DEFORMABLE MODELS AS A PHOTOGRAMMETRIC MEASUREMENT TOOL - POTENTIAL AND PROBLEMS

Eberhard Gülch

Institute of Photogrammetry, University Bonn, D-53115 Bonn, Germany, e-mail: ebs@ipb.uni-bonn.de
on leave from: Department of Geodesy and Photogrammetry, KTH, S - 10044 Stockholm, Sweden

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ABSTRACT

Deformable models or snakes are a shape extraction tool based on energy minimization functions. The contour or surface extracted through an iteration process is a compromise between the external energies defined by image features and the internal energies of the deformable model. During the last decade substantial research efforts have been reported on improvement and extension of that method. We will describe several representative approaches on open and closed contours and surfaces and analyse the encountered difficulties. We will add our own experience with 2D snakes. Snakes have been applied to all kinds of images, from satellite to medical imagery, from mono-images to image sequences in 2D and 3D. The independence from sensor type and application area makes them interesting for photogrammetry. The largest potential is currently in the usage as a refinement tool for interactive measurements with simple models. Most applications are specific and require substantial user guidance and validation. The lack of quality estimates, the quite huge number of parameters to be set by an operator and the large computational efforts still hinder a usage on a broader basis as a generic photogrammetric measurement tool.

1. MOTIVATION

Deformable models or active contour models or deformable surfaces are feature extraction tools based on energy minimization functions (Kass et. al., 1987). External energies can be image energies derived from grey values or image features. Internal energies describe the behaviour, e.g. rigidity of the deformable model. The contour or surface extracted through an iteration process is a compromise between the external energies and the internal energies of the deformable model. The method was clearly designed for interactive extraction of image contours, but has potential for more automated surface reconstruction or motion tracking as well. In this report we will examine how this method could be or is used for photogrammetric applications or related fields. We will discuss several representative approaches that have improved the performance of the original method and analyse and compare the encountered difficulties. We will add own experience with 2D snakes in satellite, aerial and medical imagery and discuss the potential and problems to introduce deformable models on a broader basis in photogrammetric applications like e.g. topographic mapping.

2. DEFORMABLE MODELS

The published reports of the last years in photogrammetry indicate intensified research on deformable models, which is still very much dominated by the computer vision field. Development started on contours and continued on surfaces.

2.1 Basic approach for deformable models

The basic approach is summarized here as presented by Kass et al., 1987. The behaviour of an active contour model (snake) is controlled by internal and external forces. The energy of a deformable model depends on where it is placed and how its shape changes locally in space. The active contour can be described in parametric representation with

$$v(s) = (x(s),y(s)) \qquad s = \text{arc length} \qquad (1)$$

The snake attracts to features of an image structure by minimizing an integral measure which represents the snake's total energy. The energy functional is:

$$E^{*}_{snake} = \int_{0}^{1} E_{int}(v(s))+E_{ima}(v(s))+E_{con}(v(s))ds \qquad (2)$$

The total energy is a sum of single energies. During minimization the snake is deformed to find an optimal compromise between the constraints introduced by internal forces (E_{int}) and external forces such as image forces (E_{ima}) and external constraint forces (E_{con}).

Internal forces open the possibility to introduce geometric constraints on the shape of the contour. The internal energy is composed of a first order term and a second order term forcing the active contour to act like a membrane or a thin plate, thus introducing material parameters:

$$E_{int}(s) = (\alpha(s) \, |v_s(s)|^2 + \beta(s) \, |v_{ss}(s)|^2 /2 \qquad (3)$$

The tension is controlled by $\alpha(s)$ and the rigidity by $\beta(s)$. Image forces attract the snake to salient features in the image. The image energy E_{ima} is written as a combination of different weighted energy terms. External constraint forces are provided by higher level image interpretation or user-interface. The user selects starting points and can apply forces interactively on the snake during the minimization process, like a spring or a repulsion force.

The energy minimization itself is done in four steps. A variational integral is set up in the continuos space, a pair of Euler equations is derived, the equations are discretized and the position vectors are solved iteratively with a given step size γ until convergence is reached.

2.2 Major extensions of deformable models

Since the initialization by Terzopoulos et al. the work on deformable models has expanded very much. Major variations and extensions of the approach have been reported. We can observe conceptual work like in Amini et al., 1988, Fua and Leclerc, 1988, Cohen, 1991, Szeliski and

Tonnesen, 1992, Malladi et. al., 1993 or Neuenschwander et al., 1995, together with other contributions that pick out single aspects for optimization and improvement. The major feature of deformable models is the ability to bridge over low texture areas. Most approaches are based on an interactive setting of seed points along the contour or on a surface. There snakes are primarily a tool for the last step in a mensuration process and not for detecting a contour or a surface from very far away. There exist many families of deformable models. Different implementations of 2D snakes are used in mono-images, like open, closed snakes or snake networks (Ruskoné et al., 1994). Approaches for 3D contour snakes (e.g. Trinder and Li, 1995) and 3D surface type snakes (e.g. Delinguette et al., 1991, Szeliski, 1992, Guéziec and Ayache, 1993) have been reported, integrating also stereoscopic correspondence and motion models for contours in the approach (e.g. Bascle and Deriche, 1993).

2.3 Application areas

In Kass et al., 1987 the major intention was the interactive specification of image contours, matching and motion tracking. Since then many new application areas have emerged. We can clearly see a large influence of development in medicine and aerial imagery for contour extraction and tracking in multiple frames, in 2D and 3D.

We can see applications in satellite and aerial imagery, in industrial scenes and medical images. Snakes work on panchromatic and colour images as well as on medical ultrasonic images, magnetic resonance images and are used in computer tomography. The method is applicable to single images, multiple images or image sequences.

The flexibility of this tool to handle different sensor types and applications makes it interesting for photogrammetric applications, like interactive topographic mapping or other mensuration problems. Considerable work has been performed the last decade to improve the performance of deformable models, still we can see only a limited, very specialized application of them in classical photogrammetric measurement tasks. We will first look at some of the improvements that have been undertaken and discuss then potential and remaining problems for further acceptance in photogrammetry.

3. IMPROVING THE PERFORMANCE

We examine the important modules of deformable models in more detail and include contributions to handle objects of unknown topology.

3.1 Regularization and energy minimization

Physically based models when applied to static data reduce to energy minimization methods, like regularization, which is a common approach to solve visual inverse problems. Different improvements for the iterative optimization have been reported.

Leclerc and Fua, 1987 describe a guided gradient ascent method for optimization, which is, however, too slow if many vertices are involved. Fua and Leclerc, 1988 give a better optimization method by using a viscous medium and solving the equation of dynamics. In Gülch, 1990 a hierarchy in the amount of contour points is applied within a coarse-to-fine strategy to gradually increase the number of vertices. Szeliski and Terzopoulos, 1991 combine physically based models with probabalistic models to a Kalman filter applied to noisy dynamic tracking data. Ronfar, 1994 designs a heuristic optimization procedure, that accounts for internal and external energy in separate steps and uses adaptive neighbourhood structures and diffusion processes for a region based strategy. Larsen et al., 1995 stabilize the variational approach by introducing a decay function for the step size that gradually reduces oscillations.

The search for alternate approaches to Lagrangian dynamics were influenced by Amini et al., 1988. They propagate 'time'delayed' dynamic programming instead of the variational method to get a more stable behaviour of 2D snakes. Hard constraints were integrated to avoid merging of points along the contour. A considerable further improvement is reported by Olstad and Tysdahl, 1993. Their approach requires some reasonable constraints on the selection of candidate points and usage of a second order energy model.

Statements on criteria for reaching convergence are rare, especially in cases of dynamic moving objects. A clear choice of best optimization method is not possible yet. The number of iterations are seldom reported, but are usually very high and can range from 10 to 400 and much more. High numbers, however, make the approach unattractive for all real-time and interactive applications.

3.2 Internal Energies

The setting of the material parameters, additional constraints and the handling of discontinuities are major problems.

Fua and Leclerc, 1988 state the inefficiency of local geometric constraints and give examples for global constraints to enforce rectilinearity or parallelity. They can set the smoothness parameter given only the initial state of the curve and a rough guess of its quality.

In Gülch, 1990 an elimination and densification step for contour points based on finite elements avoids observed uncontrolled drifting and clustering of points along the curve. It ensures an almost equal point distribution along the whole length of the contour. Breakpoints are introduced not in the internal energies but by image energies (3.3). Grün and Li, 1994 adopted the dynamic programming method and introduced an alternate restriction for point movements combined with a dynamic insertion and deletion of vertices.

Cohen, 1991 introduced an additional inflation force and a numerical choice of the material parameters and their relations to the space discretization step. It will fail when significant protrusions occur, due to the curvature minimization properties. Malladi et al., 1993 give solutions to extract also protrusions.

Samadani, 1991 develops the automated parameter setting further and introduced an adaptive control of the material parameters, based on monitoring deformation energies. However it seems that this heuristic approach requires still some user assistance for more complex scenes. Rougon and Prêteux, 1993 propose an adapting of material and damping factors based on differential group-invariant representations of local image structures.

Jasiobedzki, 1993, uses a network of active contours, where the whole network or graph can be deformed by internal and external energies, keeping the connectivity at the nodes, thus keeping the topology during the deformation process. Cohen et al., 1993 combine surfaces of different type as lake and deformable mountainous area with a separated determination of the lake border by an active contour based on a variational formulation.

Solutions for isolated problems are visible, but no overall success to automate the selection of internal energies and constraints.

3.3 Image Energies

Image energies are improved or extended to allow for less accurate initial states and to yield a better extracted shape.

The balloon model by Cohen, 1991 uses edge points previously extracted by a local edge detector to avoid instabilities due to image forces and to enlarge the area of attraction

In Gülch, 1993 a common image energy map for edge and point type features is presented based on the Förstner interest operator (Förstner and Gülch, 1987). It is an extension of an earlier approach (Gülch, 1990). The interest- and shape values, allow to localize and classify edge and point type features and the derived energy values can be geometrically interpreted. Breakpoints in the contour are enforced by image energies (corners). A theoretically well understood method is at hand that combines point and edge energies and eliminates the problem of combining image energy measures of different nature. In order to increase the area of attraction an energy pit/canyon around each point/edgel is created, based on the interest measures, with highly interesting points/edges receiving minimal energy contribution. A pit is centred at the sub-pixel position of the interesting point (fig. 1). For edges the area of attraction is chosen parallel to the edge direction and forms a canyon (fig. 2). Edges and corners are weighted in the same way and in both cases the contribution of neighbouring pixels are taken into account. By placing the centre of the attraction area at a sub-pixel position the energy map can be spatially refined. To avoid the pre-setting of a specific window size a sequence of windows is applied to the image. The derived image energies from the sequence of windows are combined to derive a final energy map as input for the snakes.

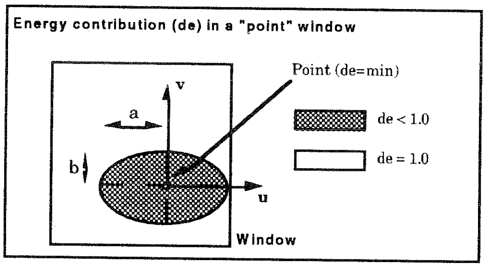

Fig. 1: Example of energy contributions (de) for a pixel in a point element window.
(de) is dependent on the location in relation to the centre of the energy ellipse.

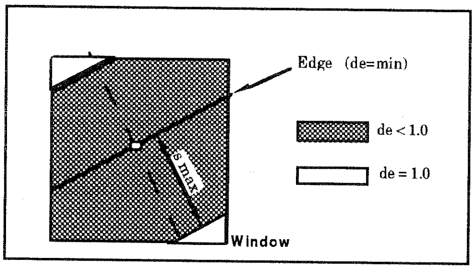

Fig. 2: Energy contributions (de) for a pixel in an edge element window.
(de) is dependent on the perpendicular distance of the pixel to the edge. s_{max} is the maximal distance for a pixel to obtain an energy contribution of de<1.0.

The energy map for an aerial scene is given in fig. 3. The distinction between edges and corners is set to be even. 60% of the weakest points and 1% of the weakest edges are excluded. The darkest areas are most attractive for the snakes. The outline of the roof structure of a large building is visible.

Fig. 3: Image energy map for an aerial scene.
In the energy map a low energy contribution is given as a black area. This is the position where the snake should attract to, bright areas show maximal energy. The outline of the roof of a building is clearly visible.

Trinder and Li, 1995 use in a similar way a chamfer image derived from the feature image, but apply it also for 3D snakes. Ronfard, 1994 uses a region-based energy criterion, rather than an edge detection step, with the argument of non-availability or expensive generation of edge maps.

3.4 Parameter settings

Besides the material parameters and parameters for the image energies (window sizes, thresholds) the parameters for the optimization have to be computed or chosen.

As one of the rare examples Delingette et al., 1991, give a clear description of parameters, their meaning and their settings.

Grün and Li, 1994, allow an interactive settings of parameters, based on empirical observations.

In Gülch, 1993 an attempt has been made to unify the image energies for contour extraction and limit the number of necessary parameters to three. Those are:

- Minimal interest operator window size in the sequence of windows
- Maximal interest operator window size in the sequence of windows
- Definition of point/edge element.

All of them can easily be related to task, size of objects and image scale. In addition to that the material parameters, a factor which weights the internal forces against the external forces and the step size have to be given as well as the sequence of starting level in the resolution hierarchy and start/end level in the desired point density.

All other approaches require the same or similar parameters, but usually few is reported on the used parameters and their possible range and selection. Very few work is done on automation of settings of parameters, except the above mentioned automated settings of the internal parameters. In very many contributions this is not regarded as a major problem and mostly neglected. Nevertheless this is one of the most critical point for photogrammetric applications.

3.5 Initial state

A major drawback of snakes is a strong dependency on the initial state. Improvements in this respect would be very welcome to relax the requirement of a very precise initial guess. We can distinguish interactive settings and automated detection by other feature extraction or segmentation tools.

3.5.1 Interactive setting of seed points and interaction. The most used form for deriving at the initial state of a snake is the interactive setting of seed points. The breakpoints of a contour have usually to be defined very precisely. Grün and Li, 1994 emphasize the importance of a convenient graphical user interface for that purpose.

Neuenschwander et al., 1995 use Ziplock snakes, given only start and end point of well defined edge segments. This method converges rather well, even with quite different initial states.

Interaction during the optimization is possible by adding suitable energy to push the snake out of a local minima to the desired position. This can be very time consuming and requires the presence of an operator during the whole extraction process.

3.5.2 Automated initial state for open contours. Snakes are applied for road extraction (e.g. Quam and Strat, 1991), where a semi-automatic road tracker applied to an aerial scene provides the initial road contours. Other approaches use region growing. In Ruskoné et al., 1994 a coarse network is provided by automated road detection and road tracking.

3.5.3 Automated initial state for closed contours. Closed snakes are used to extract buildings (e.g. Quam and Strat, 1991). Approximate values can be provided by region growing or scene partitioning methods with global thresholds (Fua and Leclerc, 1988). In general seed points or some reduced user interaction is required. This is due to the fact, that constraints on the boundaries are not easy to introduce for generic cases. In Gülch, 1990 the initial contour is given by the boundaries of the user selected largest regions which is optimized through a pyramidal approach until the final result is reached.

The extraction of natural boundaries like vegetation boundaries is possible (Quam and Strat, 1991), the modeling is however more difficult and the snakes might require more interactive seed points and attention. If the accuracy requirements are lower a more relaxed modeling can be applied. The snakes described in (Gülch, 1990, 1993) have been used in satellite imagery to extract the boundaries of logged forest stands. Rule based multi-spectral classification and analysis identifies old map polygons with newly logged forest (Johnsson, 1994). Those map polygons give the initial state for the snake. The contour has been refined in 60 iterations in a hierarchical approach (fig. 4). The contour has attracted to the major body (black) of the logged forest stand, cutting out small details. The major remaining problems are the rather coarse approximate values from the old forest maps and the difficult topological relations that can occur if the logged forest stands cover several old polygons in the map. In those case dividable snakes (cf. 3.7) would be advantageous.

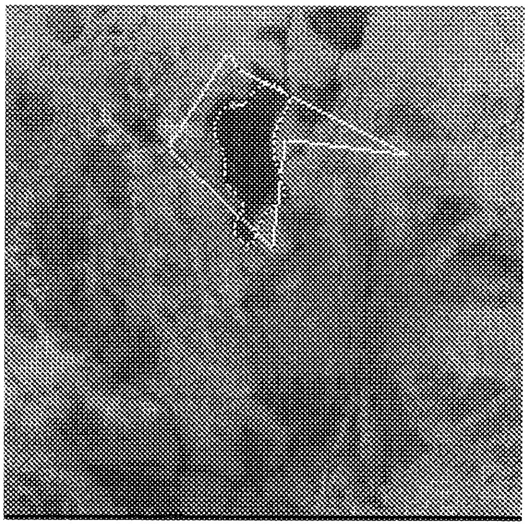

Fig. 4: Logged forest stand in satellite scene. The initial state for the contour from an automatically identified old map polygon, is shown together with the extracted boundary of the logged forest stand (60 iterations).

3.6 Objects of known topology.

In many applications the topology of objects or even size and shape are known. In those cases the snakes usually perform better than in more generic cases.

In Ruskoné et al., 1994, the accuracy of an automatically detected road network is improved by a network of snakes. The topology is given and kept fixed in this elastic network which is globally adjusted.

In Gülch, 1995 snakes refine the contour of a signalized control point, extracted by an automatic rule based region segmentation. It is assumed that the signalized point can be described by a closed boundary and that the signal appears as a homogeneous region in the image. Pixel size, the image scale and the image patch are known. Also the true type, shape, size, colour of the signals are supposed to be available. Satisfying results can be obtained with fixed parameter settings in rotated and distorted images (fig. 5).

In ultrasonic heart images an initial contour of a heart chamber has been derived by a blackboard system, based on a heart chamber model and a set of internal relations in a relaxation labelling scheme. The quite coarse approximation is sufficient for further refinement by an active contour model, as can be seen in fig. 6.

Neuenschwander et al., 1995 assume a spherical topology for 3D deformable surfaces which require a small number of seed points. The surface is initialized/optimized by automatic selection of boundary conditions and front propagation.

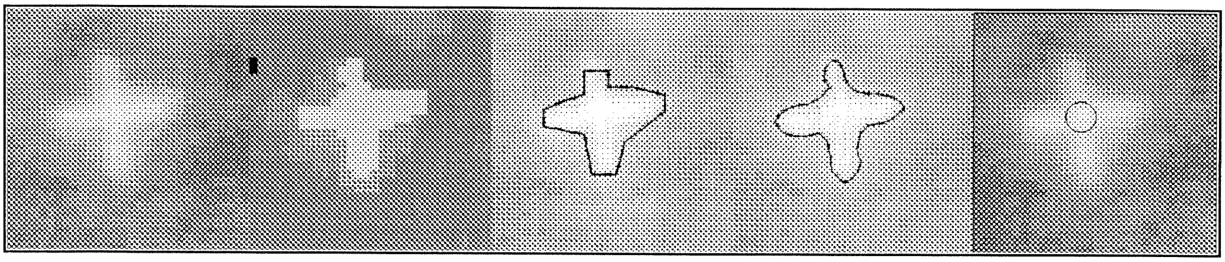

Fig. 5: Extraction of the coarse position of a signalized point (a-e = l-r) a) Image, **b)** region segmentation with signal candidate (white region), **c)** initial, shape from approximated boundary, **d)** extracted contour of the signal, (40 iterations), **e)** the coarse position of the signal (circle) is derived as a weighted centre of gravity of the contour points.

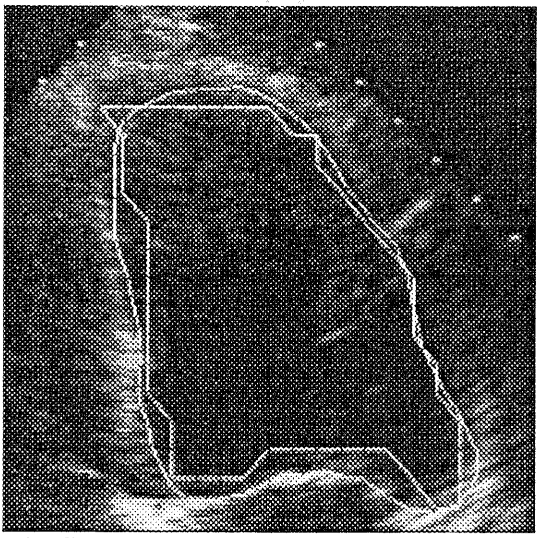

Fig. 6: Contours in ultrasonic heart images. Initial snake position (rough white polygon) and snake position after 40 iterations (rounded white curve).

3.7 Objects of unknown topology

If nothing, or very few is known about the topology of objects, like in many topographic or medical applications, approaches have to be made more robust and should include techniques for handling division of contours and surfaces.

Malladi et al., 1993 model arbitrarily complex shapes with protrusions, whereas snakes tend to prefer regular shapes. The contour can split freely if more than one object occurs independent from the initial state, due to the application of front advancement instead of optimization. This approach is not limited to 2D contours only. Larsen et al., 1995 observe a stable behaviour of their multiple and dividing snake given objects of a certain minimal size.

Delingette et al., 1991 work with free form surfaces based on points and features and a Lagrangian deformation that enables segmentation and allows coarse approximations. It requires minimal and maximal sizes of expected objects, which can be accepted in many applications. Szeliski and Tonnesen, 1992 use molecular dynamics to model surfaces of arbitrary topology, like surface fitting to sparse data. The surfaces are based on interacting particle systems, they are elastic and dynamic and can stretch and grow. In Szeliski et. al., 1993 this approach is extended by an explicitly triangulated surface based on Bézier curves to derive at a globally consistent analytic surface. Being more flexible than spline-based surfaces the particle surfaces require more computational efforts and due to discretization effects do not always allow exact mathematical shape control without further constraints.

3.8 Quality estimates

Any automated procedure should have validation features and should be thoroughly checked against ground truth.

Szeliski and Terzopoulos, 1991 describe an uncertainty measures of snake estimates. They either use a reduced description of uncertainty based on the variance of each nodal variable (neglecting covariances) or a Monte-Carlo approach to generate random examples from the posterior distribution and to derive a confidence envelope for a contour. In Gülch, 1993, information on the quality of image energies are made available, but not propagated further. In Gülch, 1995 the performance of 2D-snakes compared to mask matching and manual measurement of signalized control points shows a similar or better performance in the range of sub-pixel accuracy. Trinder and Li, 1995 report on examinations on the pull-in-range and the absolute accuracy of 2D and 3D snakes. Given a very precise initial estimate of better than 5 pixels then relative accuracies of 0.5 pixels for 2D and 1.0 pixels for 3D snakes could be achieved. Larsen et al., 1995 require a maximal movement of half of the width of the edge of the object to be tracked.

Very few is done about quality estimates, due to the fact that most often user interaction is involved at several stages in the shape extraction and visual inspection alone decides on success or failure. Poor initial values are obviously the dominating error source for deformable models.

4. POTENTIAL AND PROBLEMS

We can see that snakes have been tested for many different measurement tasks under different spezialized conditions, and proved to be a quite flexible measurement tool, but still there are some severe drawbacks.

4.1 Potential

As a summary from own and external experience it seems to be clear, that the major potential is the extraction of generic contours and surfaces in an interactive environment or guided by high-level interpretation. Deformable models differ substantially from manual contour or surface measurement. The user quickly traces a boundary or give some seed points of a surface and the measurement is refined by a dynamic solution that attracts the deformable model to the shape. The support of neighbouring contour or surface segments to bridge over low texture areas is an essential strength. The user has the possibility to guide the deformable model by implying constraints of different nature.

Simple geometric models of a road together with some automatically derived or user defined seed points allow already now the extraction of a large amount of roads in an aerial or satellite scene. Vegetation boundaries can be extracted under relaxed modeling conditions. Snakes perform rather good in the tracking of image sequences, as initial states have to be given usually just at the beginning of a sequence and the parameter don't have to be strongly adapted. The work on free form surfaces and unknown topology could be very well used in the analysis of digital surface models from automated image matching or from laser scanning.

Snakes allow the introduction of many different, image energies and can practically be trimmed to every desired internal behaviour. This manifold of possibilities is on the other hand side responsible for some the major drawbacks.

4.2 Problems

To summarize the discussion in chapter 3, we observe that different applications have different requirements and different conditions, that aren't always met by deformable models. In topographic mapping we have to deal with natural and man-made contours and surfaces. We are dealing with different sensors, with single and multiple views, with sequences and multispectral scenes. The result should be interpretable and required parameters and quality estimates connected to geometry, object, scene and task. It is very difficult to predict the behaviour of snakes in not well-defined measurement tasks. It is difficult to set stopping criteria and to get quality criteria. It is not easy to weight different image energies and it is difficult to derive approximate values in a more automated fashion. Deformable models with a high number of iterations or intensive parameter settings by the user are completely unattractive for interaction. Huge problems generally occur if generic objects should be measured.

As a general observation we can state that results reported are often difficult to compare and judge. No quality of initial state or just a view of an initial state are given nor any information on parameters. We can see promising solutions for single problems, but no overall breakthrough.

5. CONCLUSIONS

The last decade has shown considerable work on many aspects of the method of deformable models. We have made an attempt to pick out some representative improvements of this method, that, would they be combined, would bring it on the edge of a broader application in photogrammetry. Deformable models have a clear potential as a photogrammetric measurement tool but still lack some important features to be useful in more generic measurement tasks. They have currently to be user guided and well constraint. Surface snakes should be tested on the analysis of automatically derived digital surface models. Implementations in photogrammetric workstations can be predicted to support the interactive extraction of cartographic objects with contours and surfaces, if the problems of initial values, automated parameter selection and computational complexity have been solved alltogether.

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