Phototriangulation in Multi-Media Photogrammetry

Rüdiger Kotowski
INPHO GmbH
Smaragdweg 1, 7000 Stuttgart 1
Federal Republic of Germany
Commission V

1. Introduction

Central projection is the basic functional model for phototriangulation in analytical photogrammetry. A pencil of rays is considered as the geometrical equivalent of the photographic imaging process. The path of light through the lens system is neglected and replaced by a projection centre. Furthermore, object point, projection centre and corresponding image point are supposed to lie on a straight line which in fact is true for light propagating in a homogeneous and isotropic medium. For the majority of photogrammetric applications these assumptions are at least a sufficient approximation for the photographic imaging process. Deviations from ideal central projection are usually compensated by additional parameters describing the deviations effects in the image plane (e.g. lens distortion, atmospheric refraction, film unflatness and deformation).

Such a procedure becomes questionable, if light refracting surfaces between object and projection centre cause refracted rays. The collinearity condition may be fundamentally disturbed. A more detailed functional model for this photogrammetric field of activity, often called two-media or generally multi-media photogrammetry, should be applied. After the pioneer work of Zaar and Rinner in 1948 dealing with the optical and geometrical theory of two-media photogrammetry, the orientation problem in multi-media photogrammetry has often been investigated (e.g. Konecny et al. 1970, Höhle 1971, Okamoto et al. 1972, Girndt 1973). But, numerical procedures for general phototriangulation which are the topic of this paper have hardly been dicussed (Okamoto 1982, 1984; Kotowski 1987). Basic idea for an appropriate functional model is the integration of a modified ray tracing procedure into bundle triangulation. Besides image orientation and object point coordinates, parameters defining position and shape of refracting surfaces as well as relative refracting indices may be simultaneously introduced as unknowns in a phototriangulation. The model is not restricted to refracting planes or spheres as long as homologous rays can be identified and can also be adapted to geodetic direction bundles measured with a theodolite.

2. Ray tracing through refracting surfaces

The term ray tracing originates from geometrical optics and denotes a method to determine the path of light rays through an optical system with elementary laws of geometrical optics. The ray tracing procedure is closely related to the detection of image aberrations with reference to an ideal optical system. According to Born / Wolf 1964, p.143 an optical system images perfectly, if every curve in object space has a corresponding curve in image space which is geometrically similar. For the following brief description of the ray tracing procedure in geometrical optics it will be assumed that the parameters of an optical system are given. Then, a ray of light starting from an arbitrarily chosen point in object space with a given initial propagation direction is traced through the optical system by successive application of the laws of refraction and reflection. Starting from the same object point this procedure is repeated, but with different initial propagation directions. If perfectly imaged, all rays in image space should intersect in a common image point. It becomes obvious that the ray tracing procedure is able to detect deviations from stigmatic imaging (e.g. spherical aberration, coma, astigmatism, curvature of the image field and chromatic aberration). Furthermore, the application of the above mentioned procedure to a group of object points lying in a plane should lead to image points lying in a plane, as well. Considering the intersection of two planes, points on a straight line in object space should consequently lie on a straight line in image space, if perfectly imaged. A deviation from this condition is called optical distortion and may also be detected by ray tracing (Herzberger 1958, Cox 1964).

Since the ray tracing formula are well suited for programming on computers, this method has become a most helpful tool in designing imaging optical instruments perfectly, as far as possible. Refracting power, shape and relative position of each component of an optical system have to be chosen in a way that all rays propagating from a point in object space to the corresponding image point have equal optical path length according to the principle of Fermat. Inspired by this basic idea from geometrical optics, the problem of ray tracing will now be treated in a modified manner to develop an algorithm for the purposes of multi-media photogrammetry. The result will be a compact module which can be integrated into the concept of bundle triangulation.

According to fig. 1 a ray of light starts from a point P_0 (X_0, Y_0, Z_0) and propagates to P_{p+1} (X_{p+1}, Y_{p+1}, Z_{p+1}) through p refracting surfaces. The p refracting surfaces are separating homogeneous, isotropic media with the refracting indices n_i ($n_i \neq n_{i+1}$).

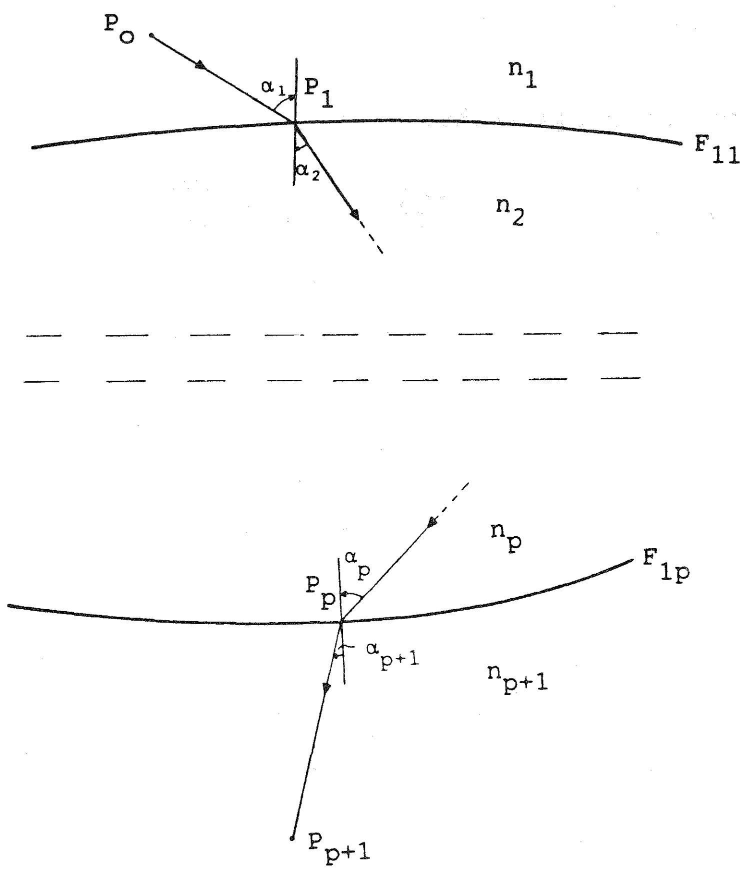

fig. 1 : ray tracing through p refracting surfaces

In general a ray between P_0 and P_{P+1} will be refracted at each refracting surface. Momentarily it will be assumed that the coordinates of P_0 and P_{P+1} are given, as well as the refracting surfaces by their implicit functions $F_{1i} = F_{1i}(X, Y, Z)$, $i = 1,..,p$. All parameters should refer to the same set of cartesian rectangular axes. Asking for the coordinates of the refraction points $P_1, P_2, .., P_P$ the following conditions can be formulated :

1.) Surface equation, each refraction point P_i lies on the corresponding refracting surface F_{1i} :

$$F_{1i}\ (X_i, Y_i, Z_i) = 0 \tag{2.1}$$

$i = 1,2,...,p$

2.) At P_i Snell's law of refraction is fulfilled :

$$F_{2i} = n_i \sin \alpha_i - n_{i+1} \sin \alpha_{i+1} = 0 \tag{2.2a}$$

resp. :

$$F_{2i} = \sqrt{1 - \cos^2 \alpha_i} - r_i \sqrt{1 - \cos^2 \alpha_{i+1}} = 0 \tag{2.2b}$$

with $r_i = n_{i+1} / n_i$

The cosines of α_1 and α_2 can be expressed by the following scalar products :

$$\cos \alpha_i = \frac{\mathbf{N}_i^T \cdot (\mathbf{X}_{i-1} - \mathbf{X}_i)}{|\mathbf{N}_i| \cdot |\mathbf{X}_{i-1} - \mathbf{X}_i|} \tag{2.2c}$$

$$\cos \alpha_{i+1} = \frac{\mathbf{N}_i^T \cdot (\mathbf{X}_i - \mathbf{X}_{i+1})}{|\mathbf{N}_i| \cdot |\mathbf{X}_i - \mathbf{X}_{i+1}|} \tag{2.2d}$$

with the vector components

$$\mathbf{N}_i^T = \left| \left(\frac{\partial F_1}{\partial X}\right)_{P_i} , \left(\frac{\partial F_1}{\partial Y}\right)_{P_i} , \left(\frac{\partial F_1}{\partial Z}\right)_{P_i} \right|$$

$$= \left| (F_{1x})_{P_1} , (F_{1y})_{P_1} , (F_{1z})_{P_1} \right|$$

$$(\mathbf{X}_{i-1} - \mathbf{X}_i)^T = \left| (X_{i-1} - X_i) , (Y_{i-1} - Y_i) , (Z_{i-1} - Z_i) \right|$$

$$(\mathbf{X}_i - \mathbf{X}_{i+1})^T = \left| (X_i - X_{i+1}) , (Y_i - Y_{i+1}) , (Z_i - Z_{i+1}) \right|$$

A substitution of (2.2c) and (2.2d) into (2.2b) leads to:

$$F_{2i} = \sqrt{1 - \frac{[\mathbf{N}_i^T \cdot (\mathbf{X}_{i-1} - \mathbf{X}_i)]^2}{|\mathbf{N}_i|^2 \cdot |\mathbf{X}_{i-1} - \mathbf{X}_i|^2}} - r_i \sqrt{1 - \frac{[\mathbf{N}_i^T \cdot (\mathbf{X}_i - \mathbf{X}_{i+1})]^2}{|\mathbf{N}_i|^2 \cdot |\mathbf{X}_i - \mathbf{X}_{i+1}|^2}} = 0 \tag{2.2e}$$

3.) Coplanarity condition,
the surface normal N_i of F_{1i} in P_i, the incident ray $\overline{P_{i-1},P_i}$ and the refracted ray $\overline{P_i,P_{i+1}}$ lie in a common plane. In a strict sense this coplanarity condition is a component of Snell's law of refraction (Herzberger 1958, p.5), but is explicitly mentioned here to emphasize the basic equations for the ray tracing algorithm :

$$F_{3i} = \begin{vmatrix} X_{i-1} - X_i & Y_{i-1} - Y_i & Z_{i-1} - Z_i \\ X_i - X_{i+1} & Y_i - Y_{i+1} & Z_i - Z_{i+1} \\ (F_1x)_{P_i} & (F_1y)_{P_i} & (F_1z)_{P_i} \end{vmatrix} = 0 \qquad (2.3a)$$

written in matrix notation :

$$(X_{i-1} - X_i)^T \; \overline{N}_{P_i} \; (X_i - X_{i+1}) = 0 \qquad (2.3b)$$

with the scew symmetrical matrix

$$\overline{N}_{P_i} = \begin{bmatrix} 0 & (-F_1z)_{P_i} & (F_1y)_{P_i} \\ (F_1z)_{P_i} & 0 & (-F_1x)_{P_i} \\ (-F_1y)_{P_i} & (F_1x)_{P_i} & 0 \end{bmatrix}$$

According to (2.1 - 2.3), three condition equations can be formulated for each refraction point P_i. By solving the resulting non-linear equation system iteratively - e.g. with the Gauss-Newton-procedure - the coordinates of the refraction points P_i can be determined. Initial values for the unknown coordinates can be derived by calculating the intersection points between the straight line $\overline{P_0,P_{P+1}}$ and the refracting surfaces F_{1i}.

Although the formulation and solution strategy of the ray tracing problem is different compared to geometrical optics, the mathematical setup is identical justifying the transfer of the term ray tracing for the purpose of multi-media photogrammetry. The basic formula (2.1-2.3) for ray tracing in multi-media photogrammetry are more a kind of framework, since the refracting surfaces are given in a general form by the implicit functions $F_{1i}(X,Y,Z)=0$ without assumptions concerning shape and spatial position. Depending on the physical situation suitable functions for the refracting surfaces can be selected (Kotowski 1987) :

* Second order surfaces :

$$F_1 = X^T A X + 2a^T X + a = 0 \qquad (2.4)$$

mit $X^T = (X,Y,Z)$, $a^T = (a_1, a_2, a_3)$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{und } a_{ij} = a_{ji} \quad \text{für} \quad i \neq j$$

Second order surfaces include the case of refracting planes, if the elements of matrix **A** are zero :

$$F_1 = d^T X + d = 0 \tag{2.5}$$

mit $d^T = 2a^T$ und $d = a$

* Undulating surfaces (cf. Okamoto 1984); defined in a suitable wave coordinate system :

$$\bar{Z} = H \sin (k (\bar{X} - \bar{X}_0)) \tag{2.6}$$

with

$k = 2\pi / \lambda$

H - amplitude

λ - wave length

$\bar{X}_0$ - phase shift

3. Integration of ray tracing into bundle triangulation

The conventional concept for bundle triangulation is based on central projection, where the image coordinates are described as a function of the interior and exterior image orientation and the object point coordinates. For the purpose of this paper it will be advantageous to split the collinearity equations into a spatial shift and rotation followed by the projection into the image plane (e.g. Wester-Ebbinghaus 1985) :

spatial shift and rotation :

$$\begin{bmatrix} X^*_{ij} \\ Y^*_{ij} \\ Z^*_{ij} \end{bmatrix} = D_j(\omega_j, \phi_j, \kappa_j) \begin{bmatrix} X_i - X_{0j} \\ Y_i - Y_{0j} \\ Z_i - Z_{0j} \end{bmatrix} \tag{3.1}$$

projection into the image plane :

$$\begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} = - \frac{c_k}{Z^*_{ij}} \begin{bmatrix} X^*_{ij} \\ Y^*_{ij} \end{bmatrix} + \begin{bmatrix} x_{0k} \\ y_{0k} \end{bmatrix} + \begin{bmatrix} dx_k \\ dy_k \end{bmatrix} \tag{3.2}$$

with :

i, j, k - index for points, bundles, cameras

$X_i^T = (X_i, Y_i, Z_i)$	-	object point coordinates
$X_{0j}^T = (X_{0j}, Y_{0j}, Z_{0j})$	-	projection centre coordinates
$D_j = (\omega_j, \phi_j, \kappa_j)$	-	orthogonal rotation matrix to transform the object system parallel to the image coordinate system
c_k	-	camera konstant of camera k
x_{ij}, y_{ij}	-	image point coordinates of object point P_i in image system j

x_{0k}, y_{0k} - principle point coordinates of camera k

dx_k , dy_k - image error functions of camera k

The correction terms dx_k, dy_k are an effective tool to compensate the effects of a wide range of deviations from central projection (e.g. lens distortion, atmospheric refraction, film unflatness and deformation). But, their efficiency for phototriangulation in multi-media photogrammetry is limited to some special configurations with refracting surfaces like a centred spherical window or planes perpendicular to the optical axis; Kotowski 1987. Violations of collinearity caused by light refracting surfaces require the development of a more detailed functional model which is able to define simultaneously the spatial position of the refracted rays. This idea can be realized by integrating the ray tracing algorithm (cf. section 2.) into the standard funtional model for bundle triangulation. With respect to practical applications it will be distinguished whether the position of the light refracting surfaces is invariant to the surveying object or to the camera.

3.1 Object invariant refracting surfaces

Refracting surfaces within a photogrammetric image block which do not change their relative position during the photogrammetric observation period are called object invariant (fig. 2). Configurations of this type arise, if the object is located in closed or inaccessible areas requiring an observation from outside. Practical applications can be found in elementary particle physics in order to calculate the spatial position of bubble chamber tracks (e.g. Bullock 1971) and in hydrodynamics to determine the flow characteristics of liquids (Höhle 1971, Jacobi 1980, Phillips 1981). In general, mapping of the ocean floor in shallow water by means of aerial photogrammetry belong to this category, as well. But, the problem becomes more complicated compared to laboratory conditions caused by a dynamic and usually undulated water surface (Okamoto 1984), by irregularities of the water's refraction index and by limited water penetration due to light absorption and scattering (e.g. Helgeson 1970, Masry et al. 1980, Fryer et al. 1985, Linke 1985).

For object invariant refracting surfaces it is advantageous to parameterize the refracting surfaces in the object coordinate system X,Y,Z. The coordinates $\underline{\bar{X}}_i{}^k$ (with k=1,2,...,p for p refracting surfaces) of the refraction points of each ray between projection centre O_j and object point P_i are calculated by ray tracing (fig. 3). The object point coordinates $\underline{X}_i$ in eq. (3.1) are then replaced by the coordinates $\underline{\bar{X}}_i{}^1$ of the refraction point lying next to the projection centre :

ray tracing in the object coordinate system :

$$\bar{X}_i{}^k = f_S\ (X_i,\ X_{0j},\ A^k,\ a^k,\ a^k,\ r^k) \qquad (3.3)$$

A^k, a^k, a^k - surface parameters of p refracting surfaces

r^k - relative refraction indices

$k = 1,2,\ldots,p$

spatial shift and rotation :

$$\begin{bmatrix} X^*_{ij} \\ Y^*_{ij} \\ Z^*_{ij} \end{bmatrix} = D_j(\omega_j,\phi_j,\kappa_j) \begin{bmatrix} \bar{X}^1_i - X_{0j} \\ \bar{Y}^1_i - Y_{0j} \\ \bar{Z}^1_i - Z_{0j} \end{bmatrix} \qquad (3.4)$$

projection into the image plane :

$$\begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} = - \frac{c_k}{Z^*_{ij}} \begin{bmatrix} X^*_{ij} \\ Y^*_{ij} \end{bmatrix} + \begin{bmatrix} x_{0k} \\ y_{0k} \end{bmatrix} + \begin{bmatrix} dx_k \\ dy_k \end{bmatrix} \qquad (3.2)$$

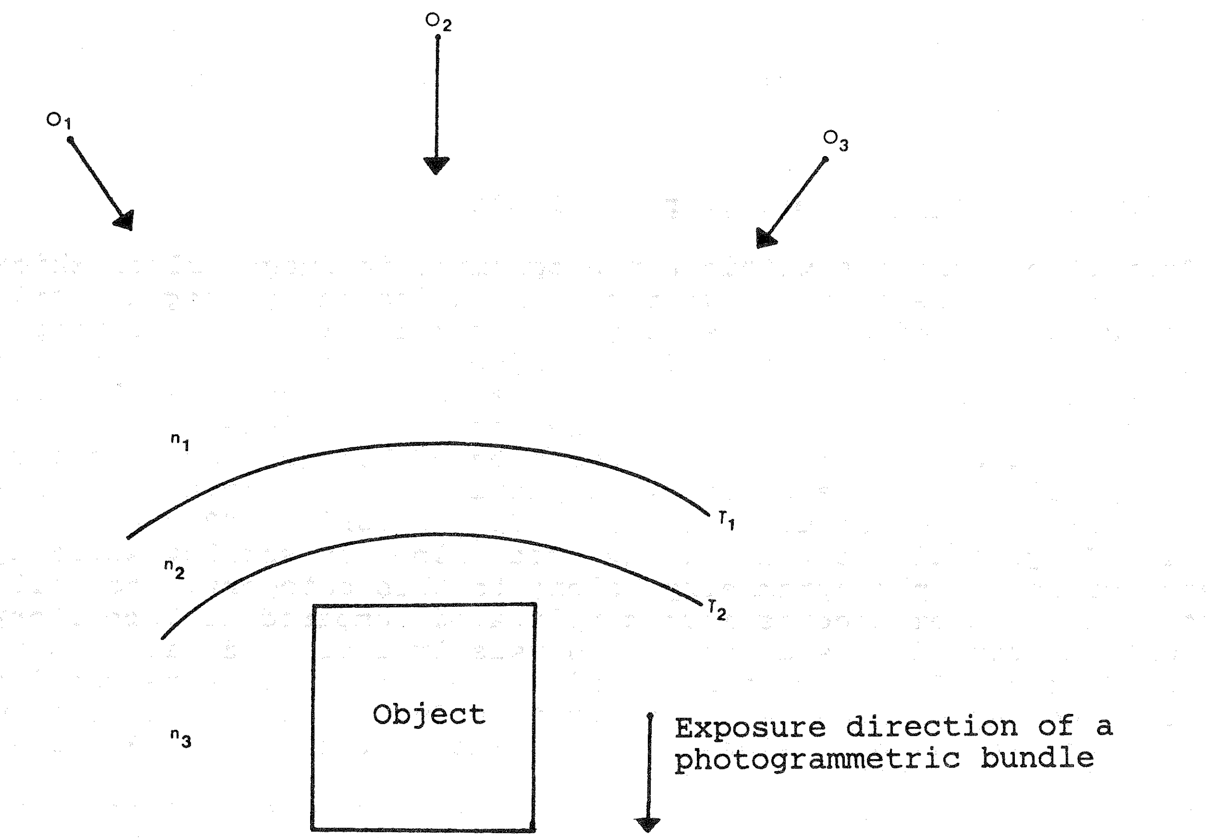

fig. 2 : object invariant refracting surfaces T_1 and T_2 in a formation of bundles O_1 , O_2 , O_3 (ground plan projection)

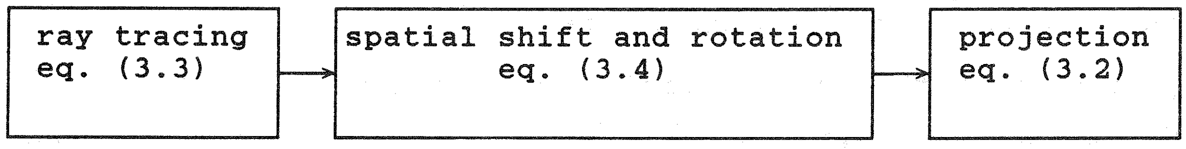

fig. 3 : ray tracing in the superior object coordinate system

3.2 Bundle invariant refracting surfaces

Refracting surfaces which do not change their relative position to each bundle during the observation period are called bundle invariant (fig. 4). These configurations are mainly found for applications in underwater photogrammetry (Newton 1984), if the camera is embedded in a watertight and pressure-proven housing (e.g. Pollio 1971, Torlegård et al. 1974, Baldwin et al. 1982, Turner et al. 1982, Baldwin 1984, Dorrer 1986, Fraser et al. 1986, Kotowski et al. 1988). Bundle invariant ray tracing within bundle triangulation seems also to be an adequate tool to describe the imaging geometry of fish-eye optics (Hellmeier 1983).

In opposition to the object invariant case it is now advantageous to parameterize the refracting surfaces in the local bundle coordinate system X^*, Y^*, Z^*. The coordinates of the refraction points are calculated by ray tracing after the spatial shift and rotation of the superior object coordinate system into the local bundle system (see fig. 5).

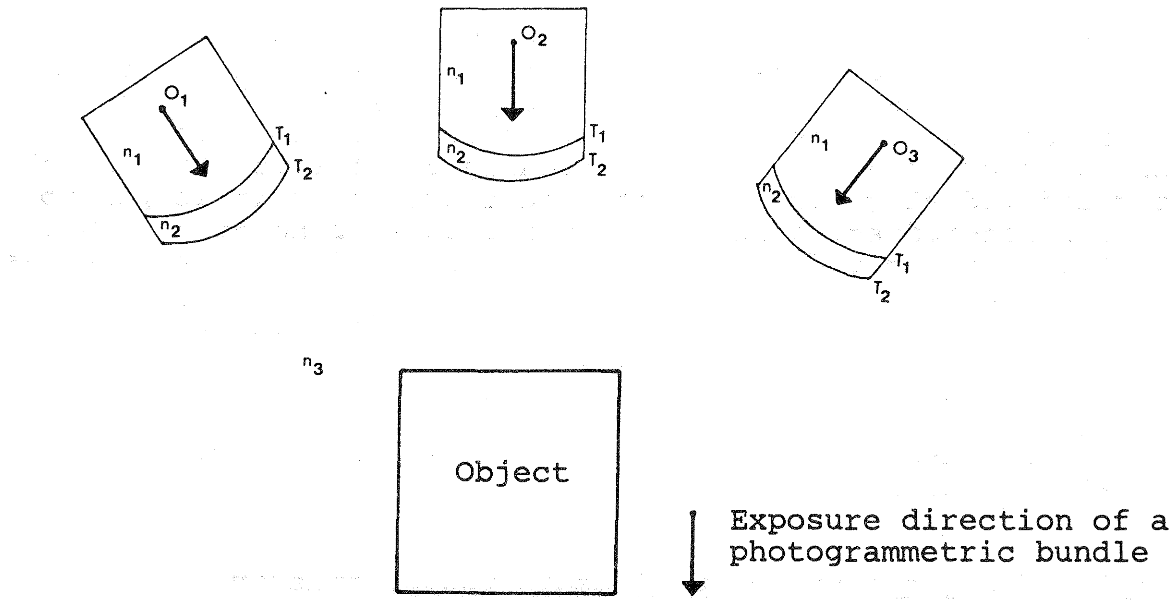

fig. 4 : bundle invariant refracting surfaces T_1 and T_2 in a formation of bundles O_1, O_2, O_3 (ground plan projection)

spatial shift and rotation eq. (3.1)	→	ray tracing eq. (3.5)	→	projection eq. (3.6)

fig. 5 : ray tracing in the local bundle coordinate system

Again the basic collinearity equations (3.1 - 3.2) can be applied, if the coordinates $\underline{X}_{ij}^{1}$ of the refraction point lying next to the projection centre are used instead of the shifted and rotated object point coordinates $\underline{X}^*_{ij}$:

spatial shift and rotation :

$$\begin{bmatrix} X^*_{ij} \\ Y^*_{ij} \\ Z^*_{ij} \end{bmatrix} = D_j(\omega_j, \phi_j, \kappa_j) \begin{bmatrix} X_i - X_{0j} \\ Y_i - Y_{0j} \\ Z_i - Z_{0j} \end{bmatrix} \qquad (3.1)$$

ray tracing in the local bundle coordinate system :

$$\bar{X}_{ij}^{k} = f_S \left(X_{ij}^{*}, \mathbf{A}^k, \mathbf{a}^k, a^k, r^k \right) \tag{3.5}$$

$\mathbf{A}^k$, $\mathbf{a}^k$, a^k - surface parameters of p refracting surfaces
r^k - relative refraction indices
$k = 1,2,\dots,p$

projection into the image plane :

$$\begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} = - \frac{c_k}{\bar{Z}_{ij}^{1}} \begin{bmatrix} \bar{X}_{ij}^{1} \\ \bar{Y}_{ij}^{1} \end{bmatrix} + \begin{bmatrix} x_{0k} \\ y_{0k} \end{bmatrix} + \begin{bmatrix} dx_k \\ dy_k \end{bmatrix} \tag{3.6}$$

The equation system (3.1)-(3.6) describe in general the functional model for phototriangulation in multi-media photogrammetry which allows to determine simultaneously position and shape of light refracting surfaces and relative refraction indices within a least squares adjustment. By switching off the ray tracing algorithm individually for each observed image point it is possible to process non-refracted rays or non-refracted bundles, as well. Thus, the standard model for bundle triangulation is included as a special case.
Although the derivations are performed for photogrammetric bundles, they can also be applied to geodetic direction bundles measured with a theodolite, if the projection into the image plane (eq. 3.2, 3.6) is replaced by projecting the rays onto the planes of the theodolite's graduated circles (e.g. Wester-Ebbinghaus 1985, Kotowski 1987).

3.3 Combined adjustment in multi-media photogrammetry

Image coordinates and measured geodetic direction bundles are usually considered as primary information for bundle triangulation. But, within practical projects various additional information can often be acquired without effort. Analytical and numerical procedures provide a proper basis for simultaneous processing of all available information. Stabilization of the network geometry and increased mutual controllability are essential reasons to recommend combined adjustment on principle. For phototriangulation in multi-media photogrammetry these aspects should be considered, as well. With respect to refracting surfaces and refraction indices the observation net has to be designed that a sufficient determinability for all multi-media parameters is guaranteed. Ill-conditioned geometry can be avoided, if a simultaneous processing of additional observation is provided. A priori information about :
- refraction indices of different media,
- shape of refracting surfaces,
- spatial position of refracting surfaces,
- relative position of different refracting surfaces,
- relative position between refracting surfaces and object points or projection centres

should participate in the adjustment with appropriate weight. Based on the combined adjustment program MOR-S (Wester-Ebbinghaus 1983, Hinsken 1985) such a program for phototriangulation in multi-media photogrammetry has been developped. With this program the presented functional model has been successfully applied in investigations with simulated and real data (Kotowski 1987, Kotowski et al. 1988).

4. Concluding remarks and future aspects

An extended functional model for bundle triangulation is presented. It allows a simultaneous parametrization and estimation of general refracting surfaces, their spatial position and different indices of refraction by implementing a generally formulated ray tracing algorithm into the model for bundle triangulation. The extended model can be applied to photogrammetric bundles and geodetic direction bundles and offers the handling of object invariant and bundle invariant refracting surfaces.

Additional refracting surfaces usually disturb stigmatic imaging and complicate the identification of homologous rays. The development of improved measurement techniques on the basis of ray tracing is recommended to avoid a drop-down of measuring accuracy. Concerning stereo restitution of multi-media images in Analytical Plotters the on-line loop should be extended by integrating a ray tracing algorithm.

References :

BuL - Bildmessung und Luftbildwesen
DGK - Deutsche Geodätische Kommission
IAP - International Archives of Photogrammetry
ISPRS - International Society of Photogrammetry and Remote Sensing
PE - Photogrammetric Engineering
PR - Photogrammetric Record
ZfV - Zeitschrift für Vermessungswesen

Baldwin, R.A.; Newton, I; 1982 : A proposed system for underwater photogrammetry from a manned submersible camera platform. IAP, 24(V/1) : 39-52.

Baldwin, R.A.; 1984 : An underwater photogrammetric measurement system for structural inspection. IAP, 25(A5) : 49-58.

Born, M.; Wolf, E.; 1964 : Principles of Optics. Second revised edition, Pergamon Press Oxford.

Bullock, F.W.; 1971 : The photogrammetry of bubble chamber tracks. PR, 7(38) : 119-134.

Cox, A.; 1964 : A system of optical design. The Focal Press, London.

Dorrer, E.; 1986 : Precision Analytical Underwater Photogrammetry. IAP, 26(5) : 176-184.

Fraser, C.; Fryer, J.G.; 1986 : On the calibration of underwater cameras. PR, 12(67) : 73-85.

Fryer, J.G.; Kniest, H.T.; 1985 : Errors in depth determination caused by waves in throughwater photogrammetry. PR, 11(66) : 745-753.

Girndt,, U.; 1973 : Analytische Behandlung von einigen Grundaufgaben der Zweimedien-Photogrammetrie. Schriften der DGK, Reihe C, Heft 196.

Helgeson, G. A.; 1970: Water depth and distance penetration. PE, 36 (2) : 164-172.

Hellmeier, H.-J.; 1983 : Fisheye-Objektive in der Nahbereichsphotogrammetrie - theoretische und praktische Untersuchungen. Schriften der DGK, Reihe C, Heft 286.
Herzberger, M.; 1958 : Modern Geometrical Optics. Interscience Publishers, Inc., New York.
Hinsken, L.; 1985 : MOR-S : Ein Anwendungsbeispiel für die Sparse-Technik in einem photogrammetrisch-geodätischen Netzausgleichungsprogramm. ZfV 110 : 416-424.
Höhle, J.; 1971 : Zur Theorie und Praxis der Unterwasser-Photogrammetrie. Schriften der DGK, Reihe C, Heft 163.
Jacobi, O.; 1980 : Photogrammetric tracking of a moving particle in running water. IAP, 23(B5) : 368-374.
Konecny, G.; Masry, S.E.; 1970 : New programs for the Analytical Plotter AP-2C. PE, 36(12) : 1269-1276.
Kotowski, R.; 1987 : Zur Berücksichtigung lichtbrechender Flächen im Strahlenbündel. Schriften der DGK, Reihe C, Heft 330.
Kotowski, R.; Meid, A.; Przybilla, H.-J.; Weber, B.; 1988 : Geometrical quality control for inspections in nuclear power stations - a procedure for high precision underwater photogrammetry. Presented paper, Commission V, 16th ISPRS-Congress, Kyoto.
Linke, K.; 1985 : Untersuchungen zur Anwendung der Zweimedien-Photogrammetrie für die Bestimmung von Wassertiefen in Flachwassergebieten. Vermessungstechnik, 33(10) : 346-349.
Masry, S.E.; Mac Ritchie, S.; 1980 : Different considerations in coastal mapping. PE, 46(4) : 521-528.
Newton, I.; 1984 : The current status of underwater photogrammetry. IAP, 25(A5) : 580-589.
Okamoto, A.; Höhle, J.; 1972 : Allgemeines analytisches Orientierungsverfahren in der Zwei- und Mehrmedienphotogrammetrie und seine Erprobung. BuL, 40(2) : 103-106 und 40(3) : 112-119.
Okamoto, A.; 1982 : Wave influences in two-media photogrammetry. Photogrammetric Engineering 37(9) : 955-968.
Okamoto, A.; 1984 : Orientation problems of two-media photographs with curved boundary surfaces. PE, 50(3) : 303-316.
Phillips, J.; 1981 : Ein photogrammetrisches Aufnahmesystem zur Untersuchung dynamischer Vorgänge im Nahbereich. Veröffentlichung des Geodätischen Instituts der Rheinisch Westfälischen-technischen Hochschule Aachen, Heft Nr. 30.
Pollio, J.; 1971 : Underwater mapping with photography and sonar. PE, 37(9) : 955-968.
Rinner, K.; 1948 : Abbildungsgesetz und Orientierungsaufgaben in der Zweimedienphotogrammetrie. Österreichische ZfV, Sonderheft 5, 46 Seiten.
Rinner, K.; 1969 : Problems of two medium photogrammetry. PE, 35(2) : 275-282.
Torlegård, K.; Lundalv, T.L.; 1974 : Underwater analytical systems. PE, 40(3) : 287-293.
Turner, J.; Leatherdale, J.D.; 1982 : Underwater photogrammetry for inspection and maintenance of North Sea oil platforms. IAP, 24(V/2) : 507-515.
Wester-Ebbinghaus, W.; 1983 : Ein photogrammetrisches System für Sonderanwendungen. BuL 51, 118-128.
Wester-Ebbinghaus, W.; 1985 : Bündeltriangulation mit gemeinsamer Ausgleichung photogrammetrischer und geodätischer Beobachtungen. ZfV 110, p. 101-111.
Zaar, K.; 1948 : Zweimedienphotogrammetrie. Österreichische ZfV, Sonderheft 4, 36 Seiten.