GPS-SUPPORTED DETERMINATION OF INTERIOR ORIENTATION ELEMENTS OF AERIAL CAMERA

Yuan Xiuxiao

School of Remote Seneing and Informatics
Wuhan Technical University of Surveying and Mapping
39 Luoyu Road, Wuhan, 430070, P.R. of China

Commission I, Working Group 3

KEY WORDS: Accuracy, GPS, Method, Aerial Camera, GPS-supported Single-image Resection, GPS-supported Bundle Adjustment.

ABSTRACT

It is common knowledge that we can solve the elements of interior orientation x_0,y_0,f of aerial camera in collinearity equation with enough ground control points. However, in the case of vertical photography, the solution of the normal equation may be faced with singularities because x_0,y_0,f and X_S,Y_S,Z_S offset each other. Therefore, the conventional aerotriangulation solves only the elements of exterior orientation $\varphi,\omega,\kappa,X_S,Y_S,Z_S$ of photograph, whereas the elements of interior orientation x_0,y_0,f of aerial camera are often determined in the laboratory. As the laboratory methods can not take into full account the actual conditions in aerophotography, large deviations often occur.

This paper raises two kinds of GPS-supported determination methods for the elements of interior orientation of aerial camera for the first time. The basic formula are derived form GPS-supported single-image resection in space and GPS-supported bundle block adjustment. A set of actual determination results obtained by camera Wild RC-20 with GPS data from the test field in Taiyuan are given and discussed. The test has confirmed that the two presented methods are correct in theory and efficient in practice. If several pictures with carrier phase measurements are taken over a high precision test field before and after an air photographic mission, the elements of interior orientation of aerial camera can be determined by using GPS-supported single-image resection in space. The determination method of interior orientation elements in GPS-supported bundle block adjustment does not pose any major problem for photogrammetric adjustment but can efficiently correct the systematic errors caused by the high dynamic aerophotography. To summarize the investigations in this paper can obtain that the dynamic determination methods for the elements of interior orientation will eventually replace the conventional photogrammetric operation method where the camera calibration is independent of photo orientation, and revolutionary changes will take place in the conventional aerotriangulation which had lasted 60 years and more.

1. INTRODUCTION

It is common knowledge that the strict mathematical relationships between the image points in a photograph and their corresponding object points can be written as follows:

$$
\begin{aligned}
x-x_0+\Delta x &= -f\frac{a_1(X-X_S)+b_1(Y-Y_S)+c_1(Z-Z_S)}{a_3(X-X_S)+b_3(Y-Y_S)+c_3(Z-Z_S)} \\
y-y_0+\Delta y &= -f\frac{a_2(X-X_S)+b_2(Y-Y_S)+c_2(Z-Z_S)}{a_3(X-X_S)+b_3(Y-Y_S)+c_3(Z-Z_S)}
\end{aligned}
\qquad (1)
$$

where, x,y are the image coordinates in plate coordinate system. X,Y,Z are the object coordinates in ground coordinate system. x_0,y_0,f are the elements of interior orientation of an aerial camera. X_S,Y_S,Z_S are the coordinates of the photographic station in ground coordinate system. a_i,b_i,c_i (i=1,2,3) are the direction cosines of the included angles between the corresponding axes of the image space coordinate system x,y,z and those of the ground coordinate system X,Y,Z. They are the function of photo rotation angles φ,ω,κ. $\Delta x,\Delta y$ are corrections of the coordinates of image points.

If several pictures are taken in accordance with certain conditions in an actual test field and the observation is performed in a high precision comparator and enough ground control points are obtained, we can solve the elements of interior orientation x_0, y_0, f in Eqs. (1). However, in the case of vertical photography, the solution of the normal equations may be faced with singularities because x_0, y_0, f and X_S, Y_S, Z_S offset each other. To overcome this weakness, the height differences between the ground control points in the test field need to be sufficiently large (***Wang Zhizhuo, 1990***). Generally speaking, it is very difficult to meet this condition in practice. Therefore, the conventional aerotriangulation solves only the elements of exterior orientation φ, ω, κ, X_S, Y_S, Z_S of photograph, whereas the elements of interior orientation x_0, y_0, f of aerial camera are always determined in the laboratory. As the laboratory methods can not take into full account the actual conditions in aerophotography, large deviations often occur.

Since GPS was applied to determine 3D coordinates (X_A, Y_A, Z_A) of the camera station during a photo flight mission, revolutionary changes have taken place in the conventional aerotriangulation which had lasted 60 years and more. The new investigations have shown that the accuracy of positioning with differential GPS carrier phase measurements is in the order of a few cm (***Ackermann,F., 1991, Friess,P., 1991***). If the GPS camera station coordinates are introduced to the adjustment of aerotriangulation, the strong correlation between x_0, y_0, f and X_S, Y_S, Z_S can be greatly weakened and the solution of normal equations is then unique. The elements of interior orientation and those of exterior orientation can be solved together in adjustment. The dynamic determination method for the elements of interior orientation can be performed in test field.

2. THE BASIC IDEAS OF GPS-SUPPORTED DETERMINATION OF INTERIOR ORIENTATION ELEMENTS

It is known from the principle of GPS kinematic carrier phase measurements that the real position determined by GPS is the center of the GPS antenna phase. The mathematical relationship between the coordinates (X_A, Y_A, Z_A) and the coordinates (X_S, Y_S, Z_S) is written as (***Li Deren, 1991***):

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} + R \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad (2)$$

where, $R = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orthogonal transformation matrix. u,v,w are the coordinates of GPS antenna phase center in image space coordinate system.

In Eqs. (1) and Eqs. (2), x, y, X_A, Y_A, Z_A are observables and x_0, y_0, f, φ, ω, κ, X_S, Y_S, Z_S, X, Y, Z, u, v, w are the parameters to be determined. After the substitution of the approximate values of these unknowns, the above equations can be linearized, after which, we can have the error equations which can be written in matrix form respectively as below:

$$\begin{array}{lllll} & & \textit{Weight} & \textit{Number of Eqs.} & \\ V_X = A\mathbf{t} + B\mathbf{x} + I\mathbf{i} & -L_X, & 1 & 2 & (a) \\ V_C = E_X\mathbf{x} & -L_C, & P_C & 3 & (b) \\ V_I = E_I\mathbf{i} & -L_I, & P_I & 3 & (c) \\ V_G = \overline{A}\mathbf{t} + R\mathbf{r} & -L_G, & P_G & 3 & (d) \end{array} \qquad (3)$$

where,

$\mathbf{t} = [\Delta\varphi\ \Delta\omega\ \Delta\kappa\ \Delta X_S\ \Delta Y_S\ \Delta Z_S]^T$ is the correction vector of exterior orientation elements.

$\mathbf{x} = [\Delta X\ \Delta Y\ \Delta Z]^T$ is the correction vector of object coordinates.

$\mathbf{i} = [\Delta x_0\ \Delta y_0\ \Delta f]^T$ is the correction vector of interior orientation elements.

$\mathbf{r} = [\Delta u\ \Delta v\ \Delta w]^T$ is the correction vector of the coordinates of u,v,w.

$L_X = \begin{bmatrix} x-(x) \\ y-(y) \end{bmatrix}$, $(x),(y)$ are the computed values of x,y when the approximate values of the unknowns have been substituted into equations (1).

$L_G = \begin{bmatrix} X_A-(X_A) \\ Y_A-(Y_A) \\ Z_A-(Z_A) \end{bmatrix}$, $(X_A),(Y_A),(Z_A)$ are the computed values of X_A, Y_A, Z_A in Eqs (2).

The meaning of coefficient matrices A and B refers to the reference [6].

E_x, E_I are unit matrix.

According to Eqs.(1), we can obtain $I = \begin{bmatrix} 1 & 0 & (x-x_0)/f \\ 0 & 1 & (y-y_0)/f \end{bmatrix}$.

Letting $\begin{bmatrix} U \\ V \\ W \end{bmatrix} = R \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix}$, we can derive from Eqs.(2) (***Yuan Xiuxiao, 1994***)

$$\bar{A}=\begin{bmatrix} -W & -V\sin\varphi & -c_3V+b_3W \\ 0 & U\sin\varphi-W\cos\varphi & c_3U-a_3W \\ U & V\cos\varphi & -b_3U+a_3V \end{bmatrix}.$$

Sometimes, there are enough ground control points in the test field and the coordinates (u,v,w) are accurately determined in ground survey. Letting **x=r=0**, Eqs. (3.a), Eqs. (3.c) and Eqs (3.d) will form the error equations for GPS-supported single image resection in space. Based on these error equations, we can solve together the elements of interior orientation and those of exterior orientation with the least squares adjustment method.

If the overall adjustment is performed by using all photographs covering the test field, Eqs. (3) is the observation equations for GPS-supported bundle block adjustment. In this case, the solution of the normal equation includes not only the elements of interior orientation and those of exterior orientation, but also the coordinates of the photogrammetric points. In order to improve the accuracy of block adjustment, it is necessary to use self-calibration technique to effectively compensate for residual systematic errors of the photogrammetric images in GPS-supported bundle block adjustment.

3. DETERMINATION OF THE ELEMENTS OF INTERIOR ORIENTATION ON CAMERA WILD RC-20

To test and verify the correctness and efficiency of the above-presented methods, the actual determination for interior orientation elements of camera Wild RC-20 was made by using a set of real photographs which were taken in the test field in Taiyuan. The details of the aerophotography and the ground control points have been described in an earlier paper (***Li Deren*** and ***Yuan Xiuxiao, 1995***). Table 1 lists the main technical data of GPS-supported photo flight.

Table 1 Technical Data of Test Photographs

Item	Data
survey aircraft	Lear Jet 36A of U.S.A
GPS receiver	4 Trimble 4000SST
aerial camera	Wild RC-20
format	23 cm X 23cm
elements of interior orientation	f=303.86mm, x_0=-0.0030mm, y_0=0.0170mm
image scale	1:5000
longitudinal overlap	62%~70%
lateral overlap	0.4%~56%
number of strips	4
number of pictures	32
number of photogrammetric points	270
number of fixed artificial points	149
antenna-camera-offset	u=1.8216m, v=0.4106m, w=1.4026m

The determination of the elements of interior orientation is made according to four cases. Case **a** solves only 6 elements of exterior orientation in conventional space resection of a single photograph. The elements of interior orientation are copied from the camera calibration certificate. Case **b**, without GPS photogrammetric coordinates, solves 6 elements of exterior orientation and 3 elements of interior orientation together on the principle of single-image resection in space and ground control points. Case **c** is GPS-supported single-image resection in space. Case **d** is GPS-supported bundle block adjustment with 4 full ground control points in the corners of the block. The coordinates of ground control points in case **a**, case **b** and case **c** are the adjusted results of conventional bundle block adjustment with 3 additional parameters, where 149 fixed artificial points were used as orientation points in photogrammetric densification. The overall accuracy of the coordinates of photogrammetric points reach $\sigma_0=\pm 9.4\ \mu m$, $m_X=\pm 4.5$ cm, $m_Y=\pm 6.4$ cm and $m_Z=\pm 5.9$ cm in the ground. The determined results of interior orientation elements are given in Table 2.

From the results of Table 2 the following conclusions can be drawn:

(1). In case **b**, the correlation between x_0, y_0, f and X_S, Y_S, Z_S is very strong (see Figure 1.a), which leads to the ill-conditioned normal equation. As a result, the determined accuracy of the orientation elements is very low. And the determination values of the orientation elements are different with different photos. The normal equation has an uncertain solution.

(2). In case **c**, the strong correlation between x_0,y_0,f and X_S,Y_S,Z_S is greatly weakened. Figure 1.b shows that their correlation coefficients are very small. Therefore, the determination values of the orientation elements are almost a constant for 32 photographs and the accuracy is very high. The overall accuracy of the elements of exterior orientation is quite close to that of case **a**, the accuracy of the elements of interior orientation reach $\pm 10\,\mu$m and all 32 determined values are not obviously different to each other.

(3). Similarly, high precision elements of interior orientation are obtained in case **d**. The results are in accordance with that of case **c**. However, the need for ground control points in case **d** are much less than that in case **c**.

4. CONCLUSION AND SUGGESTION

(1). The test has confirmed that the two kinds of GPS-supported determination methods described in this paper for interior orientation elements of aerial camera are correct in theory and efficient in practice. The camera calibration can be performed in photogrammetric adjustment.

(2). In photogrammetry, the application of GPS can not only reduce the number of ground control points and replace the hard work of field measurement, but also have enormous potentiality on camera calibration. This amply demonstrates that GPS can be widely applied in photogrammetry.

(3). If several pictures with carrier phase measurements are taken over a high precision test field before and after an air photographic mission, the elements of interior orientation of aerial camera can be determined by using GPS-supported single-image resection in space. The test field method will eventually replace the conventional photogrammetric operation method where the camera calibration is independent of photo orientation.

(4). The determination method of interior orientation elements in GPS-supported bundle block adjustment can be widely applied in photogrammetry. This processing method does not pose any major problem for photogrammetric adjustment but can efficiently correct the systematic errors caused by the high dynamic aerophotography. The dynamic method of camera calibration can take full account of the actual conditions in aerophotography. There is no question that the method is helpful for improving the accuracy of photogrammetric densification.

ACKNOWLEDGMENT

I should like to express my hearty appreciation to my supervisor Prof. Li Deren for his guidance and encouragement in the completion of the research project which leads to this paper.

REFERENCES

[1] Ackermann,F.,1991. GPS for Photogrammetry. Proceedings of ISPRS, pp 17~69.

[2] Friess,P., 1991. Aerotriangulation with GPS--Methods, Experience, Exception. Proceedings of the 43rd Photogrammetric Week.

[3] Li Deren, 1991. Application of Global Positioning System in High Accuracy Point Determination Airborne Remote Sensing. Remote Sensing of Environment (in Chinese), 6(3), pp 216~223.

[4] Li Deren, Yuan Xiuxiao, 1992. Bundle Block Adjustment with Small Format Air Photographs on Surveying and Mapping On Antarctica. Antarctic Research (in Chinese), 4(2), pp 27~35.

[5] Li Deren, Yuan Xiuxiao, 1995. GPS-supported Bundle Block Adjustment. Acta Geodaetica et Cartographica Sinica (in Chinese), 24(3), pp 1~7.

[6] Wang Zhizhuo, 1990. Principles of Photogrammetry (with Remote Sensing). Press of WTUSM, Publishing House of Surveying and Mapping, Beijing.

[7] Yuan Xiuxiao, 1994. GPS-supported Combined Bundle Block Adjustment Based on Differential Carrier Wave Phase Observations (in Chinese). MS. Dissertation, WTUSM.

Table 2 Determination Results of Interior Orientation Elements for Camera Wild RC-20

Case	Item	σ_0 (µm)	Accuracy									Elements of interior orientation		
			φ (')	ω (')	κ (')	X_s (m)	Y_s (m)	Z_s (m)	x_0 (mm)	y_0 (mm)	f (mm)	x_0 (mm)	y_0 (mm)	f (mm)
a	average value	9.0	0.40	0.49	0.08	0.22	0.18	0.05				-0.0030	0.0170	303.8600
	maximum value	12.2	0.82	1.19	0.17	0.53	0.38	0.12						
	minimum value	6.1	0.26	0.26	0.05	0.12	0.12	0.02						
b	average value	8.1	0.65	0.94	0.11	2.94	3.41	19.26	598.85	676.94	3841.02	-0.5861	-0.5996	301.7942
	maximum value	11.6	1.49	2.33	0.23	13.98	16.35	83.02	2785.90	3251.15	16509.84	0.2728	0.5334	307.4468
	minimum value	5.1	0.28	0.28	0.05	0.30	0.30	2.01	67.67	80.70	424.80	-2.8819	-9.5883	282.7140
c	average value	9.2	0.41	0.48	0.09	0.22	0.19	0.10	9.19	9.19	8.83	-0.0046	0.0167	303.8911
	maximum value	12.2	0.77	0.99	0.17	0.44	0.36	0.14	12.21	12.22	11.74	-0.0028	0.0196	303.9552
	minimum value	6.2	0.26	0.25	0.05	0.12	0.12	0.07	6.13	6.13	5.91	-0.0064	0.0140	303.8049
d	average value	10.5	0.64	0.42	0.25	0.28	0.20	0.10	10.37	10.34	10.38	-0.0035	0.0206	303.8980

φ	-0.61	-0.47	-0.61	-0.35	0.84	0.38	-0.55	0.84
	ω	0.02	0.33	0.05	-0.68	-0.15	0.28	-0.68
		κ	0.32	0.71	-0.28	-0.70	0.30	-0.29
			X_s	0.34	-0.81	-0.33	**1.00**	-0.81
				Y_s	-0.34	**-0.99**	0.33	-0.34
					Z_s	0.38	-0.76	**1.00**
						x_0	-0.32	0.38
							y_0	-0.77
								f

a. conventional single-image resection in space

φ	0.02	-0.46	-0.77	-0.02	0.01	0.00	0.29	-0.04
	ω	-0.06	-0.02	-0.85	-0.07	-0.38	0.01	0.50
		κ	0.38	0.07	0.00	0.00	-0.14	-0.01
			X_s	0.02	0.00	0.01	**0.38**	0.00
				Y_s	0.06	**-0.16**	0.00	-0.42
					Z_s	0.02	0.01	**0.56**
						x_0	0.00	-0.18
							y_0	-0.04
								f

b. GPS-supported single-image resection in space

Figure 1 Matrices of Correlation Coefficient Between Orientation Elements