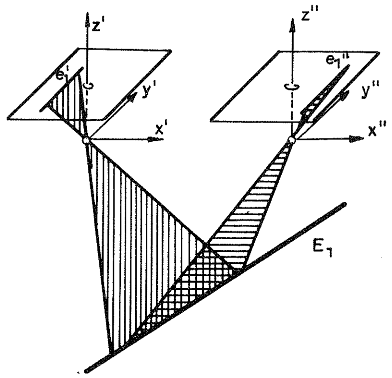

Figure 3.4. "Equivalent - Planes" Intersection

In case of two photo intersection, for example we get:

$$Xa^T=[\kappa,\phi,\omega,Xo,Yo,Zo,\kappa'',\phi'',\omega'',Xo'',Yo'',Zo''\ \lambda 1,\lambda 2,X1,Y1,Z1,l,m,n,\ldots$$

$$\ldots\lambda_1^i,\lambda_2^i,X_1^i,Y_1^i,Z_1^i,l^i,m^i,n^i,\ X,Y,Z,\ldots X^n,Y^n,Z^n]$$

In this case each feature introduces eight unknowns. Two scale factors are necessary because each photo establishes another equivalent plane (figure 3.4).

Constraints will be applied in order to eliminate rank deficience. When parameters of features (X1,Y1,Z1,l,m,n) were known then λ1 and λ2 stay as unknowns. Otherwise, when features parameters are unknowns and the aim is to compute them by equivalent-planes intersection, two absolute constraints, at least, are necessary (λ1(or λ2) and X1 (or Y1, Z1).

The observed values are:

$$Lb^T=[A1,B1,C1,\ldots Aj,Bj,Cj,\quad x1,y1,x2,y2,\ldots xn,yn$$

$$A1'',B1'',C1'',\ldots Aj'',Bj'',Cj'',\quad x1'',y1'',\ldots,xn'',yn'']$$

4. FICTICIOUS DATA TESTS.

In order to check the "equivalent planes" mathematical model a group of photocoordinates in two phoots were simulated. It was supposed that photographs were obtained in 1:8.000 with a 150 mm camera. From this photocoordinates and ground coordinates, features equations were computed and used in practical tests. In order to compute solution by least squares methods, FORTRAN programs were written and used in these tests.

In table 4.1. results in space resection problems are presented . The obtained exterior orientation parameters in eight diferents situations are showed in comparison with actual values. For each case is presented the redundancy number(S) and the χ^2 computed.

In table 4.2. errors in exterior orientation parameters for two photo intersection case are presented as well as S and χ^2 computed.

TABLE 4.1

SPACE RESECTION TRUE ERRORS (Xa-Xr)

PAR	TRUE VALUE	I	II	III	IV
K	0	-0,0000036	-0,0003257	0,0000221	-0,0003439
ø	-0,0174532	-0,0000037	0,0000096	0,0001613	0,0000906
W	0,0174532	0,0000320	0,0009285	0,0005146	0,0003969
X_o	920	0,0077	-0,0520	0,1447	0,0568
Y_o	920	-0,0537	-1,3713	-0,6339	-0,4869
Z_o	1216	-0,0030	0,0040	-0,0330	0,0680
S		8	10	6	3
X^2		7,06	4,06	1,38	2,56

PAR	V	VI	VII	VIII
K	-0,0003081	-0,0002220	-0,0004562	0,0002662
ø	0,0000498	-0,0003334	0,00053289	-0,0003943
W	0,0005780	0,0004289	0,0014751	0,0006458
X_o	0,0100	-0,3712	0,2676	-0,5254
Y_o	-0,7001	-0,5812	-2,2074	-0,7681
Z_o	-0,0470	-0,2770	0,2820	-0,1020
S	2	2	2	2
X^2	1,62	0,18	0,64	0,70

TABLE 4.2

ERRORS IN ORIENTATION PARAMETERS USING FEATURES AND SINGLE POINTS.

PAR	TRUE VALUE				
K	0	-0,0000320	0,0000043	0,0000570	0,0000066
ø	-0,0174532	-0,0000332	-0,0000869	-0,0002029	0,0000848
W	0,0174532	-0,0000570	0,0000015	0,0006949	0,00000440
X_o	920	0,0214	0,0726	-0,0212	0,0715
Y_o	920	0,1199	-0,0013	-0,8807	-0,0038
Z_o	1216	-0,0170	0,0760	-0,2280	0,0810
K"	-0,0349065	-0,0000172	-0,0000399	-0,0000728	-0,00003711
ø"	0,0174532	0,0000937	0,0001341	0,0003621	0,00012429
W"	0,0349065	0,0000014	0,0000153	-0,0004232	0,0000108
X_o"	1656	0,1280	0,2040	0,4330	0,1790
Y_o"	920	0,0217	0,0156	0,5077	0,0243
Z_o"	1216	-0,0640	-0,0560	-0,2890	-0,0620
S		3	6	12	12
X^2		0,85	6,48	4,52	7,19

5. CONCLUSION

Analysing the obtained results we may conclude that "equivalent planes" mathematical model works succesfully. The point to point correspondence is not necessary when using this model. It is showed for the case of space resection that quality of exterior orientation parameter is strongly dependent of the number, and geometrical configuration of the features. Better results will be obtained using more features, non parallel, and with great dimension. Features equation were determined by photogrammetric intersection of planes with good results, but there isn't redundancy in this intersection (number of equations (8) equal to number of pseudo-observation (6) plus number of absolute constraints (2)). Combining this model with collinearity we reached high eficience. It is recommended to study in more detail the geometrical characteristics of the presented model and its application in other problem like phototriangulation, etc.

6. REFERENCES

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