OUTLIER DETECTION IN RELATIVE ORIENTATION - REMOVING OR ADDING OBSERVATIONS

Peter Axelsson
Department of Geodesy and Photogrammetry, Royal Institute of Technology
100 44 Stockholm, Sweden
e-mail pax@geomatics.kth.se

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ABSTRACT:

Outlier detection in relative orientation has been studied using three different strategies, *(i)*, removing bad observations, *outliers*, using *data snooping*, *(ii)*, adding good observations using *Least Median of Squares estimates* and *(iii)*, finding an optimum between good and bad observations in a cost function using *minimum description length* criteria, *MDL*. To be able to compare the strategies, relative orientation algorithms based on linear or closed formulae were used. Two algorithms for calculating relative orientation without any provisional values were applied. For a *LS* solution a linear solution based on eight unknowns was used. For a direct solution with minimal point configurations the same algorithm was used but estimated without any redundancy, *i.e.*, with eight observations. In addition to this, two other algorithms were applied to some of the data in order to get the results verified and compared. In total, four algorithms tested on a large number of simulated data configurations. The results show that large fractions of outliers can be detected for the strategies *(ii)* and *(iii)* even with arbitrary image orientations, while strategy *(i)* shows very good stability for normal aerial geometry with few outliers.

1. INTRODUCTION

Procedures that do not need provisional values for the calculation of relative orientation parameters of a stereo pair of images are of great need in general applications where approximate locations and attitudes of the cameras are unknown. When automating point selection and point identification, these procedures must also be more robust against large fractions of outliers than in the case of manual measurements.

The traditional way of handling outliers in photogrammetry is by investigating diagonal elements in the least squares, *LS*, estimate of the covariance matrix of the observations and their residuals. The observations are then kept or removed depending on some statistical test, *e.g., data snooping* [Baarda,1967]. In such a procedure, all observations are part of the initial *LS* estimate and outliers are removed one at a time. A different strategy is to start with a minimum configuration of observations, adding points as long as they fulfil some criterion. The solution of the minimum point configuration is often repeated with different random sets of points and the "best" solution chosen, *e.g.*, least median squares, *LMedS* [Rousseeuw,1987] or *RANSAC* [Fichler and Bolles, 1981]. Both strategies regard outliers as observations not belonging to the model, i.e., not having the same statistical properties as good observations. A third strategy is to extend the mathematical model to include also outliers and by a cost function locate a minimum where the optimal number of outliers is found, Such a cost function has been formulated within the minimum description length, *MDL*, principle [Rissanen, 1983] and used as an estimator with robust properties [Axelsson, 1992].

2. AIM OF THE INVESTIGATION

A comparison of the three strategies was made regarding the robustness against outliers when calculating the relative orientation of a stereo pair of images. The three strategies are:

- i Removing bad observations, outliers, using *LS* estimates and *data snooping*
- ii Adding good observations using *LMedS*
- iii Finding an optimum between good and bad observations in a cost function using *MDL*

To be able to compare the strategies, only relative orientation algorithms based on linear or closed formulae were considered. For the *LS* solution a linear solution based on eight unknowns were used [Tsai and Huang, 1984], [Philip, 1989]. For the direct solution with minimal point configurations the same algorithm as for the *LS* solution was used, but estimated without any redundancy, *i.e.*, with eight observations.

In addition to this, two other algorithms were applied to some of the data in order to get the results verified and compared. In total, four algorithms were used in the investigation:

1. Linear overdetermined *LS* estimate [Philip, 1991]
2. Linear *LS* estimate with minimal point configuration.
3. Closed six-point formula [Hoffman-Wellenhof, 1979].
4. Iterative *LS* solution with five unknowns

3. METHODS

3.1 Estimating the orientation parameters

The relative orientation of two images is described as a rigid movement of the bundles of rays by a translation and a rotation

$$v = Ru + T$$

u and v being the projective coordinates $[u_x, u_y, 1]$ and $[v_x, v_y, 1]$ of an model point, R a 3x3 rotation matrix and $[t_x, t_y, t_z]$ a translation vector with known direction but unknown length.

In photogrammetric notation the equation is usually written as

$$\begin{bmatrix} x'' \\ y'' \\ -c \end{bmatrix} = R \begin{bmatrix} x' \\ y' \\ -c \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

Five parameters have to be estimated in order to solve the equation system with a minimum of five corresponding point pairs, *e.g.*, ω, φ, κ, b_y and b_z in the photogrammetric notation. This involves a linearisation of the non-linear equations and an iterative estimation technique that requires approximate values of the unknowns, making it unsuitable for general cases with arbitrary orientations.

A linear solution has been formulated in the photogrammetric society by, *e.g.*, [Stefanovic, 1973], [Thompson, 1968] and later in the computer vision society by, *e.g.*, [Tsai & Huang, 1984]. The linear solution is based on the coplanarity condition stating that the vectors *v*, *Ru* and *T* are in the same plane.

$$\begin{vmatrix} v_x & Ru_x & T_x \\ v_y & Ru_y & T_y \\ v_z & Ru_z & T_z \end{vmatrix} = v(Ru \times T) = 0$$

This can be written as [Stefanovic]

$$v^t CRu = 0$$
or
$$v^t Eu = 0$$

where C is the skew-symmetric matrix

$$C = \begin{bmatrix} 0 & t_3 & -t_2 \\ -t_3 & 0 & t_1 \\ t_2 & -t_1 & 0 \end{bmatrix}$$

and $E=CR$ is called the *essential matrix*. The *essential matrix* can be characterised in different ways, but using *singular values* is the most common in literature [Tsai & Huang, 1984]. Let $E=USV^t$, be the singular value decomposition, *SVD*, of *E*, where *S* is a diagonal matrix $S=diag(s_1, s_2, s_3)$. A matrix is an *essential matrix* if and only if $s_1=s_2$ and $s_3=0$ [1]. This also implies that the matrix is of rank (2) and that

$$EE^tE = \frac{1}{2} trace\left(EE^tE\right)E .$$

The decomposition of E into R and C is a non-trivial task, but has been solved by, *e.g.*, [Brandstätter, 1991] and in a more general form using *SVD* by [Tsai&Huang, 1984].

Since there are eight unknowns in the *essential matrix* but only five parameters needed for the relative orientation, linear dependencies might exist in the linear solution that will give a biased or singular solution. This can be seen when writing the explicit equation of the projective coordinates (x', y', 1) and (x", y", 1).

$$x'x''e_{11} + x'y''e_{12} + x'e_{13} + y'x''e_{21} + y'y''e_{22} + y'e_{23} +$$
$$x''e_{31} + y''e_{32} + 1 = 0$$

For image pairs with almost parallel optical axes and small rotations, as in the case of aerial images, the coordinates of $y' \approx y''$ and e_{23} and e_{32} will be dependent. In some extreme, but not trivial, cases the number of unknowns will reduce to five. These dependencies give rise both to numerical problems in the estimation process and to a bias in the estimated parameters.

The fact that the solution is sensitive to noise is well known and has been addressed by, e.g., [Hartley, 1995] and for the *fundamental matrix* in the uncalibrated camera case and by [Hahn, 1995] and [Philip, 1996] for the *essential matrix* and calibrated cameras. To ensure that the estimated E matrix is of rank (2) and that *the singular values* $s_1=s_2$ and $s_3=0$ is not sufficient to remove the bias in the estimate.

In the used implementations, the numerical problems with singularities are handled using the *SVD* algorithm when solving the equation system. The algorithm is numerically more stable than, *e.g.*, the *Cholesky* algorithm and singularities can be treated and eliminated during the estimation. The bias is removed by minimising the $\Sigma(\rho^2)$, where ρ is the distance to the epipolar line, *i.e.*, the y-parallax for aerial images, using the conjugate-gradient algorithm.

The difference between the results from the original linear solution and the improved solution can be seen in *table 1*. The residual's from the linear solution show a clear bias. In the improved solution the bias is removed and the standard error reduced. It is only the improved linear solution that has been used in the experiments.

[1]These requirements hold for a calibrated camera, i.e., with known principal point and principal distance. For the uncalibrated case, the matrix is called the *fundamental matrix* and the requirements on the singular values are that $s_1 \geq s_2$ and $s_3=0$.

Point number	Residuals linear solution	Residuals improved solution
1	0.0009	-0.0001
2	0.0032	-0.0034
3	0.0027	-0.0011
4	0.0042	0.0009
5	0.0070	0.0018
6	0.0020	-0.0030
7	0.0051	0.0009
8	0.0079	0.0026
9	0.0028	-0.0005
10	0.0027	-0.0014
11	0.0072	0.0024
12	0.0023	0.0006
	std error: 0.006	std error: 0.002

Table 1 The bias in the linear solution

3.2 Criteria for Outlier Classification

Once the parameters are estimated, erroneous observations should be classified as outliers by some criterium. For the *LS* estimates using all observations, statistical methods based on standardised residuals, v_i/σ_{vi}, are well established. Other estimation methods, based on different minimising functions, uses other test statistics or criteria.

3.2.1 Data Snooping: The method of *data snooping* uses the standardised residuals, v_i/σ_{vi}, for outlier detection, where the

$$\sigma_{vi} = \sigma_0\sqrt{Q_{v_i v_i}}$$

is computed from the *LS* estimated covariance matrix of the residuals,

$$Q_{vv} = Q_{ll} - A\left(A^t PA\right)^{-1} A^t$$

The matrix $A(A^tPA)^{-1}A^t$ is the estimated covariance matrix of the observations, called the A^0*-matrix* in photogrammetry and geodesy and the *hat-matrix* in statistics. When σ_0 is not known a priori but estimated from the observations the following test statistics is used [Förstner, 1985]

$$\overline{w}_i = \frac{-v_i}{\tilde{\sigma}_{0i}\sqrt{Q_{v_i v_i}}} = \frac{-v_i\sqrt{p_i}}{\tilde{\sigma}_{0i}\sqrt{r_i}}$$

The estimated σ_0 is calculated as

$$\tilde{\sigma}_0 = \frac{\left(\sum v^t pv\right) - v_i^2 p_i / r_i}{r-1}$$

The test statistics $|w_i|$ is compared to a critical value, which depends on the significance level of the test. The experiments in this study are tested on a level of 99%.

3.2.2 Least Median Squares, LMedS: The method of *LMedS* [Rousseeouw, 1987] minimises the squared sum of the medians of the residuals, $\min med(r_i^2)$. The estimate is found by a repeated search algorithm using a subset or minimum configuration of the observations. The method has a high theoretical breakdown point but the search algorithm is, in practice, only useful for low number of unknowns as in the case of relative orientation. The way the estimated σ_0 is calculated is partly based on empirical investigations. An observation is accepted if the test statistics $w_i = r_i/\sigma_0 < 2.5$, where

$$\sigma_0 = 1.4826(1 + 5/(n-p))\sqrt{\text{med } r_i^2}$$

When the number of unknowns grow, the number of possible combinations of observations grow dramatically. For a given maximum fraction of outliers, it is however possible to estimate the number of combinations required to reach a given certainty level. In the case of linear relative orientation with eight unknowns and 18 observations, there are 43758 combinations but at a maximum fraction of 40%, the number of combinations needed to get an error-free sample at a certainty level of 95% is only 177.

3.2.3 Minimum Description Length, MDL: The basic idea in *MDL* states that if the observed data are dependent or non-random, *i.e.*, is possible to model, then the expected description length of the modelled data will be less than the *description length, DL,* of the un-modelled data itself. Enough but no redundant information should be provided for decoding and restoring the data.

When using the *MDL* criterion as an estimator with robust properties, the parametric model is fixed. The different models which are compared are instead the different combinations of data belonging/not belonging to the parametric model. The data is modelled to the parametric model in such a way that the *MDL* is found.

When the parametric model is fixed and not compared with other models, several parts of the *DL* are constant, like *e.g.* the description length of the parameters. The remaining parts which have to be computed are:

$DL_{total} =$

$n_e 4lb\frac{R}{\varepsilon} +$ [2]	*DL* for the n_e outliers
$n_m 3lb\frac{R}{\varepsilon} +$	*DL* for the n_m model points
$DL(deviations)$	*DL* for the gaussian noise

where

n_e	the number of outliers
n_m	the number of model points
$n_e + n_m$	the total number of observations

[2] Here *lb* is the logarithmic function to basis 2, *i.e., lb* $x = {}^2log\ x$. The resulting unit for measuring information is called bits.

R	the range of the data (approx. the image area)
ε	the resolution of the observations

Random combinations of points were selected and the relative orientation computed with the eight and six points algorithms. For each combination, residuals were computed for all points. Points with high residuals were removed until the *DL* had reached its minimum. The combination giving the lowest *DL* was selected.

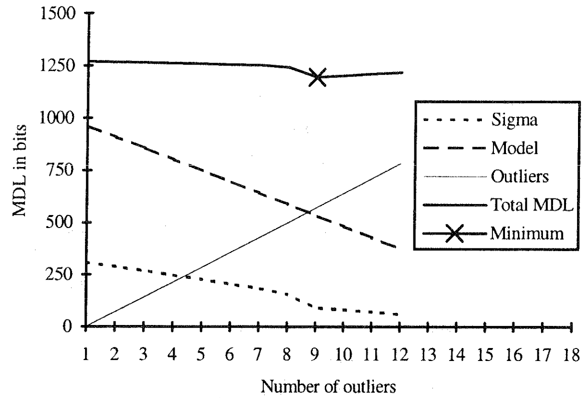

fig. 1 Illustration of the different parts of the MDL calculations

4. EXPERIMENT

For the comparison of the strategies, four data sets were generated. Two sets having random translations and orientations and two sets having typical aerial image translations with 60% overlap, *table 2*. A Gaussian noise was added to the initial simulated measurements and a random gross error was added during the estimation process, in one point at a time, until a false solution was encountered, *i.e.*, to the breakdown point.

Data Set	*Type of translation*	*Noise level*	*Type of errors*
I	*Random*	*0.5 ‰*	*Large*
II	*Random*	*2.5 ‰*	*Large*
III	*Aerial*	*0.05 ‰*	*Large*
IV	*Aerial*	*0.05 ‰*	*Small*

Table 2 Description of the four data sets

Each data set consisted of 100 point configurations, *i.e.*, sets of relative orientation points, and each point configuration consisted of 18 point pairs. The gross errors were of two kinds, large and small. The large errors were at a random position within the image area and the small errors within the neighbourhood of the observation.
The noise level of set I was approximately equivalent to a σ_0 of 10μm and for set II of approximately 50μm for a small format camera. The noise level of set III and IV was equivalent to a σ_0 of 5 μm for a 23x23 cm aerial image.

The relative orientations of the point configurations were calculated with the four different algorithms. The forth algorithm, the 5-point iterative *LS* algorithm, were only applied to set III and IV since the approximate translations and rotations were assumed to be small and known *á priori*.

5. RESULTS

The results from the calculations are presented in figures, showing the number of successful relative orientations for the different data sets. A relative orientation was classified as successful if the correct number of outliers was discovered and did not depend on how close the estimated parameters were to a true value. Both errors of *type I*, removing correct observations, and *type II*, omitting to remove erroneous observations, are indicated as failures in the histograms.

5.1 Strategy 1: Removing bad observations

The linear *LS* algorithm was used together with data snooping to remove erroneous observations (fig 2).

Linear LS solution, Data Snooping

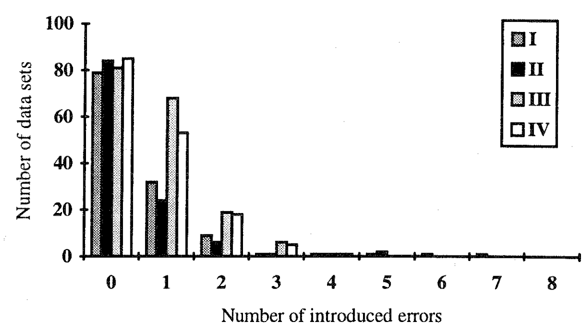

fig. 2 Removing bad observations, strategy (i), LS linear estimate, algoritm 1

As a comparison, an iterative 5 point *LS* algorithm was applied to the data set of aerial configurations, set III and IV (fig 3).The iterative *LS* solution could not be used on set I and II since arbitrary orientations could not be handled and approximate values were un-known.

Iterative LS solution, Data Snooping

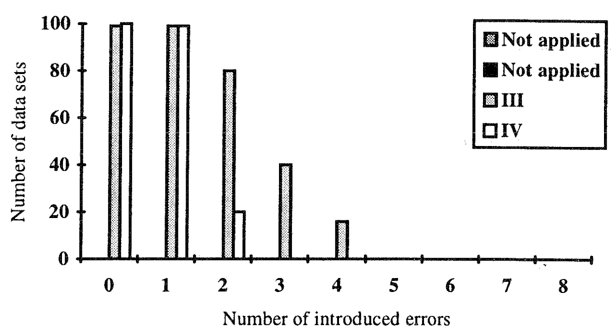

fig. 3 Removing bad observations, strategy (i), LS iterative estimate, algoritm 4

The linear solution showed a rather unstable behaviour and very few of the outliers were detected, especially in the general cases. The iterative algorithm shows a much more stable behaviour for the aerial image data, but also here very few of the outliers are detected. For "small" errors, set III, one error is detectable but not more.

5.2 Strategy II: Adding good observations

The linear solution using eight points with repeated solutions were used together with the *LMedS* criteria for including correct observations (fig 4).

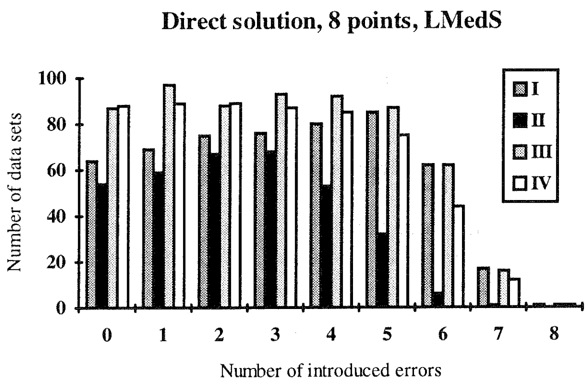

fig. 4 Adding good observations,strategy (ii), linear solution with 8 points, LMedS, algorithm 2

The *LMedS* algorithm finds the solutions and the correct observations even for large number of outliers, but shows an unstable behaviour when the numbers of outliers are small. This is mainly due to errors of *type I*, *i.e.*, correct observations have been removed. The reason for this lies partly in the behaviour of the relative orientation. If eight points are picked randomly some correct observations might very well have large residuals even if the median is low. The presence of noise influences the result in a similar way as for small gross errors. The number of errors of *type I* increases very quickly.

5.3 Strategy III: Including the outliers in the model

The linear solution using eight points with repeated solutions were used together with the *MDL* criteria for modelling correct observations and outliers in a cost function (fig 5). The results of the *MDL* cost function shows a similar behaviour as the *LMedS* algorithm, but the unfavourable behaviour for few outliers of the *LMedS* algorithm is not present.

When applying the six-point algorithm formula [Hoffman-Wellenhof, 1979].instead of the eight-point algorithm to the data some interesting observations can be made. The algorithm has a higher percentage of recovered orientations when data contains large fractions of outliers. For few outliers the algorithm makes more errors of type I, especially for "small" error in data set IV and a high noise level in data set II.

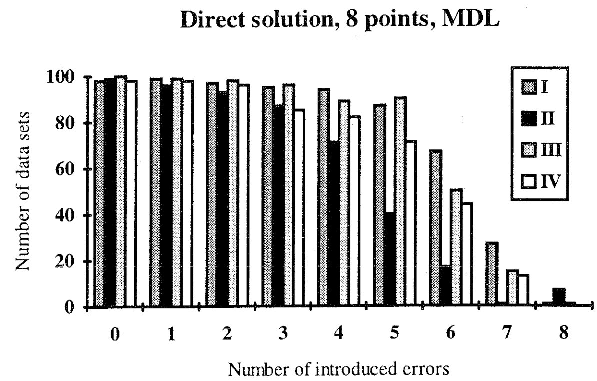

fig. 5 Modelling correct observations and outliers using MDL in a cost function, strategy (iii), eight points linear solution, algorithm 2

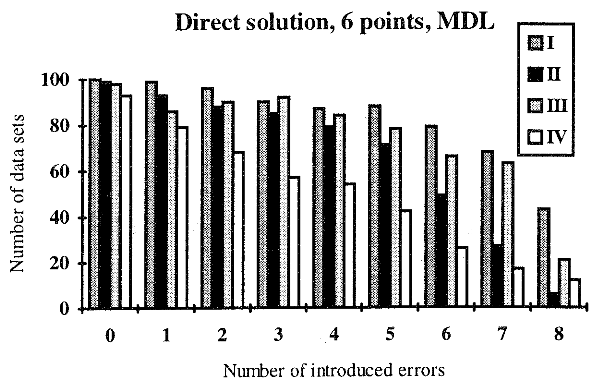

fig. 6 Modelling correct observations and outliers using MDL in a cost function, six points algorithm, algorithm 3

6. DISCUSSION

The calculation of relative orientation parameters in the general case is a difficult task, in which the geometry of the observations interact with dependencies and correlations of the estimated parameters in a very complex manner. Large fractions, and indeed also small fractions, of erroneous observations or outliers can severely disturb the solution.

The chosen algorithms in this study are not claimed to be the optimal ones, but the conclusions and comments are believed to be general.

Several methods for detecting outliers and making the estimates more robust than the *LS* estimate have been presented in the photogrammetric society over the years, most of the methods having in common that they look at the residuals originating from some type of *LS* estimate using all data. Observations meeting some criteria are then kept while others are removed or given new weights and the process is iterated until no more points are removed. The types of weight functions and test statistics range from looking at the residuals [Krarup et al 1980] to more statistically elaborate methods like *data snooping* and the *Balanced L1-norm* suggested by [Kampmann & Wolf, 1989]. The major drawback with these methods is that they will fail if the first estimate is too far away from the true solution. In the relative orientation problem a few number of outliers may be enough to make the solution degenerate completely. In the tests carried out in this study it can be seen very clearly that one, or possibly two, errors can be found. When the number exceeds two or three points, or the fraction of outliers goes beyond 10-15% these methods are very likely to fail. In these cases other strategies must be applied.

Algorithms based on repeated calculations using a small sample of data, sometimes referred to as *RANSAC* or *bootstrap* methods, are in many cases able to find a solution close to the optimal one and to identify large fractions of outliers. Since only a small sample of data is used for the solution, expectancy values and standard errors are not possible to calculate as in a *LS* estimate. Depending on the purpose of the calculation, the estimates can of course be improved by a *LS* adjustment on the remaining data after removal of the outliers. If the sample is chosen by random, as in all cases in these tests, it is very likely that the selected solution contains observations close to each other. Residuals of observations far from these tend to be high since the model coordinates are extrapolated. Due to this, errors of type I are more common than for methods using all available data. If enough precautions are taken to ensure that observations are not removed by mistake and one has an awareness of the limitations of the solution, these methods are well suited for limited tasks like the relative orientation.

The third strategy, to include the errors in the model and calculate a cost function, shows a very nice behaviour both for few numbers of outliers as well as for many. Some additional information must be provided in order to compute the *DL's* that is not needed for the other methods. This information defines the range or bounds of the observations and its resolution. The calculations of the *DL's* are not very complicated and could be considered as an alternative in some implementations.

For autonomous systems with arbitrary orientations, estimates based on linear algorithms using repeated calculations on small samples of data seem to be a fruitful way of getting robust results. For standard aerial images, these methods can be used as well, but standard methods, like *data snooping* and iterative five points algorithms, are more stable as long as the number of outliers are low. The answer to the question put forward with this study, whether to remove or add outliers, is, not very surprising, thus depending of the application and the expected types of errors and error fractions that might occur.

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