APPLICATION OF THE BAYESIAN INFERENCE IN THE DEFORMATION ANALYSIS BY CLOSE RANGE PHOTOGRAMMETRY

YEU Bock-Mo
YOO Hwan-Hee
Department of Civil Engineering
University of Yonsei
Seoul, Korea

YANG In-Tae
Department of Civil Engineering
University of Kang Weon
Chuncheon, Korea

Commision V

I. ABSTRACT

Three dimensional deformation analysis by close range photogrammetry is usually done by applying the similarity transformation to the definition of the common datum and by applying the sampling theory to the detection of unstable points.

In this paper, iterative reweighted similarity transformation method is used for a more accurate datum definition . The Bayesian inference is used in the detection of unstable points to get less sensitive and more realistic results than those acquired by the sampling theory.

II. INTRODUCTION

In industrial field , close range photogrammetry has been applied to the deformation analysis of structure . The development of on-line and real-time recording systems takes the advantages of application to close range photogrammetry in industry. Within the aerospace and the automobile industry there is increasing usage of photogrammetry for the periodic inspection of tooling. Photogrammetry offers some significant advantages over the conventional gauging techniques (Fraser (1986,1988), Borutta and Peipe (1986), Schewe (1987)).

Because the reliability of deformation analysis by means of close range photogrammetry depends on the accuracy of three dimensional coordinates of object points, the accuracy of three dimensional coordinates is very important.

Current authors are studying to improve accuracy of three dimensional coordinates (El-Hakim and Faig (1980), Fraser (1984,1987), Förstner (1985), Julia (1986), Gruen (1985), Veress and Huang (1987)) .

In the analysis of deformation, the first step is to determine the common datum by the reference points between each epochs. The second step is to identify the significant deformation by statistical techniques.

III. DATUM DEFINITION

All sets of object points are devided into sets of stable points and unstable points during all epochs. A coordinate transformation is operated by the points assumed as stable points to take same datum between epochs.

There exists datum defect between epochs because of configuration defect, the possible existence of systematic errors or outlying observations in each epochs. Iterative reweighted similarity transformation is used to define the common datum.

$$\hat{X} = S \hat{X}_1 \qquad (1)$$

and

$$Q\hat{x} = S \, Q\hat{x}_1 \, S^T \qquad (2)$$

with

$$S = (I - G (G^T W G)^{-1} G^T W) \qquad (3)$$

In equation (3), W is a weight matrix, if all points have same impotance in the definition of datum, then W = I.

But, the diagonal elements of the weight matrix W change according to the each point, if the reliability of stable points are different.

IV. UNSTABLE POINTS DETECTION

Bayesian inference method can be less sensitive and more realistic than the sampling theory in the detection of displaced points. This method which uses the central F - distribution may be simply used in the detection of unstable points.

For some time the Bayesian inference approach was considered subjective because it required introduction of prior information. In the last two decades, however, the use of vague or noninformative priors has been shown to give results equivalent to the sampling theory.

Riesmeier (1984) and Koch (1984) indicated that the Bayesian inference approach was more advantageous than the inequality in the hypothesis testing.

In the Gauss - Markof model to estimate unknown parameters,

$$AX = E (L), \quad E (L) = \sigma^2 I \qquad (4)$$

where, X is random vector, σ^2 is random variable. If L vector is given, the probability density P (X, σ | L) of X and σ is found by using the Bayes theorem.

$$P (X, \sigma | L) \propto P (X, \sigma) \, P (L | X, \sigma) \qquad (5)$$

In equation (5), P (X, σ | L) is a posteriori probability density proportional to a priori probability density, P (X, σ) and a posteriori probability density of the vectors of unknown parameters has multivariate t-distribution.

$$X \sim t(\hat{X} , \hat{\sigma}^2 (A^T A)^{-1} , n - u) \qquad (6)$$

If X vector multiply by random matrix H , HX has also multivariate t-distribution.

$$HX \sim t(H\hat{X} , \hat{\sigma}^2 H(A^T A)^{-1} H^T , n - u) \qquad (7)$$

where, $\det (H (A^T A)^{-1} H^T) \neq 0$.

By means of the multivariate t-distribution,(6) or (7) , any statistical inference concerned with the unknown parameters or linear transformations of the parameters may be performed . If the parameter space is restricted by the inequality constraints H X > W , the probability associated with this restricted space is given by,

$$P(HX > W \mid L) = \int_B P(X \mid L) \, dX \quad , \quad B = \{ X : HX > W \} \qquad (8)$$

The changed form of the eq. (8) into the hypothesis testing by inequality is ,

$$P(HX > W \mid L) > 1 - \alpha \qquad (9)$$

In eq. (7) , the second term of the right has central F-distribution . Therefore it is identical with the hypothesis testing of the sampling theory .

$$P(HX \in B \mid L) = 1 - \alpha \qquad (10)$$

where,

$$B=\{HX:(H\hat{X}-HX)^T (H(A^T A)^{-1} H^T)^{-1} (H\hat{X}-HX)/(r \hat{\sigma}^2) \sim F_{r,n-u;1-\alpha} \} \qquad (11)$$

From eq. (9), carrying out hypothesis testing by inequality gives ,

$$P_T (HX \in A) = \begin{cases} (1- \int_0^{T_r} F(r,n-u)dF)^{-1} \left(\int_0^{T_\ell} F(r,n-u)dF - \int_0^{T_r} F(r,n-u)dF \right) & , \quad T_r < T_\ell \\ 0 & , \quad T_r > T_\ell \end{cases} \qquad (12)$$

where,

$$T_r = (H\hat{X}-W_r)^T (H(A^T A)^{-1} H^T)^{-1} (H\hat{X}-W_r)/(r \hat{\sigma}^2)$$
$$T_\ell = (H\hat{X}-W)^T (H(A^T A)^{-1} H^T)^{-1} (H\hat{X}-W)/(r \hat{\sigma}^2)$$

The region T_r has to be chosen and this choice is arbitrary value depending on the allowable displacement of the stable points.

V. EXPERIMENTS AND RESULTS

(1) Data acquisition

In order to examine the three dimensional movements of the objects by photogrammetry , 12 fixed points were distributed to the edges and 16 unstable points within the objects .

Table 1 shows the simulated displacement of each unstable points . The displacements are occurred along the x-axis direction in Pt.1 , Pt.2 , Pt.3 , and Pt.4 , along y-axis direction in Pt.5 , Pt.6 , Pt.7 , and Pt.8 , along z-axis direction in Pt.9 , Pt.10 , pt.11 , and Pt.12 , and along the three dimensional directions in Pt.13 , Pt.14 , Pt.15 , and Pt.16 .

The three dimensional coordinates of these points between epochs were computed by bundle adjustment with additional parameters, using photo coordinates obtained by convergent imaging configuration at imaging distance 2.5 m from objects.

Although the biases are reduced by systematic error correction, the varience of the least square estimator of unknown parameters tend to increase. The application of least square method should be considered carefully in applying self calibration. Therefore, in this study, biased estimator by generalized ridge estimation was applied to achieve 7.3 μm root mean square error of real position by photo scale.

(2) Datum definition

P value in eq. (13) was applied to 2, 1.5, 1 as a weight condition, when being used iterative reweighted similarity transformation method to define common datum.

$$\Sigma|d|^{P} \Rightarrow \min. \qquad 1 \leq P \leq 2 \qquad (13)$$

Table 2, 3, 4 show the computed displacements in each point obtained by using the weight conditions $\Sigma|d|^2 \Rightarrow$ min., $\Sigma|d|^{1.5} \Rightarrow$ min., $\Sigma|d| \Rightarrow$ min., respectively. RMSE equals to 0.216 mm when weight condition is $\Sigma|d|^2 \Rightarrow$ min., 0.160 mm when $\Sigma|d|^{1.5} \Rightarrow$ min., and 0.125 mm when $\Sigma|d| \Rightarrow$ min.. The accuracy of weight condition $\Sigma|d| \Rightarrow$ min. is better than the other cases. In the common datum defintion, iterative reweighted similarity transformation appling weight condition $\Sigma|d| \Rightarrow$ min. is less influenced on systematic error or outlying observations than the weight conditions $\Sigma|d|^2 \Rightarrow$ min., $\Sigma|d|^{1.5} \Rightarrow$ min., and the accuracy of displacement can be improved.

(3) Unstable points detection using Bayesian inference

Bayesian inference which is not sensitive to geometric accuracy of object points was used in order to detect the unstable points. The displacement computed by the weight condition $\Sigma|d| \Rightarrow$ min., was used in detecting the unstable points. In the detection of unstable points by the Bayesian inference, the tolerent displacements of the fixed points between epochs were regarded as twice the amount of position errors of

3-D coordinates. The 3-D coordinates position error, σ_P = 7.3 μm by photo scale, was computed through the process of data acquisition.

Table 5 shows error ellipsoid (α = 0.05) of object points and the unstable points detected by the Bayesian inference.

Although Pt.1 to Pt.16 had been displaced, some of these points were detected as fixed points under the consideration of the magnitude of displacement and confidence region. In table 5, Pt.1 and Pt.2 are unstable points of which displacement is 0.3 mm and 0.4 mm, respectively, along the X-axis. They are regarded as the fixed points by the Bayesian inference method and these displacements exist within the error ellipsoid (α = 0.05). Thus Pt.1 and Pt.2 have to be considered as fixed points.

Also, Pt.5 is regarded as fixed point although displacement is 0.3 mm along the Y-axis. Consequently, it is possible to detect more than 0.4 mm displacemment along X-Y plane.

Since Pt.9, Pt.10 and Pt.11 of which displacements are 0.3 mm, 0.4 mm, and 0.5 mm, respectively, along the Z-axis, they were detected as fixed points. Detectable displacement along Z-axis was larger than in X-Y planes. The detection of unstable points by Bayesian inference has a little difficulty in anticipating the allowable displacement at fixed points.

Since the varience of coordinates in photogrammetry can be accurately obtained, the Bayesian inference can be applied to the detection of unstable points. Comparing the Bayesian inference method and the confidence region method for unstable point detection, it was found that Bayesian inference was less sensitive and the more realistic in unstable point detection when carring out deformation analysis by photogrammetry.

VI. CONCLUSIONS

Common datum definition and unstable point detection are analyzed through the deformation analysis in the close range photogrammetry. In using iterative reweighted similarity transformation to define common datum, weight condition $\sum|d| \Rightarrow$ min.produced better results than weight condition $\sum|d|^2 \Rightarrow$ min.or $\sum|d|^{1.5} \Rightarrow$ min.. This is because the influence of systematic errors, outlying observations and configuration defects is effectively minimized when weight condition is done by the $\sum|d| \Rightarrow$ min..

In detection of unstable points by photogrammetry, Bayesian inference can be effectively applied because its results are more realistic and less sensitive than those obtained by the sampling theory.

References

Borutta, H. and Peipe, J., 1986, Deformation Analysis of Three-Dimensional photogrammetric point Fields, Int. Arch. Photo., Vol.26, No.5, pp. 165-174.

El-Hakim,S.F. and Faig,W., 1980, The General Bundle Adjustment Triangulation (GEBAT) system-theory and Applications, 14th Congress of ISPRS, pp. 296-307.

Förstner,W.,1985, The Reliablity of Block Triangulation, P.E. & R.S., Vol.51, No.6 ,pp.1137-1149.

Fraser, C.S., 1986, Network Design Considerations for Non-topograpic Photogrammetry, P.E. & R.S., Vol.50, No.8, pp. 1115- 1126.

Fraser, C.S., 1984, Microwave Antenna Measurement, P.E. & R.S., Vol. 52, No. 10, pp. 1627-1635.

Fraser, C.S., 1987, Limiting Error Propagation in Network Design , P.E. & R.S., Vol. 53, No. 5, pp. 487-493.

Fraser, C.S., 1988, Periodic Inspection of Industrial Tooling by Photogrammetry, P.E. & R.S., Vol. 54, No.2, pp. 211-216.

Gruen, A.W., 1985, Algorithmic Aspects in on-line Triangulation, P.E. & R.S., Vol. 51, No. 4, pp. 419-436.

Julia, J.E., 1986, Development with the COBLO Block Adjustment Program, Photo. Record.,12 (68), pp. 219-226.

Koch, K.R., 1984, Statistical Tests for Detecting Crustal Movements Using Bayesian Inference, NOAA Technical Report NOS NGS 29.

Riesmeier, K., 1984, Test Von Ungleichungshypothesen in linearen Modellen mit Bayes-Verfahren, Deutsche Geodatische Kommission, C. München.

Schewe, H., 1987, Automatic Photogrammetric Car-Body Measurement, Proceeding of the 41st photogrammetric week at Stuttgart University, pp 47-55.

Veress, S.A. and Huang, Y., 1987, A Method for Improving the Efficiency of the Sequential Estimation Procedure in Photogrammetry, P.E. & R.S., Vol. 53, No.6, pp. 613-616.

Table 1 Simulated Displacement (mm)

Pt	dx	dy	dz	ds
1	0.300	0.000	0.000	0.300
2	0.400	0.000	0.000	0.400
3	0.500	0.000	0.000	0.500
4	0.600	0.000	0.000	0.600
5	0.000	0.300	0.000	0.300
6	0.000	0.400	0.000	0.400
7	0.000	0.500	0.000	0.500
8	0.000	0.600	0.000	0.600
9	0.000	0.000	0.300	0.300
10	0.000	0.000	0.400	0.400
11	0.000	0.000	0.500	0.500
12	0.000	0.000	0.600	0.600
13	0.300	0.300	0.300	0.520
14	0.400	0.400	0.400	0.693
15	0.500	0.500	0.500	0.866
16	0.600	0.600	0.600	1.039

Table 2 The displacement computed by weight condition $\sum |d|^2 \Rightarrow$ min.

mm

Pt.	computed displacements				discrepancy
	dx	dy	dz	ds	
1	0.093	0.009	0.008	0.094	0.206
2	0.516	0.190	0.290	0.622	0.222
3	0.634	0.123	0.296	0.710	0.210
4	0.411	0.097	0.012	0.422	0.178
5	0.120	0.010	0.252	0.279	0.021
6	0.137	0.253	0.164	0.331	0.069
7	0.166	0.322	0.127	0.384	0.116
8	0.178	0.376	0.028	0.417	0.183
9	0.013	0.070	0.070	0.099	0.201
10	0.080	0.109	0.068	0.151	0.249
11	0.115	0.098	0.248	0.290	0.210
12	0.141	0.066	0.208	0.260	0.340
13	0.232	0.040	0.189	0.301	0.219
14	0.309	0.229	0.017	0.385	0.305
15	0.422	0.480	0.129	0.652	0.218
16	0.514	0.656	0.250	0.870	0.170
RMSE					0.216

Table 3 The displacement computed by weight condition $\sum |d|^{1.5} \Rightarrow$ min.

mm

Pt.	computed displacements				discrepancy
	dx	dy	dz	ds	
1	0.165	0.261	0.023	0.310	0.010
2	0.294	0.012	0.092	0.308	0.092
3	0.375	0.031	0.194	0.423	0.077
4	0.421	0.123	0.186	0.476	0.124
5	0.121	0.016	0.233	0.263	0.037
6	0.121	0.245	0.073	0.283	0.117
7	0.167	0.285	0.010	0.331	0.169
8	0.157	0.567	0.175	0.614	0.014
9	0.033	0.115	0.103	0.158	0.142
10	0.053	0.091	0.077	0.131	0.269
11	0.097	0.100	0.322	0.351	0.149
12	0.128	0.080	0.441	0.466	0.134
13	0.225	0.014	0.234	0.325	0.195
14	0.314	0.213	0.035	0.381	0.309
15	0.449	0.496	0.172	0.691	0.179
16	0.545	0.676	0.334	0.930	0.110
RMSE					0.160

Table 4 The displacement computed by weight condition $\Sigma|d| \Rightarrow$ min.

mm

Pt.	computed displacements dx	dy	dz	ds	discrepancy
1	0.217	0.263	0.021	0.341	0.041
2	0.348	0.011	0.149	0.378	0.022
3	0.449	0.031	0.289	0.535	0.035
4	0.470	0.128	0.302	0.573	0.027
5	0.079	0.019	0.238	0.252	0.048
6	0.167	0.304	0.025	0.348	0.052
7	0.126	0.359	0.094	0.391	0.109
8	0.098	0.563	0.301	0.646	0.046
9	0.036	0.112	0.124	0.171	0.129
10	0.013	0.090	0.090	0.128	0.272
11	0.038	0.102	0.379	0.394	0.116
12	0.074	0.084	0.641	0.651	0.051
13	0.286	0.011	0.237	0.372	0.148
14	0.362	0.210	0.069	0.424	0.266
15	0.518	0.497	0.230	0.754	0.116
16	0.617	0.677	0.431	1.012	0.028
RMSE					0.125

Table 5. Unstable Points Detection

Object Pt.No.	error ellipsoid (α = 0.05) σ_{max} (mm)	σ_{min} (mm)	σ_z (mm)	Bayesian inference Significant Deformation at α =0.05
1	0.367	0.283	0.509	No
2	0.360	0.378	0.510	No
3	0.342	0.303	0.512	Yes
4	0.369	0.269	0.500	Yes
5	0.349	0.290	0.498	No
6	0.336	0.270	0.481	Yes
7	0.358	0.291	0.516	Yes
8	0.353	0.293	0.509	Yes
9	0.347	0.284	0.511	No
10	0.345	0.288	0.522	No
11	0.338	0.282	0.484	No
12	0.352	0.291	0.509	Yes
13	0.337	0.288	0.502	Yes
14	0.342	0.284	0.505	Yes
15	0.351	0.288	0.504	Yes
16	0.336	0.287	0.515	Yes