

# CONJUGATE POINTS IN THE ORIENTATION OF ACROSS AND ALONG TRACK STEREOMODELS

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## ABSTRACT:

Conjugate points have long been used in the orientation of aerial and spaceborne imagery. In this study, a physical and polynomial model were used for testing the use of conjugate points in linear array imagery from higher altitude platforms. SPOT and OPS data were used for the tests, with the ultimate objective of reduction of the ground control needed for their orientation. The orientation models are summarised and the use of conjugate points is assessed. A summary of the results obtained is presented for different types of control configurations adopted. The several problems found using conjugate points in the orientation of linear array imagery are discussed. The main draw backs found were that the algorithms become very time expensive and occasional problems of convergence arise. The use of weight matrices and the information from the co-variance matrix does not help solve the problem. The results obtained show no advantage in adopting such method when enough control exists. However, this technique may be used to ensure a model orientation when the ground control is not enough on its own. The information extracted from conjugate points seems to be more helpful for the physical than for the polynomial orientation algorithm. The relative orientation of the model shows little improvement for the former, while no improvement is registered for the latter.

## 1. CONJUGATE POINTS AND ALGORITHM DEVELOPMENT

Algorithms developed for the orientation of airborne and spaceborne imagery usually make use of control points with known ground coordinates. However, the cost of a map product is directly dependent upon the cost of the imagery orientation procedure, hence upon the number of control points used. Reducing the ground control to a minimum is therefore a major concern during algorithm development.

Most researchers direct their effort into the use of on-board registered data. Such data vary with sensor and platform, usually consisting of the position and velocity of the platform for regular intervals of time. These are usually very sparse in time and have to be interpolated for the time of image acquisition. Other data usually also measured on-board are the platform attitude variations with time. The use of these data is proving to be very efficient when introduced to the orientation algorithms. It improves the initial solution for the orientation parameters, especially those modelling the attitude of the platform.

Other techniques for the reduction of ground control have been explored with success in photography, such as the use of conjugate points. Conjugate points are common points identified on the photographs of a

stereopair, for which the ground coordinates are unknown. Once introduced to the orientation model, conjugate points help improving the relative orientation of the model.

Linear array imagery is becoming increasingly important for map production and revision. Although this type of imagery is more common from higher altitude platforms, and has lower resolution, robust orientation algorithms have been widely tested, with successful results. They have been described in various bibliographies, and the results compared [Neto and Dowman, 1991]. In the case of SPOT, one main conclusion from these tests was that a minimum number of 6 ground control points would be advisable for a best orientation without extra data. Similar results were obtained for other data using simulation studies [Neto, 1993].

IGN has considered in the past the use of conjugate points for the orientation of SPOT imagery [Rodriguez *et al.*, 1988]. With the move toward along track linear array satellite sensors, it becomes necessary to consider such data in any study of this type. For this reason, the author studied the effect of using conjugate points in the orientation of SPOT and OPS data, and analysed the effect on the reduction of the necessary ground control. This study was carried out using a physical and polynomial model of the orbit of the platforms. The physical model is summarised in this paper.

## 2. ORIENTATION MODELS

### 2.1 Physical model

The model used for the orientation of linear array imagery from satellite sensors is described in Neto [1992, 1993] but is summarised here. It differs slightly for across and along track stereopairs.

A geocentric co-ordinate systems is used and the satellite orbit is described using Eulerian parameters (  $a, e, i, \Omega, \omega, F$  ) to fix the position of the satellite in space where  $a$  is the semi-major axis,  $e$  the eccentricity,  $i$  the inclination,  $\Omega$  the longitude,  $\omega$  the argument of perigee and  $F$  is the true anomaly of the orbit.

The major components of dynamic motion are the Earth's rotation and the satellite movements along the orbit path. These motions have been modelled as linear angular changes of  $F$  and  $\Omega$  with time.

For across track stereopairs, the collinearity equations are used for the orientation of a single image and the rotations are described by the orbital elements and the attitude of the sensor.

For alongtrack stereopairs an additional parameter is required, namely a variable to represent the time displacement,  $\Delta t$ . Hence if the first image is arbitrarily chosen,  $\Delta t$  sets the position of the second image relative to the first, as shown in figures 1 and 2.

The two images of an alongtrack stereopair are taken during a common orbit and have the same values for semi-major axis, inclination, longitude of the ascending node and argument of perigee. If the origin of the first image is taken as origin of the second image, the values of the other orientation parameters are also common to both images, as they are set for the origin. However, the points are identified in the second image by their line and sample values, and the line number being a measurement of time on the second image is not related to the origin of the first image. The time displacement  $\Delta t$  acts as the translation in time suffered by the second image relative to the first, so that the orientation parameters for each line of the second image are affected by the line position  $x$  (measurement of time) plus  $\Delta t$ . The orientation parameters for each position become then dependent on  $\Delta t$  and correlation occurs.

An attitude model can be initially formed using the attitude data file provided. The attitude parameters used in the iterative process adjust this attitude model to the ground control.

An across track stereopair can be oriented using 6 control points per image, providing 12 observation equations, for the 10 parameter solution. An along track stereopair can be oriented using as few as 3 control points per image,

providing 12 observation equations, for an 11 parameter solution. The number of control points may be decreased if a poorer orientation is accepted, for example when 2 control points per image are used, then a 7 parameter orientation is possible with some redundancy.

The model can accept information on the position and attitude of the sensor if it is available, but this is not a prerequisite for a solution.

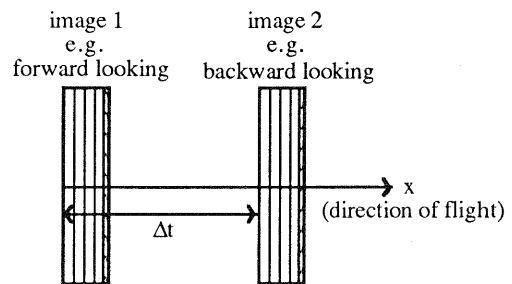


Figure 1. Time displacement  $\Delta t$  between two images taken during the same orbit (represented on the image).

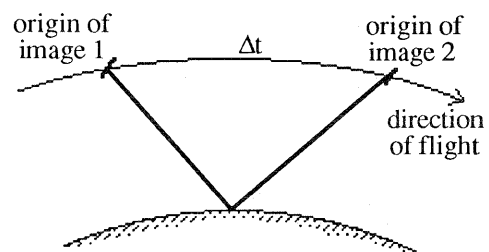


Figure 2. Time displacement  $\Delta t$  between two images taken during the same orbit (orbit representation).

### 2.2 Polynomial model

The polynomial model varies from the physical model in the way that the position of the sensor is modelled. In this case, first order polynomials were adopted to describe the position of the sensor in space, instead of using the orbital parameters. Higher order polynomials were tested, with no improvement in the final accuracy of the models. Due to the smooth characteristics of satellite orbits, this model can be applied to short arcs of orbits. Tests with simulated data for long arcs of orbits show that the accuracy worsens and the model is not as stable.

For the orientation of along track stereoscopic pairs, the use of the time displacement parameter is still used but the algorithm needs more iterations for convergence. The physical model is clearly more stable than the polynomial approach.

In this case, an across track stereo pair can be oriented using as few as 7 control points per image, providing 14

observation equations, for the 12 parameter solution. The number of control points may be decreased if a poorer orientation is accepted. An along track pair of images, taken in the same orbit, can theoretically be oriented with only 4 ground control points per image, for the 13 parameter solution, giving a solution with some redundancy. A reduction in the number of ground control used and of the orientation parameters settings, results in a non-convergence of the algorithm or unacceptable results.

### 2.3 Conjugate points

Conjugate points are control points identified in more than one image, whose ground coordinates are assumed unknown. Each ray pair gives one coplanarity equation.

The ground coordinates  $(X_A, Y_A, Z_A)$  of a conjugate point A are unknown and the method consists in orienting a pair of images so that a conjugate point forms two ray pairs coming from each image that will intersect in space. These have thus to lay in the same plane, which is described by the so-called coplanarity equation.

In figure 3,  $\vec{A}_i$  and  $\vec{A}_j$  denote de vectors originating at exposures centers of each image, i and j, passes through the images at points  $a_i$  and  $a_j$  and ends at point A. The vector  $\vec{B}$  extends from one projection center to the other. For the two rays vectors  $\vec{A}_i$  and  $\vec{A}_j$  to intersect in space the following condition must apply:

$$(\vec{A}_i \times \vec{A}_j) \cdot \vec{B} = 0 \quad [1]$$

Still from figure 3, the vectors  $\vec{A}_i$  and  $\vec{A}_j$  can be expressed as:

$$\begin{cases} \vec{A}_i = (X_A - X_s^i) \cdot \vec{i} + (Y_A - Y_s^i) \cdot \vec{j} + (Z_A - Z_s^i) \cdot \vec{k} \\ \vec{A}_j = (X_A - X_s^j) \cdot \vec{i} + (Y_A - Y_s^j) \cdot \vec{j} + (Z_A - Z_s^j) \cdot \vec{k} \end{cases} \quad [2]$$

Let

$$\begin{cases} u_i = r_{11} \cdot x + r_{12} \cdot y - r_{13} \cdot f \\ v_i = r_{21} \cdot x + r_{22} \cdot y - r_{23} \cdot f \\ w_i = r_{31} \cdot x + r_{32} \cdot y - r_{33} \cdot f \end{cases} \quad [3]$$

then [2] can be written in the form

$$\begin{cases} \vec{A}_i = \lambda_i \cdot (u_i \cdot \vec{i} + v_i \cdot \vec{j} + w_i \cdot \vec{k}) \\ \vec{A}_j = \lambda_j \cdot (u_j \cdot \vec{i} + v_j \cdot \vec{j} + w_j \cdot \vec{k}) \end{cases} \quad [4]$$

and

$$\vec{B} = (X_s^j - X_s^i) \cdot \vec{i} + (Y_s^j - Y_s^i) \cdot \vec{j} + (Z_s^j - Z_s^i) \cdot \vec{k} \quad [5]$$

The coplanarity condition of equation [1] is then satisfied if the following determinant is nil, as in [6].

The lack of coplanarity results in a residual  $F_i$ , which can be expressed as the observation equation [7].

$$\begin{vmatrix} (X_s^j - X_s^i) & (Y_s^j - Y_s^i) & (Z_s^j - Z_s^i) \\ u_i & v_i & w_i \\ u_j & v_j & w_j \end{vmatrix} = 0 \quad [6]$$

$$\begin{aligned} & (X_s^j - X_s^i) \cdot (v_i \cdot w_j - v_j \cdot w_i) + \\ & (Y_s^j - Y_s^i) \cdot (w_i \cdot u_j - w_j \cdot u_i) + \\ & (Z_s^j - Z_s^i) \cdot (u_i \cdot v_j - u_j \cdot v_i) = F_i \end{aligned} \quad [7]$$

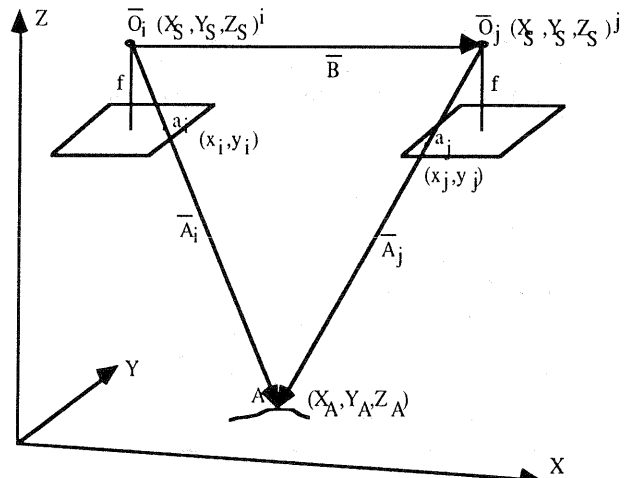


Fig 3- Coplanarity vectors.

This information is expected to improve the relative orientation of the model. It does not influence the absolute orientation of the model, and it can be used in two different ways.

First, it can be used in conjunction with ground control. The ground control can be reduced and the number of observation equations required for the model orientation may be completed using conjugate points. In this case, only the relative orientation of the model is expected to be improved.

Second, conjugate points can be used alone and a relative orientation of the model should be possible. However, the program is rather different from classical methods of orienting aerial photography, and this was impossible to achieve with the algorithm described.

A minimum of two control points per image is advisable for a good absolute orientation of the model. However, more control is needed for an orientation considering all the parameters. Two ground control points give eight observation equations for each image of an across track stereopair. If only conjugate points are to be added to this set of control, a minimum of four points is required to obtain the four lacking equations. For the case of along track imagery, two ground control points give four observation equations, and an extra eight conjugate points are necessary to obtain the eight more equations needed.

#### 4. RESULTS OF TESTS

##### 4.1 Tests with SPOT data

###### SPOT data

The SPOT data used covered an area in South East France, used in earlier orientation studies [Dowman *et.al.*, 1991]. The main characteristics of the data are summarised in table 1. A set of 106 ground control is known for the area, and on-board registered data is available.

Altitude	830km
No of CCDs per line	6 000
CCD size	13µm
Pixel size	10m x 10m
Principal distance	1082mm
Across track angle	Image 1 22.3° Image 2 20.5°
B:H	0.8

Table 1 - Characteristics of SPOT sensor and data.

###### Results of tests with SPOT data

Several tests were carried out with the SPOT data for different control configurations. Table 3 summarises some of the results obtained using the physical orientation model, with an indication of the number of control and of conjugate points used. Some of the results obtained using the polynomial model are summarised in table 4.

The use of the on-board registered data gives a good initial orientation model, reducing the number of iterations necessary. Although worse results were obtained using the polynomial approach, it still proved to adapt well to the short arcs of the orbit. However, the polynomial algorithm did not converge where less than three control points were used.

The final SPOT models were analysed and compared. Although a slight improvement in the relative orientation characterised by smaller errors in the skewness parameters was observed for the models oriented using conjugate points, this improvement was not significant when compared to the errors found. Contrary to what was initially expected, the relative orientation was not significantly improved in the case of SPOT data, and the algorithm becomes more time expensive when conjugate points are used.

No iter	No contr.	No conj.	rms (m) [in UTM projection]				
			E	N	H	2D	3D
4	6	0	9.7	8.3	6.6	12.8	14.4
3*	6	0	9.8	8.5	6.5	12.9	14.5
5*	6	4	9.6	7.9	6.4	12.4	14.0
6*	4	8	12.0	10.5	8.1	15.9	17.9
7*	2	12	13.4	14.1	8.7	19.4	21.3
7*	2	16	13.2	14.3	8.5	19.5	21.2

Table 3 - SPOT model accuracy after orientation with several control configurations [\*using header data], using the physical orientation algorithm.

No iter	No contr.	No conj.	rms (m) [in UTM projection]				
			E	N	H	2D	3D
4	7	0	10.9	11.5	7.1	15.8	17.3
4*	7	2	10.8	11.5	7.0	15.8	17.2
4*	6	5	12.3	14.1	8.3	18.7	20.4
5*	4	10	14.4	14.5	9.9	20.4	22.7
6*	4	16	14.4	14.1	9.6	20.2	22.3

Table 4 - SPOT model accuracy after orientation with several control configurations [\*using header data], using the polynomial orientation algorithm.

##### 4.2 Tests with OPS data

###### OPS data

A summary of the OPS data used is given in table 5. The data covered an area in the French Alps around the town of GAP. As earlier reported by Dowman and Neto [1994], 40 control points on the two images and their ground coordinates were extracted from 1:25,000 maps of the region. However, many problems were experienced during the identification process of which most were related to the difficulty of finding well defined points on the imagery. The ground control available was concentrated over the area in three main clusters, which is not an ideal control configuration for the orientation process.

Altitude	568km
No of CCDs per line	4 096
CCD size	7 $\mu$ m
Pixel size	18.3m x 24.2m
Principal distance	213.5mm
Along track angle	Image 1 0° Image 2 15.3°
B:H	0.3

Table 5 - Characteristics of OPS sensor and data.

### Results of tests with OPS data

The initial computation of the data set resulted in problems of convergence, mainly affecting the computation of the platform's altitude for the physical model. An exhaustive study resulted in the use of only eight of the orientation parameters. It was necessary to do this because of the small B/H which results in high correlations between the orientation parameters. Also, the given value for the eccentricity of the orbit was not well defined by the literature. The only information offered was that it should be smaller than 0.0015 in all cases.

The errors presented were found in a very small number of check points. Of the 40 ground control points, those which were not used as control, were adopted for checking the model's accuracy. The accuracies found for some of the tests are presented in tables 6 and 7, using the physical and the polynomial orientation models, respectively.

No iter	No contr.	No conj.	rms (m) [in UTM projection]				
			E	N	H	2D	3D
6	6	0	62	84	96	106	131
7	6	4	58	86	79	104	130
6	5	0	56	91	78	107	132
7	5	8	62	89	89	108	140
7	4	12	67	92	98	114	150

Table 6 - OPS model accuracy after orientation with several control configurations, using the physical orientation algorithm.

No iter	No contr.	No conj.	rms (m) [in UTM projection]				
			E	N	H	2D	3D
6	6	0	71	86	97	112	148
7	6	4	71	89	95	114	148
6	5	0	74	92	93	118	150
8	5	8	72	91	96	116	151
8	5	12	85	103	101	134	167

Table 7 - OPS model accuracy after orientation with several control configurations, using the polynomial orientation algorithm.

Similarly to the tests with SPOT data, the tests carried out with the OPS along track imagery resulted in time expensive solutions when conjugate points were adopted. The physical approach for the orientation gave the best results, both for relative and absolute orientations of the model. However, the algorithms took more iterations to converge for a minimum number of control points.

The results concur with other reports presented on the study of the accuracy of OPS data in height [Maruyama, 1993]. The large errors found are most likely due to the large errors in the control data and the problems found in the identification of control. The small angle of convergence can give rise to instability in the orientation of stereo pairs. The B/H=0.3 is the major constraint for the orientation of the data with this algorithm because it is extremely influential on the ellipses of error, hence on the correlations between some of the orientation parameters.

Besides the savings in time spent in the orientation process, it gives the same kind of accuracy with less control points.

## 5. CONCLUSIONS

The use of conjugate points were tested with SPOT and OPS imagery for the two different models. The algorithm becomes very time expensive with the use of conjugate points.

First, it introduces a few more equations to the calculation, resulting in the inversion of larger matrices. However, this would not be important if an improvement in the final accuracy of the models was registered.

Second, the set of observation equations is not as stable as when only ground control is used. A deeper study showed that this is due to the different scales of the residuals in the observation equations derived from ground control and conjugate points.

No significant improvement in the orientation of the model stems from adding conjugate points to the control. It was also found that the convergence of the algorithm depends on the number of observation equations formed. The absolute orientation of the models gets worse when the number of ground control is decreased independently from the number of conjugate points being used. However, conjugate points may be used to ensure a model orientation when the ground control is not enough on its own.

If a good initial approximation is obtained for the orientation parameters, conjugate points can be used with success to improve the relative orientation of the three-dimensional models. However, the results obtained with this study were not significant.

There is concern when orienting push broom type data using an iterative solution that correlations between the unknowns may cause instability in the solution. Tests were carried out to determine the effect of weighting on the solution. Weighting the quality of the conjugate points was thus tested with only a small improvement in the final relative orientation.

It is expected that three-line stereo systems will overcome most of the problems with convergence. A larger number of equations per point identified in the images will contribute toward a more stable solution.

In general, the polynomial model has more problems of convergence with conjugate points than the physical model.

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