SCANNER RESECTION USING TRAJECTORY DATA

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KEY WORDS: Satellite, Scanner, Precision, Calibration, Orientation, Vision, Geometry, SPOT.

ABSTRACT

A new solution to the problem of determining the parameters of satellite scanner exterior orientation is presented. The scanner problem is simplified to that of the frame camera by making use translational and rotational trajectory data recorded during the scanninging period. An accurate solution to the frame camera is then presented which works even with poorly-distributed ground control points.

1 Introduction

Current scanner resection solutions are divided into two main classes — those that ignore translational and rotational variation over the imaging period, and those which use polynomial approximations of motion variation [Shevlin, 1996]. The latter are significantly more accurate than the former but owing to the amount of approximation required they do not achieve optimal estimates of the unknowns. As far as this author can determine, no currently-published resection solution uses actual trajectory data in finding the unknown parameters of exterior orientation. This paper explains how the use of trajectory data can facilitate the near optimal determination of the parameters of exterior orientation.

With the advent of space-qualified GPS attitude and orbit determination receivers the problem of satellite scanner resection will not be as important in the near future as it is today. At the current time, however, resection is still required for remote sensing platforms such as SPOT whose interior and exterior image geometry needs to be known precisely for photogrammetric applications. Trajectory data supplied with imagery typically consists of samples of angular velocity recorded by the attitude and orbit control system throughout the imaging period and estimates of orbital position determined from Doppler analysis of telemetry signals in conjunction with orbital models.

It has been shown by the author that a using suitable parameterisations of rotation, angular velocity samples can be splined and integrated to yield a rotational trajectory (specified as a set of discrete rotations $\mathbf{R}_i, i = 1, \dots, n$ for n scanlines) relative to the unknown orientation $\mathbf{R}_0$ at the start of the imaging period [Shevlin, 1994; Shevlin, 1995]. Since the estimates of position are approximated using orbital models they cannot be considered correct in terms of absolute coordinates but they can be used to give an accurate approximation of relative translation (specified as a set of discrete translations $\mathbf{t}_i$) over the imaging period. Hence rotational and translational trajectories over the imaging period (which can be considered as the parameters of interior orientation in scanned imagery) are known, the unknowns are the parameters of exterior orientation - position $\mathbf{p}_0$ and orientation $\mathbf{R}_0$ at the start of the imaging period.

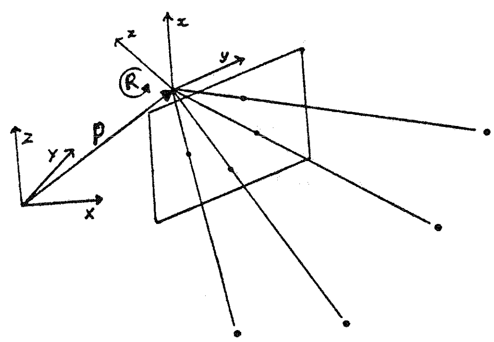
2 Problem statement

Many different frame camera resection solutions have been proposed. A dissertation from 1958 documents over 80 different approaches (referenced in [Haralick *et al.*, 1989]). Considering that this was before the advent of computer vision and digital photogrammetry it gives some idea of how many solutions exist in the literature (see [Tsai, 1987; Tsai, 1989] for comprehensive classification and review).

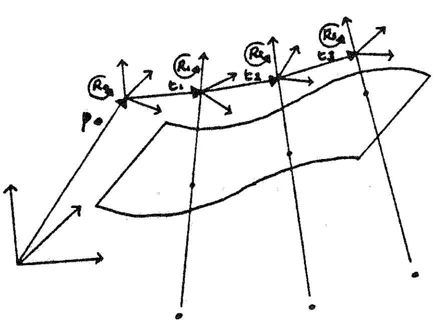
The vast majority of solutions rely on the same constraints relating the imaging and scene coordinate systems — collinearity, coplanarity, and coangularity. Different equations specifying these constraints in terms of the unknowns are formulated and a wide variety of techniques applied to solve them. Currently published scanner resection solutions *all* seem to be based on the collinearity constraint specified through the equations of perspective projection. Primarily due to the way in which motion is modelled these solutions are not as accurate as they could be [Shevlin, 1995]. The aim of the work presented here is to use an accurate model of scanner motion to achieve resection of higher accuracy than that of current techniques.

In approaching this problem the author did not want to duplicate or modify existing techniques since most are already minor modifications of a few well-established ones. A new perspective of the problem was sought. This was eventually achieved with the observation that scene point projections on the focal plane and the focal point (as well as other interior orientation parameters) are sufficient to form a *bundle* of lines in the imaging coordinate system. These lines specify the paths travelled by image-forming light rays reflected off scene objects. This is shown for the frame camera and scanner geometries in figure 1. The collinearity condition for resection could then be considered as *fitting* the bundle of lines to the scene points, or more formally—

Given the relative positions and orientations of a set of image-forming rays in an imaging coordinate system and a corresponding set of observed control points in a scene coordinate system, determine the exterior orientation of the former system with respect to the latter such that the perpendicular distances between the rays and the corresponding control points are minimised.



(a) Frame camera



(b) Scanner

Figure 1: Image-forming ray bundles

3 Parameterisations

A computational process is required to find the least-squared error solution of this problem. In order to determine such a process the problem must be analysed using manipulable parameterisations of the problem domain elements. The elements are points, lines, point-line distance, translation, and rotation (the latter two are required to describe both the known relationship between lines and the unknown relationship between coordinate systems). The *motor* algebra [Brand, 1947] provides convenient parameterisations of all these elements. A point can be represented by a vector $\mathbf{s}$, a line can be written in Plücker coordinates as $\hat{\mathbf{l}} = \mathbf{n} + \epsilon \mathbf{p} \times \mathbf{n}$, and a moment (proportional to point-line distance) is $\hat{\mathbf{l}} \otimes \mathbf{s} = \mathbf{p} \times \mathbf{n} - \mathbf{s} \times \mathbf{n}$ where $\mathbf{p}$ is a vector denoting a point on the line and $\mathbf{n}$ is a unit direction vector.

4 Scanner problem analysis

In order to gain some insight into the scanner resection problem it will be temporarily assumed that the position and orientation of the scanner imaging coordinate system with respect to the scene is known. The squared error between image-forming rays $\hat{\mathbf{l}}_i$ and corresponding scene points $\mathbf{s}_i$ is written,[1]

$$\sum_{i=1}^{n} |\hat{\mathbf{l}}_i \otimes \mathbf{s}_i| = \sum_{i=1}^{n} |\mathbf{p}_i \times \mathbf{n}_i - \mathbf{s}_i \times \mathbf{n}_i|. \qquad (1)$$

The vector difference on the right hand side can be rewritten,

$$(\mathbf{p}_i - \mathbf{s}_i) \times \mathbf{n}_i. \qquad (2)$$

This shows how the moment magnitude and direction is a function of the vector between the scene point and a point on the line. Note that the vector is unaffected by a translation of its end points by $\mathbf{t}_i$,

$$\mathbf{p}_i - \mathbf{s}_i = (\mathbf{p}_i - \mathbf{t}_i) - (\mathbf{s}_i - \mathbf{t}_i) = \mathbf{p}_i - \mathbf{t}_i - \mathbf{s}_i + \mathbf{t}_i = \mathbf{p}_i - \mathbf{s}_i. \qquad (3)$$

The positions $\mathbf{p}_i$ of points on scanner rays are specified by *known* translations $\mathbf{t}_i$ with respect to an initial unknown position $\mathbf{p}_0$, $\mathbf{p}_i = \mathbf{p}_0 + \mathbf{t}_i$. Rewriting equation (2) gives gives,

$$(\mathbf{p}_0 + \mathbf{t}_i - \mathbf{s}_i) \times \mathbf{n}_i. \qquad (4)$$

Making use of observation (3) gives,

$$\mathbf{p}_0 + \mathbf{t}_i - \mathbf{s}_i = (\mathbf{p}_0 + \mathbf{t}_i - \mathbf{t}_i) - (\mathbf{s}_i - \mathbf{t}_i) = \mathbf{p}_0 - (\mathbf{s}_i - \mathbf{t}_i). \qquad (5)$$

Since both $\mathbf{s}_i$ and $\mathbf{t}_i$ are known, *artificial* scene control points $\mathbf{s}'_i = \mathbf{s}_i - \mathbf{t}_i$ can be calculated and equation (1)

[1] $|\mathbf{a}|$ is used to denote the square of vector magnitude, calculated as $\mathbf{a} \cdot \mathbf{a}$.

can be rewritten explicitly in terms of the unknown initial position $\mathbf{p}_0$,

$$\sum_{i=1}^{n} |\mathbf{p}_0 \times \mathbf{n}_i - \mathbf{s}'_i \times \mathbf{n}_i|. \tag{6}$$

This statement of the sum to be minimised for the *scanner* bundle-fitting problem is the *same* as that required for the *frame camera* bundle-fitting problem assuming scene control points are labelled $\mathbf{s}'_i$. As far as the author can determine, this is the *first time* that the scanner problem has been simplified to that of the frame camera.

Although it seems that conventional frame camera resection solutions (such as those based on the collinearity condition) may now be applied to solve the problem, this is not the case. The ground control points have been translated towards one another, their distance being a function of the orientation of the scanner at the time of imaging. Despite the fact that they may have been well-distributed across the scene initially they have been translated to be spatially clustered. Tests with SPOT trajectory data have shown that ground control points whose image observations were initially thousands of pixels apart are translated such that the image observations are only tens of pixels apart [Shevlin, 1995] and that conventional resection solutions fail to converge to accurate estimates when using these clustered points [Shevlin, 1996].

The unknown initial orientation of the imaging coordinate system $\mathbf{R}_0$ has yet to be introduced into the problem. Let $\mathbf{n}_i = \prod_n^1 \mathbf{R}_i \mathbf{n}_0$ be the unit direction vectors of lines in the imaging coordinate system specified by *known* rotations $\mathbf{R}_i$ of an initial vector $\mathbf{n}_0$ (which could be the optical axis, for instance). These vectors are transformed into the scene coordinate system by the *unknown* rotation $\mathbf{R}_0$. Using this to rewrite equation (6) gives an expression explicitly in terms of all unknowns,

$$\sum_{i=1}^{n} |\mathbf{p}_0 \times \mathbf{R}_0\mathbf{n}_i - \mathbf{s}'_i \times \mathbf{R}_0\mathbf{n}_i|. \tag{7}$$

The author has made several attempts to find $\mathbf{p}_0$ and $\mathbf{R}_0$ which minimise this sum, but without success. This lead to the bundle-fitting formulation being put aside. However the analysis was in no way a waste of effort since it facilitated the simplification of scanner problem to that of the frame camera for the first time.

Since conventional frame camera resection techniques are not sufficient for this geometry a new one has been derived. In general kinematic analysis is greatly facilitated by the fact that translation and rotation can be treated separately therefore aim was to formulate two separate resection problems, one for position and one for orientation. Since reducing the dimensionality of the unknown parameter space results in fewer parameters being sought together, the probability of their optimal determination is increased.

5 Coplanarity condition

The coangularity condition (Church's condition [Ghosh, 1988, p. 104]) to constrain the resection problem is that the angle θ between a pair of rays in the imaging coordinate system is the same as that between the rays in the scene coordinate system. This can be written as $\cos\theta_{ij} = \cos\theta_{IJ} = \mathbf{n}_i \cdot \mathbf{n}_j = \mathbf{n}_I \cdot \mathbf{n}_J$ where the upper and lower case subscripts of angles θ and unit vectors $\mathbf{n}$ refer to the rays in the image and scene coordinate systems respectively. Analytic solutions based on this constraint are presented in [Ghosh, 1988; Wolf, 1983, pp. 104, 240 resp.], however they are not least-squared error solutions.

This approach inspired the realisation that the problem is simplied by specifying the image-forming rays with respect to the known scene points instead of the unknown focal point. Instead of applying the coangularity condition through equations written in terms of direction vectors it was decided to investigate the formulation which results using the scalar product of Plücker lines $\hat{\mathbf{l}}_i \cdot \hat{\mathbf{l}}_j = \hat{\theta} = \theta + \epsilon d$. The *dual angle* $\hat{\theta}$ comprises the length d of a perpendicular joining the two lines and a rotation of angle θ about the perpendicular.

Given lines $\hat{\mathbf{l}}_i = \mathbf{R}\mathbf{n}_i + \epsilon \mathbf{s}_i \times \mathbf{R}\mathbf{n}_i$ and $\hat{\mathbf{l}}_j = \mathbf{R}\mathbf{n}_j + \epsilon \mathbf{s}_j \times \mathbf{R}\mathbf{n}_j$ the scalar product $\hat{\mathbf{l}}_i \cdot \hat{\mathbf{l}}_j$ is found by distributing across the real and dual parts,

$$\begin{aligned}&\hat{\mathbf{l}}_i \cdot \hat{\mathbf{l}}_j = \theta + \epsilon d = \\ &\mathbf{R}\mathbf{n}_i \cdot \mathbf{R}\mathbf{n}_j + \epsilon\,(\mathbf{R}\mathbf{n}_i \cdot \mathbf{s}_j \times \mathbf{R}\mathbf{n}_j + \mathbf{R}\mathbf{n}_j \cdot \mathbf{s}_i \times \mathbf{R}\mathbf{n}_i).\end{aligned} \tag{8}$$

This gives a new idea for a constraint — that the perpendicular distance d between the rays at the focal point should be zero,

$$\begin{aligned} d &= \mathbf{R}\mathbf{n}_i \cdot \mathbf{s}_j \times \mathbf{R}\mathbf{n}_j + \mathbf{R}\mathbf{n}_j \cdot \mathbf{s}_i \times \mathbf{R}\mathbf{n}_i \\ &= \mathbf{s}_j \cdot \mathbf{R}\mathbf{n}_j \times \mathbf{R}\mathbf{n}_i + \mathbf{s}_i \cdot \mathbf{R}\mathbf{n}_i \times \mathbf{R}\mathbf{n}_j \\ &= \mathbf{s}_j \cdot \mathbf{R}(\mathbf{n}_j \times \mathbf{n}_i) + \mathbf{s}_i \cdot \mathbf{R}(\mathbf{n}_i \times \mathbf{n}_j) \\ &= (\mathbf{s}_j - \mathbf{s}_i) \cdot \mathbf{R}(\mathbf{n}_i \times \mathbf{n}_j) = 0. \end{aligned} \tag{9}$$

This expression is one of *coplanarity*. A vector between two scene control points is coplanar with vectors defining the directions of their image projections, see figure 2. The coplanarity constraint is more often used for the *relative orientation* problem. The constraint equation (9) looks interesting for the resection problem because the *only* unknown present is orientation.

Letting[2] $\mathbf{g}_i = ||\mathbf{s}_i - \mathbf{s}_j||$ and $\mathbf{c}_i = ||\mathbf{n}_i \times \mathbf{n}_j||$, the squared error function of camera orientation $\mathbf{R}$ to be minimised is,[3]

$$\sum_{i=1}^{n} \langle \mathbf{g}_i, \mathbf{R}\mathbf{c}_i \rangle^2 \tag{10}$$

[2] $||\mathbf{a}||$ is used to denote the unit norm of vector $\mathbf{a}$, calculated as $\mathbf{a}/\sqrt{|\mathbf{a}|}$.

[3] $\langle \mathbf{a}, \mathbf{b} \rangle$ is used to denote the inner product, calculated as $\mathbf{a} \cdot \mathbf{b}$.

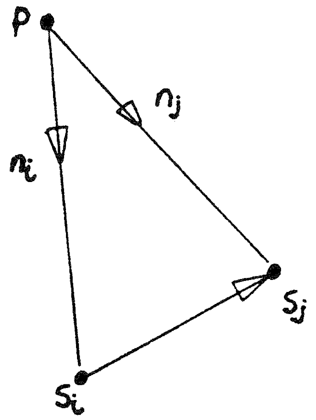


Figure 2: Coplanarity of image rays

Since achieving this the author has discovered a similar minimisation equation to (10) in [Liu *et al.*, 1990], but it was found through observation of problem geometry rather than analytically as shown above.

6 Rotation solution

The elements of an orthonormal rotation matrix **R** are non-linear functions of the unknown Euler angles of rotation ω, ϕ, κ. This makes equation (10) highly non-linear in terms of the three unknowns and thus a closed-form solution for them is improbable. Various approaches to minimisation are outlined in [Shevlin, 1996]. The one presented here uses the *quaternion* q parameterisation of rotation [Horn, 1987]. Equation (10) can be rewritten,

$$\sum_{i=1}^{n} \langle \mathbf{g}_i, q\mathbf{c}_i\bar{q}_i \rangle^2 = \sum_{i=1}^{n} \langle q\mathbf{c}_i, \mathbf{g}_i q \rangle^2 . \qquad (11)$$

Let $\mathbf{N}_i = \bar{\mathbf{G}}_i^\top \mathbf{C}_i$ where $\bar{\mathbf{G}}_i$ and $\mathbf{C}_i$ are orthogonal matrices formed from vectors $\mathbf{g}_i$ and $\mathbf{c}_i$ (see [Horn, 1987]). Let $\mathbf{N} = \sum_{i=1}^{n} \mathbf{N}_i$. Writing the quaternion q as a vector $\mathbf{q} = [q_0\, q_1\, q_2\, q_2\, q_3]^\top$ and denoting quaternion multiplication as a matrix by vector product gives the following matrix expression equivalent to equation (11),

$$(\mathbf{q}^\top \mathbf{N} \mathbf{q})^2. \qquad (12)$$

This could be minimised by finding solving the quadratic form $F(\mathbf{q})$ for the four quaternion variables (subject to the unit quaternion constraint $q_0^2 + q_1^2 + q_2^2 + q_2^2 + q_3^2 = 1$),

$$F(\mathbf{q})\colon\ \mathbf{q}^\top \mathbf{N} \mathbf{q} = 0. \qquad (13)$$

Instead of attempting to solve a quadratic in four variables subject to a constraint, it was decided to form an overdetermined system of simultaneous non-linear equations $F_i(\mathbf{q}) : \mathbf{q}^\top \mathbf{N}_i \mathbf{q} = 0$ (for each observation $i = 1, \ldots, n$). Each of the $F_i(\mathbf{q})$ can be linearised in the neighbourhood of a known $\mathbf{q}$ using the first two terms of a Taylor's series expansion,

$$F_i(\mathbf{q} + \Delta\mathbf{q}) = F_i(\mathbf{q}) + \frac{\partial F_i(\mathbf{q})}{\partial q_0}\Delta q_0 + \frac{\partial F_i(\mathbf{q})}{\partial q_1}\Delta q_1 + \frac{\partial F_i(\mathbf{q})}{\partial q_2}\Delta q_2 + \frac{\partial F_i(\mathbf{q})}{\partial q_3}\Delta q_3 \quad (14)$$

The Newton-Raphson method can be used to solve iteratively for corrections $\Delta\mathbf{q}$ which minimise the least-squared error.

An important advantage of using the quaternion parameterisation in the solution of the problem (as opposed to Euler angles and hence rotation matrices as used in [Liu *et al.*, 1990]) is the ease with which successful initial values for the iterative solution can be found. The following set of coefficients provide a regular tesselation of the unit quaternion hemisphere and it can be shown that convergence to the four possible solutions is guarenteed,

$$\left\{ \begin{bmatrix}0.5\\0.5\\0.5\\0.5\end{bmatrix}, \begin{bmatrix}0.5\\0.5\\0.5\\-0.5\end{bmatrix}, \begin{bmatrix}0.5\\0.5\\-0.5\\0.5\end{bmatrix}, \begin{bmatrix}0.5\\0.5\\-0.5\\-0.5\end{bmatrix}, \begin{bmatrix}0.5\\-0.5\\0.5\\0.5\end{bmatrix}, \begin{bmatrix}0.5\\-0.5\\0.5\\-0.5\end{bmatrix}, \begin{bmatrix}0.5\\-0.5\\-0.5\\0.5\end{bmatrix}, \begin{bmatrix}0.5\\-0.5\\-0.5\\-0.5\end{bmatrix} \right\} \quad (15)$$

The negation of the elements of this set provide diametrically-opposed points on the other hemisphere and so could also be used as starting values. The solutions found through this linearisation may then be used as start points in a non-linear optimisation search to minimise equation (12). A very accurate and efficient means to perform such a search on the unit quaternion sphere is the *spherical optimisation search* outlined in [Kanatani, 1993, p. 123].

7 Translation solution

Once orientation has been found the least-squared error solution for position $\mathbf{p}$ can be found in closed-form using the pseudo-inverse method. The position of the i^{th} image-forming ray is specified with the scene control point $\mathbf{s}_i$ in the Plücker line $\hat{\mathbf{l}}_i = \mathbf{n}_i + \epsilon\mathbf{s}_i \times \mathbf{n}_i$. Since each image-forming ray passes through the focal point $\mathbf{p}$,

$$\mathbf{p} \times \mathbf{n}_i = \mathbf{s}_i \times \mathbf{n}_i. \qquad (16)$$

An overdetermined system of linear equations in terms of the unknown $\mathbf{p}$ can be formed,

$$\left.\begin{aligned} \mathbf{p} \times \mathbf{n}_1 &= \mathbf{s}_1 \times \mathbf{n}_1 \\ \mathbf{p} \times \mathbf{n}_2 &= \mathbf{s}_2 \times \mathbf{n}_2 \\ &\vdots \\ \mathbf{p} \times \mathbf{n}_n &= \mathbf{s}_n \times \mathbf{n}_n \end{aligned}\right\} \qquad (17)$$

Rewriting this using a skew-symmetric matrix product instead of the cross product on the left-hand side and evaluating the product of known vectors on the right-hand side as $\mathbf{v}_i$ gives,

$$\left.\begin{aligned} \bar{\mathbf{N}}_1\mathbf{p} &= \mathbf{v}_1 \\ \bar{\mathbf{N}}_2\mathbf{p} &= \mathbf{v}_2 \\ &\vdots \\ \bar{\mathbf{N}}_n\mathbf{p} &= \mathbf{v}_n \end{aligned}\right\} \tag{18}$$

Rewriting as a matrix equation gives,

$$\begin{bmatrix} \bar{\mathbf{N}}_1 \\ \bar{\mathbf{N}}_2 \\ \vdots \\ \bar{\mathbf{N}}_n \end{bmatrix} \mathbf{p} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_n \end{bmatrix} \text{ or } \bar{\mathbf{N}}\mathbf{p} = \mathbf{v}. \tag{19}$$

Hence the solution for $\mathbf{p}$ is as follows where $\bar{\mathbf{N}}^\dagger$ denotes the pseudo-inverse of $\bar{\mathbf{N}}$,

$$\mathbf{p} = \bar{\mathbf{N}}^\dagger \mathbf{v}. \tag{20}$$

8 Conclusion

A new simplification of the free-moving scanner resection problem has been formulated and accurate solutions presented. The new approach makes use of angular and linear velocity data typically recorded during the imaging period (c.f. the SPOT satellite attitude and orbit control system [Spo, 1991]) to simplify the scanner resection problem to the simpler case of the frame camera. A robust solution to frame camera resection is required since ground control point vectors are translated closer together in order to achieve the simplification. The frame camera problem has been separated into two — one for the determination of rotation, and one for position. This facilitates the accurate determination of unknowns despite the poorly-distributed ground control.

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