

# DEM Estimation with Simulated Annealing Based on Surface Reconstruction Method

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## KEYWORDS:

Algorithms, DEM/DTM, Theory, Cartography

## ABSTRACT:

A method for Digital Elevation Model (DEM) estimation based on Surface Reconstruction with Simulated Annealing is proposed.

The proposed method allows us to designate the region for minimizing the difference so that the estimated DEM is optimum in the sense of minimizing all the difference between actual radiance and estimated radiance based on the models for the region of interest. The existing surface reconstruction method focuses on the pixel of interest, not the region. When the pixel of interest, however, is suffered from occlusion, then the estimation accuracy of DEM get worth. Turns out, the proposed method takes into account the region, not the only one pixel of interest, minimizing the total difference in the region so that a good estimation accuracy is expected in such case.

## 1. INTRODUCTION

Surface Reconstruction method is one of well known methods for DEM estimation (Ref. 1). Surface Reconstruction method, however, does not work so well for the areas with

a variety of local minimal in the least square method which is featured in Surface Reconstruction method because of non-linearity of the equation. Further the previously proposed Surface Reconstruction method does not take into account a geometric relationship among the sun, target surface and two sensors which makes stereo pair of images results in degradation of DEM estimation accuracy.

In Surface Reconstruction method, actual radiance at sensor is compared to estimated radiance based on a geometric relationship among the surface of interest, sensor onboard satellite and the sun, and a surface roughness model. The estimated radiance is a function of not only the geometric relationship and surface model but also DEM. Then DEM is estimated to minimizing the difference between the actual radiance and the estimated radiance based on the models. In order to minimize the difference, Least Square Methods are used in general. It, however, has a non-linear nature in the process on the estimation of the model derived radiance so that singular DEMs sometime are estimated through Surface Reconstruction method, in particular, for the case that there are many local minimal in the least square based estimation.

On the other hand, non-linear least square method such as Levenberg

Markard method would give us a solution to avoid local minimal. Furthermore, Simulated Annealing also would give us another solution. Two candidates of the methods are proposed in this paper for overcoming the aforementioned situation.

The proposed method allows us to designate the region for minimizing the difference so that the estimated DEM is optimum in the sense of minimizing all the difference between actual radiance and estimated radiance based on the models for the region of interest. The existing surface reconstruction method focuses on the pixel of interest, not the region. When the pixel of interest, however, is suffered from occlusion, then the estimation accuracy of DEM get worth. Turns out, the proposed method takes into account the region, not the only one pixel of interest, minimizing the total difference in the region so that a good estimation accuracy is expected in such case.

## 2. SURFACE RECONSTRUCTION WITH SIMULATED ANNEALING

### 2.1 Surface Reconstruction Method for DEM Estimation

Heinrich Enber and Christian Heiphe(1988) and the others proposed Surface Reconstruction Method for estimation of Digital Elevation Model (DEM) with stereo pair of images. In the method, a relationship between the surface of interest and the intensity of the pixel corresponding to the surface is assumed, namely the pixel intensity is a function of DEM and the other factor. Thus the difference between real pixel intensity and the estimated pixel

intensity is expressed by the following equation,

$$d = g - \hat{g}(z, \rho) \quad (1)$$

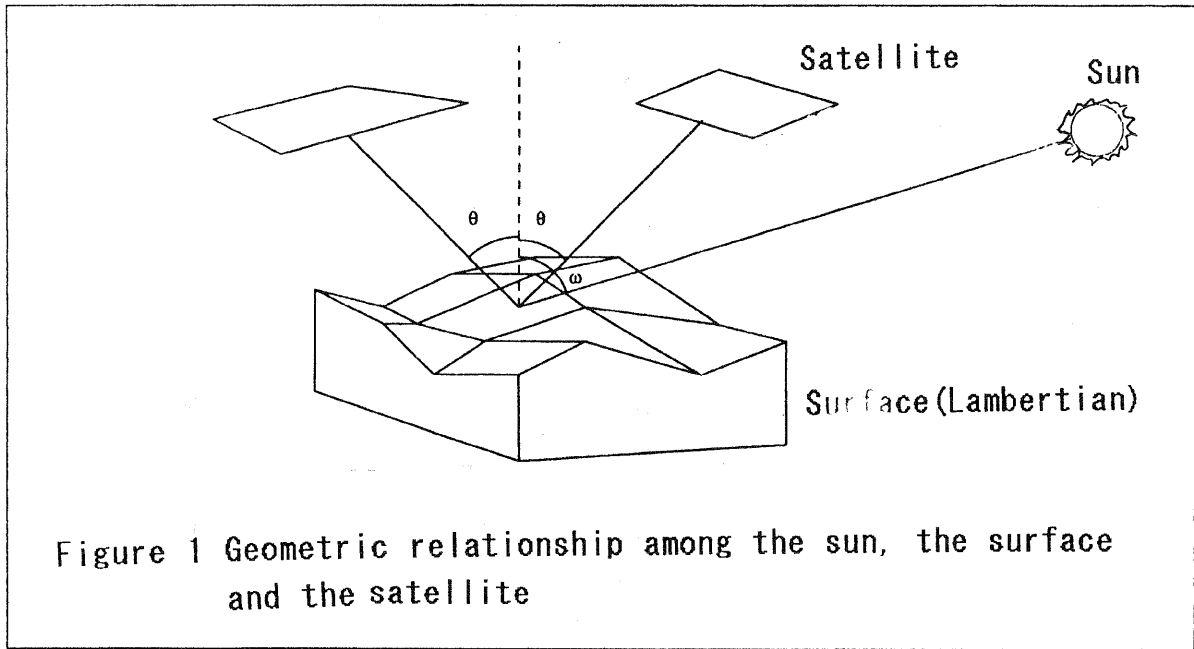
where  $d, g, \hat{g}, z, \rho$  are the difference between real and estimated pixel intensities, real pixel intensity, estimated pixel intensity, estimated DEM and the other factor, respectively. If the difference can be minimized, then the DEM of the pixel of interest can be estimated. This is the fundamental principle for Surface Reconstruction method. This equation is a non-linear equation so that non-linear optimization methods are applicable to solve this equation. On the other hand, the equation can be linearized with Gauss method or Taylor expansion,

$$d = \hat{g} - (\partial g / \partial z) \Delta z - (\partial g / \partial \rho) \Delta \rho - g(z_0, \rho_0) \quad (2)$$

where  $z_0, \rho_0, \Delta z, \Delta \rho$  are initial values and unknown variables for DEM and the other factors, respectively. If co-linear condition can be assumed, then the initial values are determined.

### 2.2 Methodology

In this proposed method, equation (1) will be solved as a non-linear optimization problem. There are several well known optimization problem solving methods. In this study, some of the typical methods are attempted. One of the methods is non-linear least square method like



a Levenberg Markard followed by simulated annealing.  $\sin(\nu) = -\cos(\delta)\sin(h)/\sin(x)$  (4)

### 2.3 Physical Models

On the other hand, the factor p employed physical models for the geometric relationship among the satellite, ground surface and the sun, the typical atmospheric effect described with MODTRAN-3, and the surface conditions that is ground cover target for the pixel of interest is forest and is followed by Lambert's Law (Lambertian Surface).

#### 2.3.1 Geometric Relationship Among the Satellite, the Surface and the Sun

The geometric relationship proposed here is shown in Figure 1.

In the figure solar zenith angle and azimuth angle are expressed by the following equation,

$$\cos(x) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(h) \quad (3)$$

where  $x, \nu, \phi, \delta, h$  are solar zenith angle, solar azimuth angle, Latitude, solar latitude, and solar time angle, respectively.

#### 2.3.2 Atmospheric Transmittance

U.S. Standard Atmosphere is assumed in this study. MODTRAN-3 allows us to calculate atmospheric transmittance as a function of location, solar zenith angle, solar azimuth angle, season and so on so that it is applicable for calculation of arbitrary atmospheric transmittance. Up-welling radiance from the atmosphere should be taken into account. Only the atmospheric transmittance, however, is considered.

#### 2.3.3 Surface Conditions

In terms of surface condition, almost flat plane of the Lambertian surface is assumed in this study so that azimuth and elevation angles and nadir surface reflectivity are the factors for characterization of the surface.

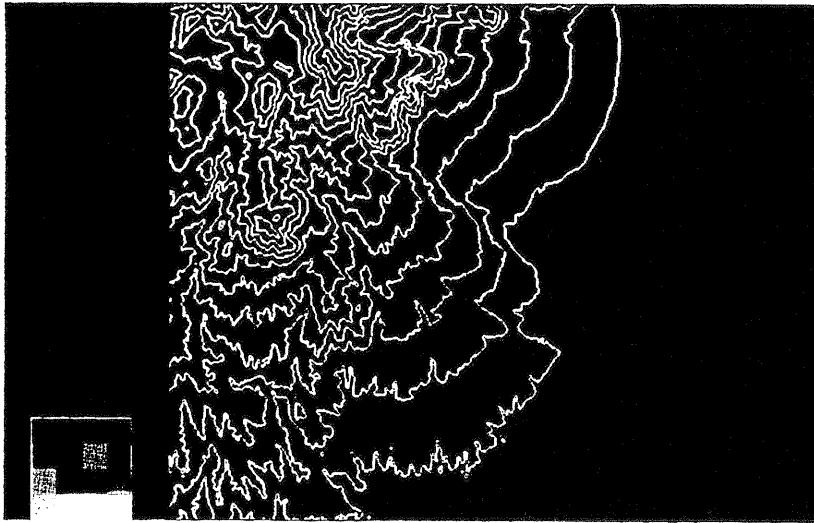


Figure 2 Real DEM data of Unzen, Kyushu in Japan  
Contour line was derived from 50 x 50 m of meshed data

## 2.4 Simulated Annealing

The simulated annealing is one of the optimization methods which allows us to determine a parameter with minimizing the energy,  $E$  which is corresponding to the difference between real and model derived at sensor radiance.

Let me assume that parameter set,  $S$  which is corresponding to the DEM.

## 3. Experiments

The actual DEM data with 50 meter of mesh size of Unzen area in Kyushu Japan, which is provided by Geographical Survey of Japan is used to show the effectiveness of the proposed method through a comparison between the existing surface reconstruction method with non-linear least square method (Levenberg Markard) and the proposed method. Figure 2 shows the DEM data which consists of 512 x 512 pixels corresponding to the 25 x 25 Km<sup>2</sup>.

## 3.1 Geometric Relationship Among the Satellite, the Surface and the Sun

The 256 line is corresponding to the solar azimuth angle with the elevation angle of 45 degree that implies the sun direction is middle of the DEM data. This Solar azimuth and elevation angles are variables and can be designated. On the other hand, satellite altitude is designated at 800 Km. The earth is assumed as the sphere with the diameter of 6,378 Km.

## 3.2 Atmospheric Transmittance

Assuming that wavelength region of the mission instrument is  $0.7\mu\text{m}$ , atmospheric transmittance for nadir viewing is calculated as 0.86. Meanwhile atmospheric transmittance for the arbitrary viewing angle is calculated with MODTRAN-3 in the same manner.

## 3.3 Surface Conditions

The target area is mostly covered with forest so that Lambertian surface with 0.55 of spectral

Table 1 A Comparison Between RMS Errors for the Existing and the Proposed Methods

| Method                                       | Root Mean Square Error in Unit of Meter |
|--|---|
| The Existing Method with Least Square Method | 44.8                                    |
| The Proposed Method with Simulated Annealing | 28.3                                    |

reflectance at the  $0.7\mu\text{m}$  is assumed.

### 3.4 Model derived at sensor radiance image

Model derived at sensor radiance image can be generated throughout from simulated annealing. Figure 3 shows an example for the image which is corresponding to the Unzen, Kyushu in Japan. Through a comparison between real image and the model derived at sensor radiance image, it is clear that the both images are very similar except the detailed portions around steep areas.

### 3.5 RMS error

The root mean square errors on DEM estimation for the existing method and the proposed method are shown in Table 1.

The results show that the proposed method is superior to the existing method in terms of root mean square error of DEM estimation accuracy. One of the reasons for this is that the existing method does not take into account the physical model and

that there are too many unknown variables in the minimization of the real and model derived at sensor radiance. On the other hand, the proposed method take into account the physical models of the geometric relationship among the satellite, the surface and the sun, atmospheric transmittance and the surface reflectance so that these models generate constraints for the solution space in the optimization processes.

## 4. CONCLUDING REMARKS

Further study is required for sensitivity analysis and the others.

Sensitivity of the DEM is not large enough compared to the other factors so that other constraints are required for improvement of the DEM estimation accuracy.

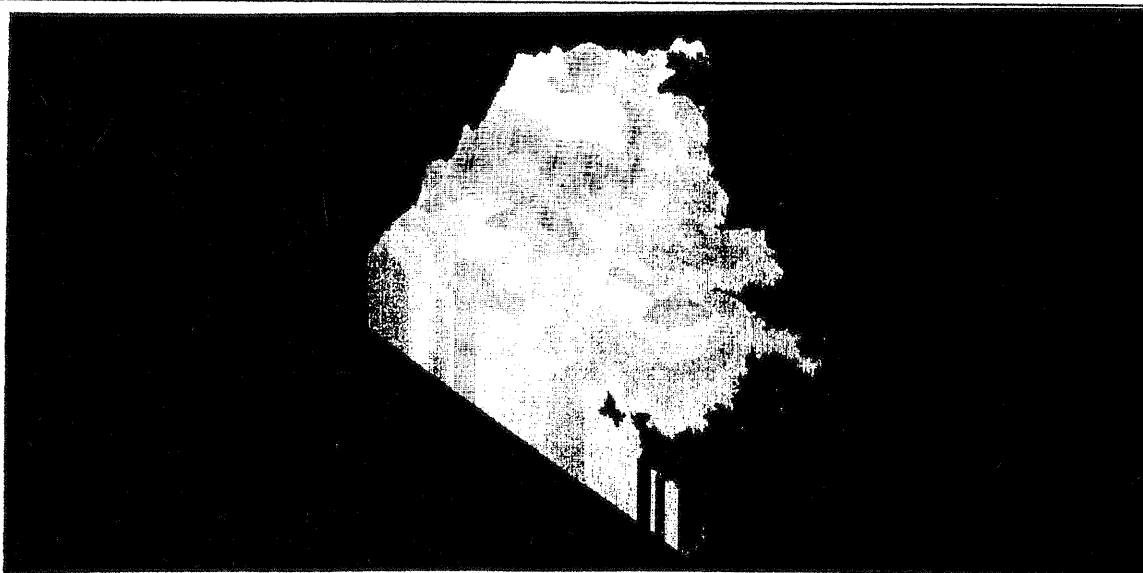


Figure 3 Model derived at sensor radiance image of Unzen, Kyushu in Japan  
(Viewing angle is 30 degree)

## REFERENCES

(1) Heinrich Enber and Christian Heipke, Integration of digital image matching and object surface reconstruction, 1988.

(2) Saul A. Teukolski and William T. Vettering, William H. Press, Numerical Recipes in C.