Gross Error Detection by Robust Estimation in DLT

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Abstract

When gross errors exist in observations, the localization of the gross errors by the least squares is difficult. Thus it is necessary to use orther estimation methods which allow more correct estimations under the given conditions. The Robust estimation has been introduced in photogrammetry by several authors as an effective way which yields results uninfluenced by the gross errors.
This paper deals with the efficiency of the Robust estimation in DLT.

1. Introduction

In the adjustment of photogrammetric observations by the least squares, gross errors are considered as random errors and distributed evenly to all observations. This results in the reduced reliability of the final results and therefore it is essential that gross errors should be eliminated prior to the final stage of computation.

Direct Linear Transformation (DLT) has been variously applied to the close range photogrammetry using non-metric cameras (Karara, 1974; Brandow et al., 1976).
The advantages of the DLT method are direct noniterative resection and the ability of proceeding without prior interior orientation of the image (Brill, 1987).

The original DLT program was developed by Marzan and Karara (1975), but the accuracy of origianl DLT is lower than that of the simultaneous bundle adjustment. Several modifications for DLT program have been made by Chen(1985). He introduced the data snooping schemes for the gross error elimination, the calculation of orientation parameters, and more iterations in order to improve the accuracy.

The Baarda's data snooping is a sensitive method in the case when the partial redundance range is wide. But the major disadvantage of the data snooping is that only one gross error can be detected in one adjustment.

In recent years, alternative methods to least squares have been introduced in photogrammetry by several authors(Krarup et al., 1980; Benciolini et al., 1982; Klein and Foerstner, 1984).
These robust estimation methods are relatively insensitive to gross errors, and more than one gross error can be detected in the same adjustment.

The objective of this paper is comparative analysis between robust estimation, Danish method and data snooping in DLT.
Robust estimators used in this paper are Huber's descending M-estimator and bisquare estimator.

2. Robust M-estimation

Least squares is an optimal procedure in many cases when the errors in a regression model have a Gaussian distribution or when linear estimates are required (Gauss-Markov Theorem). But least squares is not optimal in many non-Gaussian situations with longer tails. Andrews et al., (1972) clearly demonstrates the inefficiency of least squares for a wide variety of distributions, as compared to robust estimates of location.

Consider the model

$$Y = Ax + e \qquad (1)$$

where y and e are n by 1, A is n by m, and x is m by 1.

A robust estimate for x, $\hat{x}$, minimizes

$$\sum \rho (Vi / s) \qquad (2)$$

where ρ is a robust loss function, Vi is a residual, S is a known or estimated scale parameter. Such an estimate is called a M-estimate.

If we let ψ be equal to derivative of ρ , $\hat{x}$ should satisfy

$$\sum Aij \psi(Vi / S) = 0 \quad \text{for } j = 1, \ldots\ldots m \qquad (3)$$

Equation (3) is a set of nonlinear equations and iterative methods are required.

Three iteration schemes for finding the solution of equation (3), are Newton's method, Huber method, and reweighted least squares method (RLS) (Holland and Welsch, 1977). The RLS method only requires a computation procedure of the weight function W(z), defined as $\psi(z) / z$, and then it is possible to use an existing weighted least squares algorithm.

The reweighted least squares method for obtaining M-estimates is iterative, and therefore requires a starting solution. An ordinary least squares estimate is a sufficiently good starting solution.

After the iterative process is converged, proper weights are determined to all observations. An errorneous data have weights approximately equal to zero and have no influence on the result of the adjustment. These residuals will show the true errors.

3. Comparison analysis in DLT

The gross error detection methods used in this paper are Huber's descending M-estimator, bisquare estimator, Danish method, and Baarda's data snooping.

Huber's Descending M-Estimator (Huber, 1981)

$$W_i = \begin{cases} 1 & |r_i| \leq a \\ (b / r_i) \tanh [b (c - |r_i|) / 2 \} & a < |r_i| \leq c \\ 0 & |r_i| > c \end{cases}$$

where r_i is the normalized residual ($= V_i / \sigma$),
$a = 1.982$, $b = 1.991$, $c = 5$

Bisquare Estimator (Veress and Huang, 1987)

$$W_i = \begin{cases} (1 - \mu^2)^2 & | \mu | < 1 \\ 0 & | \mu | > 1 \end{cases}$$

where $\mu_i = V_i / (K \cdot S)$, $S = \text{median} |V_i|$,
K = turning constant (= 6)

Danish Method (Kubik et al., 1988)

$$W_i = \begin{cases} 1 & |V_i| < 2\sigma \\ \exp [- |V_i|^2 / (2\sigma)] & |V_i| \geq 2\sigma \end{cases}$$

In this paper, the convergence criterion used above three methods, was

$$\sum | W_i(new) - W_i(old) | < 0.02 \sum W_i(new)$$

In the iterative data snooping, a critical value was 3.29.

The data used in this paper is the observations in the thesis of Marzan(1976). The result of this data according to a least squares adjustment is shown in Table 1. These results show that the gross errors are greatly influenced on the computation of the object space coordinates.

Table 2 summarizes the results according to the data snooping with the maximum 5 iterations. The iterative data snooping with total 12 interations eliminated the two gross errors of points 1411 and 1401.

Tables 3, 4 and 5 are the results of Huber's descending M- estimation, bisquare estimation, and Danish method, respectively. The results of Huber's descending M-estimation and bisquare estimation, show that three points 1401, 1411, 1417 are gross errors. In the case of Danish method, only one point 1411 was detected, thus it can be inferred that other gross error influenced the final results.

The RMSE of the object space coordinates according to Huber's descending M-estimation and bisquare estimation were approximately equal to those of iterative data snooping. But the iteration numbers of Huber's descending M-estimation was 8, and bisquare estimation was 10.

In order to analyze the sensitivity of gross error detection methods according to the magnitude of gross errors, a total of 18 points, excluding the two points 1411 and 1401, were used.
And three levels of gross error, 5σ, 10σ and 20σ, were added to the x and y image coordinates of the point 1410. The results are shown in Table 6 and 7.
As shown in the results of a least squares method, the gross error included in x image coordinate greatly influenced the final results in DLT than those in y image coordinate. The iterative data snooping only detected the magnitude of 20σ in x coordinate, and the Danish method detected the magnitude of 20σ in x and y coordinates. But in the case of Huber's descending M-estimation and bisquare estimation, the gross errors of 10σ and 20σ in x and y coordinates were correctly detected, and gross errors of 5σ resulted in decreased weights. Thus the Robust estimation such as Huber's descending M-estimation and bisquare estimation is an effective gross error detection method in DLT.

4. Concluding remarks

The least squares method yield unreliable results if gross errors exist in the observations. The Robust estimation is an effective way which yields the results uninfluenced by the gross errors.
In DLT, the Robust estimation is more effective than Data Snooping and Danish method. However the choice of weight function and its constant value remains to be investigated.

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Table 2 Adjustment by the iterative data snooping

Number of Points = 20 Iter. = 2
Rejected Point No. = 1411

	Left Image					Right Image				
No.	Vx	Vy	Tx	Ty		Vx	Vy	Tx	Ty	
1411	-.557	-.007	5.219	.061	<-RE	.318	.001	5.100	.013	<-RE

Number of Points = 19 Iter. = 5
Rejected Point No. = 1401

	Left Image					Right Image				
No.	Vx	Vy	Tx	Ty		Vx	Vy	Tx	Ty	
1401	-.029	.003	4.778	.561	<-RE	-.001	-.004	.070	.275	<-OK

Number of Points = 18 Iter. = 5
All Points Accepted

	Left Image					Right Image				
No.	Vx	Vy	Tx	Ty		Vx	Vy	Tx	Ty	
1402	.004	-.003	.291	.250	<-OK	-.001	-.002	.066	.207	<-OK
1403	.001	-.004	.109	.302	<-OK	.001	.001	.057	.071	<-OK
1404	-.006	.006	.560	.541	<-OK	-.002	.002	.156	.146	<-OK
1405	.002	-.002	.190	.199	<-OK	.001	-.001	.075	.101	<-OK
1406	.003	-.003	.223	.245	<-OK	.001	.002	.072	.220	<-OK
1407	.004	.003	.288	.248	<-OK	-.001	-.002	.076	.168	<-OK
1408	-.002	.002	.206	.138	<-OK	-.001	-.002	.060	.129	<-OK
1409	-.002	.003	.155	.278	<-OK	.002	.003	.146	.199	<-OK
1410	-.003	.000	.219	.001	<-OK	.000	.001	.032	.055	<-OK
1412	.001	.001	.068	.101	<-OK	.001	.001	.085	.070	<-OK
1413	-.004	-.002	.302	.177	<-OK	.001	-.004	.093	.344	<-OK
1414	-.003	-.001	.218	.094	<-OK	.001	-.002	.038	.178	<-OK
1415	-.001	.001	.054	.086	<-OK	-.001	.001	.050	.081	<-OK
1416	.001	-.000	.060	.029	<-OK	-.001	.003	.057	.204	<-OK
1417	.001	-.002	.124	.163	<-OK	.002	-.005	.145	.436	<-OK
1418	-.002	-.002	.135	.138	<-OK	-.001	.002	.060	.137	<-OK
1419	.000	.001	.013	.063	<-OK	-.003	.003	.247	.212	<-OK
1420	.005	.002	.499	.226	<-OK	.001	.000	.048	.038	<-OK

No.	MX	MY	MZ	POS
1402	-.001	.000	-.005	.005
1403	.000	.000	-.001	.001
1404	.001	.000	.005	.005
1405	.000	.000	-.001	.002
1406	-.001	-.001	-.002	.002
1407	-.001	.000	-.004	.004
1408	.001	.000	.002	.002
1409	.000	-.001	.005	.005
1410	.000	.000	.003	.003
1412	.000	.000	.000	.001
1413	.000	.000	.006	.006
1414	.000	.000	.003	.003
1415	.000	.000	.000	.000
1416	.000	.000	-.002	.002
1417	.000	.001	.000	.001
1418	.000	.000	.001	.001
1419	.001	.001	-.004	.004
1420	.000	.001	-.005	.006

Table 3 Adjustment by Huber's descending M-estimation

	Left Image					Right Image				
No.	Vx	Vy	Wx	Wy		Vx	Vy	Wx	Wy	
1401	-.043	.000	0.00	1.00	<<	-.001	-.004	1.00	0.61	
1402	.003	-.002	1.00	1.00		.000	.001	1.00	1.00	
1403	.001	-.003	1.00	1.00		.001	.002	1.00	1.00	
1404	-.007	.007	0.44	0.39		-.002	.002	1.00	1.00	
1405	.002	-.002	1.00	1.00		.001	-.002	1.00	1.00	
1406	.003	-.003	1.00	1.00		.002	.001	1.00	1.00	
1407	.003	.003	1.00	1.00		.000	-.002	1.00	1.00	
1408	-.002	.001	1.00	1.00		.000	.000	1.00	1.00	
1409	-.002	.003	1.00	1.00		.002	.003	1.00	0.92	
1410	-.003	.000	1.00	1.00		.001	.000	1.00	1.00	
1411	-.665	.000	0.00	1.00	<<	.371	.001	0.00	1.00	<<
1412	.001	.001	1.00	1.00		.001	.000	1.00	1.00	
1413	-.004	-.002	1.00	1.00		.002	-.004	1.00	0.54	
1414	-.002	-.001	1.00	1.00		.001	-.002	1.00	1.00	
1415	-.001	.001	1.00	1.00		-.001	.001	1.00	1.00	
1416	.001	-.001	1.00	1.00		-.001	.002	1.00	1.00	
1417	.002	-.001	1.00	1.00		.002	-.006	1.00	0.00	<<
1418	-.001	-.002	1.00	1.00		-.001	.002	1.00	1.00	
1419	.001	.001	1.00	1.00		-.004	.003	0.75	0.85	
1420	.006	.002	0.57	1.00		.001	.001	1.00	1.00	

No.	MX	MY	MZ	POS
1401	.012	.012	.048	.050
1402	-.001	-.001	-.004	.004
1403	.000	.000	.000	.001
1404	.001	.000	.005	.005
1405	.000	.001	-.001	.001
1406	-.001	.000	-.001	.001
1407	-.001	.000	-.003	.003
1408	.001	.000	.003	.003
1409	.000	-.001	.004	.004
1410	.000	.000	.003	.003
1411	.000	.000	.999	.999
1412	.000	.000	.001	.001
1413	.000	.000	.006	.006
1414	.000	.000	.003	.003
1415	.000	.000	.000	.001
1416	.000	.000	-.002	.002
1417	.000	.001	.000	.001
1418	.000	.000	-.001	.001
1419	.001	.001	-.006	.006
1420	-.001	.001	-.006	.006

Table 4 Adjustment by bisquare estimation

	Left Image				Right Image			
No.	Vx	Vy	Wx	Wy	Vx	Vy	Wx	Wy
1401	-.042	.000	0.00	1.00 <<	.000	-.002	0.99	0.78
1402	.004	-.002	0.79	0.92	-.001	.001	0.98	0.93
1403	.001	-.003	0.98	0.91	.000	.002	1.00	0.82
1404	-.007	.008	0.35	0.29	-.002	.000	0.74	1.00
1405	.002	-.002	0.94	0.96	.001	-.003	0.97	0.46
1406	.002	-.002	0.91	0.93	.001	.001	0.91	0.78
1407	.004	.002	0.80	0.91	-.001	-.002	0.98	0.78
1408	-.002	.001	0.94	1.00	.001	.001	0.98	0.99
1409	-.002	.003	0.96	0.89	.002	.002	0.73	0.85
1410	-.002	.000	0.91	1.00	.000	.000	1.00	1.00
1411	-.665	.000	0.00	1.00 <<	.370	.001	0.00	0.93 <<
1412	.001	.001	0.99	0.97	.001	.000	0.96	0.99
1413	-.004	-.002	0.79	0.95	.001	-.004	0.96	0.27
1414	-.003	-.001	0.89	0.99	.000	-.001	1.00	0.96
1415	-.001	.001	1.00	0.98	-.001	.001	0.92	0.97
1416	.001	.000	0.98	1.00	-.001	-.001	0.98	0.97
1417	.002	-.001	0.96	0.97	.002	-.011	0.73	0.00 <<
1418	-.002	-.002	0.97	0.97	-.002	-.002	0.74	0.84
1419	.000	.001	1.00	0.99	-.005	.002	0.78	0.85
1420	.005	.002	0.70	0.95	.001	.001	1.00	0.91

No.	MX	MY	MZ	POS
1401	.011	.011	.048	.050
1402	-.001	-.001	-.005	.005
1403	.000	.000	-.001	.001
1404	.001	-.001	.006	.006
1405	.000	.001	-.001	.002
1406	-.001	.000	-.001	.001
1407	-.001	-.001	-.004	.004
1408	.000	.000	.003	.003
1409	.000	-.001	.005	.005
1410	.000	.000	.002	.002
1411	.000	.000	.998	.998
1412	.000	.000	.000	.000
1413	.000	.000	.006	.006
1414	.000	.000	.003	.003
1415	.000	.000	-.001	.001
1416	.000	.000	-.002	.002
1417	-.001	.001	.001	.001
1418	.001	.001	-.001	.001
1419	.001	.001	-.006	.007
1420	.000	.001	-.005	.005

Table 5 Adjustment by Danish method

	Left Image				Right Image			
No.	Vx	Vy	Wx	Wy	Vx	Vy	Wx	Wy
1401	-.029	.003	1.00	1.00	.024	.011	1.00	1.00
1402	.013	.000	1.00	1.00	.001	.003	1.00	1.00
1403	.007	-.002	1.00	1.00	-.011	-.004	1.00	1.00
1404	-.004	.006	1.00	1.00	-.022	-.007	1.00	1.00
1405	.002	-.004	1.00	1.00	-.017	-.022	1.00	1.00
1406	.003	-.003	1.00	1.00	-.059	.011	1.00	1.00
1407	.007	.003	1.00	1.00	-.037	.014	1.00	1.00
1408	.008	.001	1.00	1.00	-.002	-.007	1.00	1.00
1409	.005	.000	1.00	1.00	-.010	.002	1.00	0.00
1410	-.002	-.001	1.00	1.00	-.046	.000	1.00	1.00
1411	-.665	-.002	0.00	1.00 <<	.318	.001	0.00	1.00 <<
1412	-.001	-.002	1.00	1.00	-.015	-.002	1.00	1.00
1413	-.006	-.004	1.00	1.00	-.013	.013	1.00	1.00
1414	-.005	.001	1.00	1.00	-.059	-.011	1.00	1.00
1415	-.002	.002	1.00	1.00	-.040	-.015	1.00	1.00
1416	.004	-.004	1.00	1.00	-.007	.008	1.00	1.00
1417	.002	-.001	1.00	1.00	.019	-.021	1.00	1.00
1418	-.003	.000	1.00	1.00	-.003	.003	1.00	1.00
1419	-.002	.003	1.00	1.00	-.013	.012	1.00	1.00
1420	.004	.006	1.00	1.00	-.011	.010	1.00	1.00

No.	MX	MY	MZ	POS
1401	.007	.013	.062	.063
1402	-.003	-.003	-.013	.014
1403	.001	-.004	-.021	.021
1404	.006	-.006	-.022	.023
1405	.005	.000	-.023	.023
1406	.012	-.008	-.061	.063
1407	.003	-.008	-.043	.044
1408	-.002	-.001	-.010	.011
1409	-.001	.000	-.019	.019
1410	.006	.000	-.044	.045
1411	.014	.000	.936	.936
1412	.004	.001	-.017	.017
1413	.004	.002	-.008	.009
1414	.013	.007	-.054	.056
1415	.005	.005	-.038	.039
1416	-.001	.003	-.014	.014
1417	-.001	-.004	.021	.021
1418	.001	.000	.000	.001
1419	.003	.002	-.013	.014
1420	.003	.002	-.017	.017

Table 6

	σ	Least squares method MX	MY	MZ	POS	Data snooping Vx	Vy	Tx	Ty
	0	.0005	.0004	.0035	.0035				
	5	.0007	.0005	.0042	.0043	-.010	.000	1.900	.011
x	10	.0009	.0006	.0054	.0055	-.016	.000	.863	.014
	20	.0016	.0008	.0083	.0085	-.029	.000	3.525	.011
	5	.0005	.0006	.0034	.0035	-.003	-.007	.170	.410
y	10	.0005	.0007	.0034	.0035	-.003	-.013	.156	.793
	20	.0005	.0021	.0039	.0044	-.003	-.027	.163	1.670

Table 7.

	σ	Huber' descending M-estimator Vx	Vy	Wx	Wy	Bisquare estimator Vx	Vy	Wx	Wy
	5	-.011	.000	.019	1.000	-.011	.000	.006	.999
x	10	-.020	.000	.000	1.000	-.019	.000	.000	.999
	20	-.035	.000	.000	1.000	-.036	.000	.000	.999
	5	-.003	-.007	1.000	.665	-.003	-.008	.920	.463
y	10	-.003	-.017	1.000	.000	-.003	-.017	.911	.000
	20	-.003	-.034	1.000	.000	-.002	-.034	.920	.000

Danish Method

	σ	Vx	Vy	Wx	Wy
	5	-.011	.000	.014	1.000
x	10	-.021	.000	.002	1.000
	20	-.038	.000	.001	1.000
	5	-.003	-.007	1.000	1.000
y	10	-.003	-.017	1.000	.043
	20	-.003	-.035	1.000	.007

Table 1 RMS error by ordinary least squares

No.	MX	MY	MZ	POS
1401	.008	.009	.067	.068
1402	-.002	-.004	-.008	.010
1403	.001	-.004	-.020	.020
1404	.007	-.003	-.014	.016
1405	.003	-.002	-.079	.079
1406	.000	-.022	-.153	.155
1407	-.010	-.017	-.105	.107
1408	-.002	.001	-.011	.012
1409	-.001	.002	-.019	.019
1410	-.009	.001	-.011	.012
1411	.000	.001	.826	.826
1412	.001	.003	-.094	.094
1413	.002	.004	-.068	.068
1414	.001	.013	-.148	.148
1415	-.007	.008	-.100	.100
1416	.000	.000	-.010	.010
1417	.002	.000	.032	.032
1418	.002	-.001	.006	.006
1419	.003	.001	-.015	.016
1420	.003	.002	-.017	.017

MX = RMS Error of X Object Space Coordinate
MY = RMS Error of Y Object Space Coordinate
MZ = RMS Error of Z Object Space Coordinate
POS = Positioning Error
Iter. = Iteration Numbers
Vx = Residual of x Image Coordinate
Vy = Residual of y Image Coordinate
Tx = Test Statistics of x Image Coordinate
Ty = Test Statistics of y Image Coordinate
<-RE = Rejected Point
<-OK = Accepted Point
Wx = Weight of x Image Coordinate
Wy = Weight of y Image Coordinate
<< = Point with 0 Weight