ORTHOGONAL 3-D RECONSTRUCTION USING VIDEO IMAGES

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ABSTRACT

This paper deals with the projective block adjustment method developed at Helsinki University of Technology. Especially, the question how the radial distortion is corrected in the method is discussed. The method is based on the projective singular correlations between the images in the block, and it can be used to obtain a 3-D orthogonal coordinate system for an arbitrary block of images. Linear distortions do not cause any problems in the computation of the singular correlations, and they can be determined from the singular correlation parameters afterwards. Nonlinear image errors, such as radial distortion, instead, cause significant error in the determination of the correlation parameters. This effect makes it also possible to determine the radial distortion simultaneously with the singular correlation parameters.
The solution of the radial distortion requires approximate interior orientation to be known, and an improved interior orientation is computed afterwards from the singular correlation parameters. The cycle can then be repeated, and a rigid solution can be obtained, bringing the whole block into an orthogonal 3-D coordinate system which is free from nonlinear distortions caused by the radial distortion of the images. The resulting coordinate frame can then be utilized in further 3-D reconstruction of the scene, for example, using digitized video images.

1. INTRODUCTION

This article describes a method to obtain full orthogonal 3-D model coordinate system using pure image information only. The method is based on the projective singular correlation between the images taken from the same object. Singular correlation is also known in the computer vision society as the epipolar transformation /Maybank et al., 1992/.

The principle of the method was presented in /Niini, 1994/, and in /Niini, 1995/. Now, the method has been extended so that possible radial distortion of the images can also be computed. This is a significant improvement since radial distortion is usually quite large when video cameras are used.

The original projective block adjustment method has four sequential parts: solution of the singular correlations, solution of the interior orientations, solution of the rotation matrices, and the solution of the relative positions of the images along with the model coordinates. The original method assumed the images to be free from nonlinear distortions, or possible nonlinear distortions had to be corrected in advance /Niini, 1994,1995/.

2. NEW BLOCK ADJUSTMENT

The solution of the radial distortion requires approximate interior orientation to be known. This means that the solution of radial distortion and singular correlations, and the solution of interior orientations should actually proceed simultaneously.

In the new block adjustment method, however, to keep the method similar to the older version, they are solved in a cyclic iteration. First, solve the singular correlations with the radial distortion coefficients using approximate interior orientation. Second, solve the interior orientation only. Usually with small radial distortion, only the approximate image center is needed. Repeat the procedure until the radial distortion and interior orientation do not any more change significantly.

Starting from reasonable approximate values, the total iteration converges satisfactorily in a few loops. However, the total number of iterations may still be much larger due to the nested iterations since both steps are also iterative.

The radially undistorted image coordinates are also obtained, and the resulting interior orientation corresponds to the one which transforms the undistorted image coordinates to the ideal image coordinates which strictly fulfil the collinearity conditions. The least squares solution of the system is based on the minimization of the noise in the original image observation space.

The third and fourth parts of the block adjustment method, the solution of the image rotation matrices, base vectors and model coordinates remain the same as before, except that the image coordinates used in these steps are now free from the radial distortion making it possible to obtain pure orthogonal 3-D model coordinate system without control points or a priori information about the block geometry.

Finally, the block parameters can further be enhanced in a general, simultaneous adjustment, presented in chapter 5.

3. COMPENSATION OF THE RADIAL DISTORTION

The solution of the radial distortion is based on the fact that the singular correlation between two images should produce zero when only linearly deformed images are used. Any deviation from zero, exceeding of course, the random noise due to the observational errors, can then be interpreted as the effect of nonlinear errors /Niini, 1990/. For example, using video cameras, the radial distortion is usually significant. In practice, it has also been shown to be sufficient to model the nonlinear distortions with one parameter only, namely with the parameter corresponding to the third power of the radius /Melen, 1994/. Decentering or tangential distortion need not to be considered here, because they are practically too dependent on the principal point which, in turn, is allowed to change in this block adjustment.

3.1 The camera model

A physical camera model is assumed. The properties of the camera are: first, the lens may cause nonlinear (radially symmetric) distortion to the ray of light when it passes through the lens; second, the distortion caused by a poorly known sensor geometry is linear. Thus, all linear distortions happens only after the nonlinear ones. The compensation of the distortion should then take place in the reverse order, not simultaneously.

The physical model used here is not exactly the same as the traditional "physical model" in /Kilpelä et al., 1981/. The traditional model treats linear and nonlinear distortions simultaneously, which works well using aerial images with small linear distortions, but it may be considered erroneous using video images with apparent linear distortion /Melen, 1994/.

3.2 Elliptic radial distortion

When computing the singular correlations, it is not necessary to compensate the linear distortions in advance since the interior orientation can always be computed after the singular correlations have been computed but not before. Instead, it is preferable just to find a suitable expression for the nonlinear distortion in terms of the linearly distorted observations.

To obtain the required expression, it is first studied how the final, radially and linearly distorted observations x_{lr}, y_{lr}, are formed from the ideal, undistorted image observations x_i, y_i:

$$\begin{aligned} x_r &= x_i + k_1 x_i (r_i^2 - r_0^2) + k_2 x (r_i^4 - r_0^4) + \ldots, \\ y_r &= y_i + k_1 y_1 (r_i^2 - r_0^2) + k_2 x (r_i^4 - r_0^4) + \ldots, \end{aligned} \tag{1}$$

and

$$\begin{aligned} x_{lr} &= x_r + x_p - \frac{\beta}{\alpha} y_r , \\ y_{lr} &= y_p + \frac{1}{\alpha} y_r . \end{aligned} \tag{2}$$

Above, $r_i = \sqrt{x_i^2 + y_i^2}$ is the radius computed from the principal point, and k_1, k_2,... are the radial distortion parameters, r_0 is the radial distance where radial distortion effect is wanted to be zero. x_p, y_p, are the principal point coordinates, α is affinity (scale ratio between the x- and y-axes of the image coordinate system), and ß corresponds to the non-orthogonality angle between the image coordinate-axes. The radial distortion model is adopted from /Karara, 1989/.

The observations x_{rl}, y_{rl} are measured and they are subject to the random observational errors. The term r_0 with a certain nonzero value is useful in the adopted model because it prevents the system to collapse into a trivial solution where the magnitude of the radial distortion correction becomes as large as the image coordinates themselves. Fixing r_0 also fixes the image coordinate scale since, for a certain radial distortion profile, the camera constant depends on r_0, and of course, on the radial distortion coefficients k_i /Brown, 1968/. A suitable value for r_0 is, say 200 pixels, in a 512 x 512 pixel^2 image.

By replacing the inverse of (2) into (1), and after some manipulations it is obtained

$$\begin{aligned} x_{rl} &= x_c + k_1 (x_c - x_p)(r_e^2 - r_0^2) + \ldots, \\ y_{rl} &= y_c + k_1 (y_c - y_p)(r_e^2 - r_0^2) + \ldots, \end{aligned} \tag{3}$$

which express radially distorted observations as functions of the radially corrected (but linearly distorted) observations x_c, y_c and of the interior orientation parameters. Especially,

$$r_e = \sqrt{(x_c - x_p)^2 + (\alpha^2 + \beta^2)(y_c - y_p)^2 + 2\beta (x_c - x_p)(y_c - y_p)}$$

is the elliptical radius computed from the point x_c, y_c. Equation (3) presents the elliptical radial distortion model, because for all equidistant image points the corresponding radial distortion pattern is elliptically symmetric, as seen in figure 1. It also expresses the radial distortion effect directly in the linearly deformed image coordinate system as required.

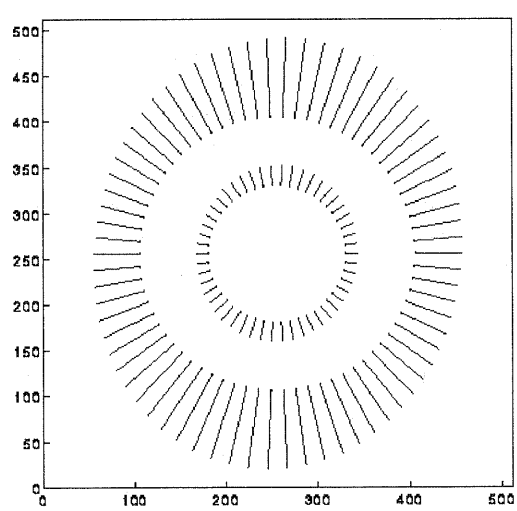

Figure 1. Elliptical radial distortion.

From now on, only the first term of the distortion function is taken into account because it is, using video cameras, the only significant parameter /Melen, 1994/. Nothing really prevents one to include the other terms in the model, too.

The approximate radial distortion correction is

$$\begin{aligned} dx &= k(x_c - x_p)(r_e^2 - r_0^2) \\ dy &= k(y_c - y_p)(r_e^2 - r_0^2) \end{aligned} \tag{4}$$

and solving the equation (3) for x_c, y_c, it is also obtained

$$\begin{aligned} x_c &= x_{rl} - dx \\ y_c &= y_{rl} - dy \end{aligned} \tag{5}$$

which give approximate values of the radially corrected image coordinates during iteration.

3.3 Solution of radial distortion

Let

$$[x_c' \ y_c' \ 1] \ M \begin{bmatrix} x_c'' \\ y_c'' \\ 1 \end{bmatrix} = 0. \tag{6}$$

be the singular correlation equation of two images which are only linearly deformed. M is the 3x3-singular correlation matrix containing seven independent parameters. See /Niini, 1994/ and /Niini, 1995/ for details concerning the parameterization of M.

Now, expressing x_c', y_c', x_c'', y_c'' with equations (5) and (4), and replacing them into (6), the resulting equation is obtained

$$\begin{bmatrix} x_c' - (x_c' - x_p)k(r_e^{2'} - r_0^2) \\ y_c' - (y_c' - y_p)k(r_e^{2'} - r_0^2) \\ 1 \end{bmatrix}^T M \begin{bmatrix} x_c'' - (x_c'' - x_p)k(r_e^{2''} - r_0^2) \\ y_c'' - (y_c'' - y_p)k(r_e^{2''} - r_0^2) \\ 1 \end{bmatrix} = 0 \tag{7}$$

This is the singular correlation condition containing the radial distortion parameters. Differentiating equation (7) with respect to the unknown parameters and observations, a general system of type Ax+Bv=l is obtained, which can be solved as explained previously in /Niini, 1994 and 1995/. The only exception is that in each iteration step, new radially corrected observations x_c, y_c have to be iterated, because they are also used in the computation of the correlations.

Using two images only, it is possible to solve the common radial distortion if the approximate interior orientation parameters are known. Because radial distortion is a fixed part of the interior orientation, like the camera constant, the solvability of it follows the rules concerning the solution of interior orientation from singular correlations.

For example, if the interior orientation is not known, at least three images are required to solve the unknown interior orientation and the radial distortion of the images. With two different cameras, at least two images must have been taken with both cameras. Using more images and thus more singular correlations in the block, it is also possible to solve different radial distortions of different cameras, provided that the minimum requirements for each different camera are fulfilled, and the block geometry constraints the solution sufficiently.

Special cases. Large radial distortion can be utilized to solve for the approximate principal point in the very first step of the adjustment. This is reasonable if the approximate image center, for some reason, is too far from the true principal point.

Decentering terms of the radial distortion are derived from equation (3), with one radial term only:

$$\begin{aligned} x_{rl} &= x_c + k_1 x_c (r_e^2 - r_0^2) - k_1 x_p (r_e^2 - r_0^2) \\ y_{rl} &= y_c + k_1 y_c (r_e^2 - r_0^2) - k_1 y_p (r_e^2 - r_0^2) \end{aligned} \tag{8}$$

and taking $p_1 = k_1 x_p$, $p_2 = k_1 y_p$ as additional decentering terms. Then the principal point is computed from $x_p = p_1/k$ and $y_p = p_2/k$. Thus, if the radial distortion parameter k_1 is large enough, a substantially better approximate center for the image is easily obtained. In later iterations, these decentering terms can be ignored, and the determination of the principal point can be left in its original place in the block adjustment.

4. COMPENSATION OF LINEAR DISTORTION

After radial distortion and singular correlation parameters have been determined, the interior orientation can be computed, using the iterative way presented in /Niini, 1993/ or /Niini, 1994/.

The resulting linear transformation from linearly distorted coordinates to the ideal image coordinates is the following.

$$\begin{aligned} x_i &= (x_c - x_p) + \beta(y_c - y_p) \\ y_i &= \alpha(y_c - y_p) \\ z_i &= -c_p \end{aligned} \tag{9}$$

where c_p, the camera constant, is now also dependent on the choice of r_0. Other terms are the same as in equation (2).

5. GENERAL BLOCK SOLUTION

Finally, a general and one step adjustment can be made for all the block parameters based on the following factorization of the singular correlation matrix, adopted from /Niini, 1994/.

$$M = C_1^T R_1^T (B_2 - B_1) R_2 C_2 \tag{10}$$

This factorization expresses the singular correlation matrix between two images in terms of the upper triangular interior orientation matrices C_1, C_2, orthogonal rotation matrices R_1, R_2, and skew symmetric base matrices B_1, B_2, both of which contain the three projection center coordinates of the corresponding image. All matrices are 3x3-matrices and their initial values are obtained from the projective block adjustment.

This form of M can be put into the equation (7) which gives the observation equations used to solve all the block parameters simultaneously, including the radial distortion. This mathematical model is coherent and more stable than the sequential method because it directly contains the physical orientation parameters of the block, without any intermediate or fictitious parameters. A least squares solution based on (7) gives also the cofactors and weight coefficients of all the parameters at once which would be hard to compute using the sequential method alone.

This method resembles the bundle block method, except that no bundles have to be formed. Further, no model or object coordinates are needed. They can be solved afterwards by intersecting the corresponding projection rays.

6. EXAMPLE

The method was used to calibrate six real images taken automatically with the Mapvision (TM) photogrammetric station. The images were of size 512 x 512 pixels. The common calibration of the cameras was determined with the method described here, except that the final enhancement using (7) was not made, yet. A total of fifteen singular correlations were obtained. All correlations were taken into account in the computations, and a reference variance of s_0=0.157 pixels were obtained. The interior orientation values obtained were x_p=273.710, y_p=257.425, c_p=827.623, α=0.6848, and k=-0.2473*10^{-6} (r_0=100 pixels). ß was fixed to zero. The root mean square error of the model coordinates was 0.261 pixels. All values are reasonable, compared to previous experiences with the Mapvision calibration with the free net bundle block method /see Haggrén et al., 1989/ especially designed to calibrate the Mapvision system.

7. CONCLUSIONS

This article deals with the projective block adjustment method, based on the singular correlation. The method has been extended to contain the radial distortion parameters. The projective block adjustment can be used to solve the relative orientation of a image block, without knowing any information of the block geometry in advance.

Radial distortion is solved using an elliptical radial distortion model, and it requires at last the approximate principal point to be known in advance. The solution is obtained in the very first part of the method, in a two step iterative process. The first step contains the determination of both the radial distortion and singular correlation parameters using approximate interior orientations. The second step contains the determination of improved interior orientations. These two steps are repeated until the solution converges. The determination of the rest of the block parameters, such as the image rotation matrices and projection center coordinates, is the same as presented in earlier articles. Finally a general and simultaneous least squares adjustment can be made to the block parameters which, typically, will slightly improve the sequentially obtained solution, and is can be considered useful because it gives the weight coefficients and cofactors of all parameters at once.

The method seems to work satisfactorily with both synthetic and real data, provided that the block geometry is well defined.

The whole block adjustment method is intended to be a part of a video based measurement procedure /Niini, 1995/, where building interiors are digitized from video stereopairs and transformed to a common orthogonal 3-D coordinate system, determined with the method presented here.

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