# UPDATING THE CONTENT OF A GIS DATABASE BY THE ROBUST FUSION OF DIFFERENT LOW COST GEOMETRIC SOURCES IN GENERALIZED ANALYTICAL MIXED MODELS

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#### **ABSTRACT**

The paper proposes an original method for the process of simultaneously georeferencing the digital version of several cartographic units, belonging either to the same or to different map projections, and updating the same cartographic product by the restitution of the particulars taken on the ground with amateur or digital photogrammetric sensors.

The georeferencing procedure is performed by a classical 8-parameter transformation with the introduction of constraint equations for the tie points of contiguous map units, while the updating is carried out by Direct Lirear Transformation (DLT).

Both analytical processes are joined in a common mixed model of estimation and prediction. The coordinates of the points to be updated are submitted to a BLUU estimate, together with the orientation parameters of the images, the cartographic transformation coefficients and the tie point coordinates of cartographic and photographic layers, used to orient the pictures.

The coordinates of the cartographic control points, possibly seen on some pictures, are predicted.

Some numerical experiments of updating the digital version of a technical map are finally reported.

#### 1. INTRODUCTION

The wide spread of GIS in the last years requires the availability of reliable and up-to-date digital mappings, at different scales.

Although the best and completest cartographic products can be today obtained by photogrammetric survey at a proper scale, a correct scanning of existing cartography submitted to a robust georeferencing process, can give acceptable cartographic products for GIS applications.

For these reasons, the kind of updating and testing procedures to adopt become prominent. These must be highly productive, of low cost and easy realization, of sufficient precision and, just for the testing, able to verify the accuracy of global cartographic entities and not only of isolated points (static or pseudo-static mode) or lines and trajectories (kinematics mode) as can be done for both methods with GPS techniques.

The use of amateur or digital sensors for updating the map with the necessary particulars by large scale pictures taken on the ground, is a technique that can dramatically reduce the costs and improve the productivity, if point coordinates used to orient the images are extracted from an already existing digital map or from a digital version, obtained by scanning procedure, of a map.

In this case a careful analysis of error sources characterizing non metric images and the point coordinates used to orient them must be done, and a specific mathematical model able to predict such kinds of error, must be applied.

The paper proposes an original method for updating a digital map and testing its quality, by the analytical fusion, in a mixed model of estimation and prediction, of the georeferencing process of several kinds of digital maps, with the object reconstruction of the necessary parts, obtained by Direct Linear Transformation (DLT) of non metric pictures.

Among all possible estimations and predictions, the paper considers the ones satisfying optimality conditions in terms of "mean square error", that must be the minimum among all possible linear estimators and predictors, and in terms of "uniform umbiasedness", that must be valid not only for the

true but unknown values of the terms to be estimated or predicted, but for any value of them.

The coordinates of cartographic points to be updated and tie points, together with all transformation parameters, are submitted to a BLUU estimate, while cartographic control points that can be seen on some pictures are predicted.

For control points, GPS measurements allow to fix a correct datum to the digital mapping, since the transformation from the GPS reference system to the cartographic one is precise enough for small areas.

The computation of a multiple georeferencing procedure and the simultaneous solution of a DLT algorithm for updating the contents of the map layers allows to consider the correlation existing between the different data acquisition techniques, and, at the same time, makes the computation procedures more robust and reliable than single transformations.

Particular emphasis is given to the correct weighting of the functional model relating to each transformation. In this regard, a robust method of assigning the observational weights according to the residual values of the observational equations is proposed (Krarup, Juhl and Kubik, 1980).

Some numerical experiments relating to the updating of the digital version of the cartographic product of the Friuli-Venezia Giulia regional administration at the scale 1:5.000, put in evidence the capability and the limits of the proposed method.

# 2. ANALYTICAL MODEL

Let's take into account an 8-parameter transformation as analytical model for simultaneously georeferencing the digital version of contiguous cartographic units or different map layers obtained by scanning procedures:

$$x_{i} = \frac{a_{l}X_{i} + b_{l}Y_{i} + c_{l}}{uX_{i} + vY_{i} + 1}$$
(1)

$$y_{i} = \frac{a_{2}X_{i} + b_{2}Y_{i} + c_{2}}{uX_{i} + vY_{i} + 1}$$

where:

 $\mathbf{x}_i$ ,  $\mathbf{y}_i$  are the coordinates of point i in the instrumental reference system;

 $\boldsymbol{X}_i$  ,  $\boldsymbol{Y}_i$  are the coordinates of point i in the cartographic reference system;

 $a_j$ ,  $b_j$ ,  $c_j$  (j = 1,2), u, v are the unknown coefficients of an 8-parameter transformation.

To solve system (1) for the unknown coefficients of the homologous transformation and for the tie point coordinates of two contiguous map units, it is necessary to linearize system (1) in its fixed part and its stochastic part:

$$l_1 + v_1 = A_{11}x_1 + A_{14}x_4 + A_{15}s$$

where:

 $A_{11}$  is the coefficient matrix of the unknown transformation parameters and cartographic tie point coordinates;

 $A_{14}$  is the coefficient matrix of tie point coordinates for cartographic and photographic layers;

A<sub>15</sub> is the coefficient matrix relating to the stochastic cartographic control point coordinates;

 $\mathbf{x}_1$  is the unknown vector of an 8-parameter transformation coefficients and cartographic tie point coordinates;

 $\mathbf{x}_4$  is the unknown vector of cartographic-photographic tie point coordinates;

s is a stochastic vector of cartographic control point coordinates.

To perform the updating of an already existing digital mapping by pictures taken on the ground with amateur cameras, Crosilla and Visintini (1996) have already proposed to apply the socalled Direct Linear Transformation (DLT):

$$x_{j} = \frac{L_{1}X_{j} + L_{2}Y_{j} + L_{3}Z_{j} + L_{4}}{L_{9}X_{j} + L_{10}Y_{j} + L_{11}Z_{j} + 1} + x_{j}'K_{1}r_{j}^{2}$$

$$y_{j} = \frac{L_{5}X_{j} + L_{6}Y_{j} + L_{7}Z_{j} + L_{8}}{L_{9}X_{i} + L_{10}Y_{i} + L_{11}Z_{i} + 1} + y_{j}'K_{1}r_{j}^{2}$$
(2)

where:

 $x_i$ ,  $y_i$  are the image coordinates of point j;

 $X_j$ ,  $Y_j$  are the coordinates of point j in the cartographic reference system;

Z; is the orthometric height of point j;

 $L_1$ , ...,  $L_{11}$  are the DLT parameters, function of the traditional internal orientation parameters  $(x_0, y_0, c)$  and the external  $(X_c, Y_c, Z_c, \omega, \phi, k)$  ones;

 $K_1$  is the coefficient describing image deformation due to objective radial distortion, being  $x_j = x_j' - x_0$ ,  $y_j' = y_j - y_0$ ,  $r_j^2 = x_j'^2 + y_j'^2$ .

Also in this case to solve (2) for the unknown DLT parameters and coordinates of points necessary to update the map, system (2) must be linearized in the following way:

$$I_2 + V_2 = A_{22}X_2 + A_{23}X_3 + A_{24}X_4 + A_{25}S$$

where:

A<sub>22</sub> is the coefficient matrix of DLT parameters and unknown height Z of cartographic points;

A<sub>23</sub> is the coefficient matrix relating to cartographic point coordinates to be updated;

A<sub>24</sub> is the coefficient matrix of tie points for cartographic and photographic layers, used to orient the pictures;

A<sub>25</sub> is the coefficient matrix relating to cartographic stochastic control point coordinates appearing on the photos;

x<sub>2</sub> is the unknown vector of DLT parameters and orthometric height Z of cartographic points;

x<sub>3</sub> is the unknown vector of point coordinates to be updated.

It seeems worth to consider both systems (1) and (2) in the same computation process since in this way tie points for cartographic and photographic layers, which are necessary to orient the images, improve the redundancy of the global linear systems and make it possible to take advantage of the correlation existing between variables of a georeferencing process and those of a DLT transformation.

The global system can be written as a "mixed linear model" (Dermanis, 1990):

$$1 + v = Ax + Bs \tag{3}$$

where:

$$l = \begin{vmatrix} l_1 \\ l_2 \end{vmatrix} \quad \mathbf{v} = \begin{vmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{vmatrix} \quad \mathbf{A} = \begin{vmatrix} \mathbf{A}_{11} & \varnothing & \varnothing & \mathbf{A}_{14} \\ \varnothing & \mathbf{A}_{22} & \mathbf{A}_{23} & \mathbf{A}_{24} \end{vmatrix} \quad \mathbf{B} = \begin{vmatrix} \mathbf{A}_{15} \\ \mathbf{A}_{25} \end{vmatrix}$$

and with:

$$\begin{split} E(\mathbf{v}) &= 0 \qquad E(\mathbf{v} \cdot \mathbf{v}^T) = \mathbf{Q} \qquad E(\mathbf{s}) = \mu_{\mathbf{s}} \\ E\Big[(\mathbf{s} - \mu_{\mathbf{s}}) \cdot (\mathbf{s} - \mu_{\mathbf{s}})^T\Big] &= \mathbf{Q}_{\mathbf{s}\mathbf{s}} \qquad E\Big[(\mathbf{s} - \mu_{\mathbf{s}}) \cdot \mathbf{v}^T\Big] = \mathbf{Q}_{\mathbf{s}\mathbf{v}} = 0 \end{split}$$

As a consequence it holds that:

$$\begin{split} \boldsymbol{\mu}_{l} &= \boldsymbol{E}(l) = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{\mu}_{s} & \boldsymbol{Q}_{ll} &= \boldsymbol{B}\boldsymbol{Q}_{ss}\boldsymbol{B}^{T} + \boldsymbol{Q} \\ \boldsymbol{Q}_{sl} &= \boldsymbol{Q}_{ss}\boldsymbol{B}^{T} & \boldsymbol{Q}_{vl} &= \boldsymbol{O} \end{split}$$

According to the general theory of estimation and prediction vector  $\mathbf{x}$  will be estimated while the stochastic vector  $\mathbf{s}$  will be predicted.

The best linear uniformly unbiased (BLUU) estimate  $\hat{\mathbf{x}}$  of  $\mathbf{x}$  can be obtained by:

$$\hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{Q}_{\mathbf{H}}^{-1} (\mathbf{I} - \alpha \mathbf{B} \boldsymbol{\mu}_{\mathbf{s}}) \tag{4}$$

while the BLUU compatible prediction  $\tilde{s}$  of s is given by (Dermanis, 1990):

$$\tilde{\mathbf{s}} = \alpha \mu_{\mathbf{s}} + \mathbf{Q}_{\mathbf{s}\mathbf{s}} \mathbf{B}^{\mathrm{T}} \mathbf{Q}_{\mathrm{H}}^{-1} \mathbf{H} (\mathbf{I} - \alpha \mathbf{B} \mu_{\mathbf{s}}) \tag{5}$$

and  $\tilde{\mathbf{v}}$  follows from:

$$\tilde{\mathbf{v}} = \mathbf{I} - \mathbf{A}\hat{\mathbf{x}} - \mathbf{B}\tilde{\mathbf{s}} \tag{6}$$

where:

$$\mathbf{H} = \mathbf{I} - \mathbf{A} \mathbf{N}^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{Q}_{\mathrm{H}}^{-1} \qquad \mathbf{N} = \mathbf{A}^{\mathrm{T}} \mathbf{Q}_{\mathrm{H}}^{-1} \mathbf{A}$$

The parameter  $\alpha$  takes the values:

$$\alpha = 1$$

for a non homogeneous estimation or prediction (inhomBLUUE/P)

$$\alpha = \frac{\mu_s^T \mathbf{B}^T \mathbf{Q_{II}}^{-1} \mathbf{H} \mathbf{I}}{\mu_s^T \mathbf{B}^T \mathbf{Q_{II}}^{-1} \mathbf{H} \mathbf{B} \mu_s}$$

for an homogeneous estimation or prediction (homBLUUE/P)

In the case  $\mu_s = 0$  formulae (4) and (5) can be simplified as:

$$\hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{Q}_{\mathbf{I}}^{-1} \mathbf{I}$$

$$\tilde{\mathbf{s}} = \mathbf{Q}_{\mathbf{s}\mathbf{s}} \mathbf{B}^{\mathrm{T}} \mathbf{Q}_{\mathbf{I}}^{-1} \mathbf{H} \mathbf{I}$$
(7)

In order to make the estimation of vector  $\mathbf{x}$  more robust, especially for the part relating to the coordinates of points to be updated by photogrammetric measurements, it is necessary to introduce some conditional equations in system (3):

$$l + v = Ax + Bs$$

$$c = Dx$$
(8)

where:

**D** is the coefficient matrix of the conditional equations; **c** is the known vector of conditional equations.

System (8) can be considered as a mixed linear model with constraints and the BLUU estimate of vector **x** can be obtained by (Crosilla and Visintini, 1996):

$$\hat{\mathbf{x}} = \mathbf{N}^{-1} \left[ \mathbf{A}^{\mathrm{T}} \mathbf{Q_{II}}^{-1} (\mathbf{I} - \alpha \mathbf{B} \boldsymbol{\mu}_{\mathbf{s}}) + \mathbf{D}^{\mathrm{T}} \mathbf{n} \right]$$
(9)

where:

$$\mathbf{n} = \left[\mathbf{D}\mathbf{N}^{-1}\mathbf{D}^{\mathrm{T}}\right]^{-1}\!\!\left[\mathbf{c} - \mathbf{D}\mathbf{N}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Q}_{ll}^{-1}(l - \alpha B\boldsymbol{\mu}_{s})\right]$$

Again in case  $\mu_s = 0$ , formulae (9) can be simplified as:

$$\hat{\mathbf{x}} = \mathbf{N}^{-1} \Big[ \mathbf{A}^{\mathrm{T}} \mathbf{Q}_{\mathbf{I}}^{-1} \mathbf{I} + \mathbf{D}^{\mathrm{T}} \mathbf{n} \Big]$$
 (10)

where:

$$\mathbf{n} = \left[\mathbf{D}\mathbf{N}^{-1}\mathbf{D}^{\mathrm{T}}\right]^{-1}\!\!\left[\mathbf{c} - \mathbf{D}\mathbf{N}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Q}_{ll}\mathbf{l}\right]$$

# 3. CONSIDERATIONS ABOUT THE METHOD PROPOSED

### 3.1. Photogrammetic conditional equations

As was reported previously to make the estimation of vector x more robust, system (8) can take into account some conditional equations relating to the sub-system of the DLT.

$$\begin{split} \mathbf{v}_1 = & \Big[ (L_1{}^2 + L_2{}^2 + L_3{}^2) - (L_5{}^2 + L_6{}^2 + L_7{}^2) \Big] (L_9{}^2 + L_{10}{}^2 + L_{11}{}^2) + \\ & - (L_1L_9 + L_2L_{10} + L_3L_{11})^2 + (L_5L_9 + L_6L_{10} + L_7L_{11})^2 = 0 \\ & \mathbf{v}_2 = (L_1L_5 + L_2L_6 + L_3L_7)(L_9{}^2 + L_{10}{}^2 + L_{11}{}^2) + \\ & - (L_1L_9 + L_2L_{10} + L_3L_{11})(L_5L_9 + L_6L_{10} + L_7L_{11}) = 0 \end{split}$$

These two first conditional equations follow from the fact that among the eleven  $L_j$  (j=1,...,11) DLT parameters just nine are independent, that is those relating to the three internal orientation and the six external orientation parameters for each image (Bopp and Krauss, 1978).

$$v_{3} = x_{0} - \frac{L_{1}L_{9} + L_{2}L_{10} + L_{3}L_{11}}{L_{9}^{2} + L_{10}^{2} + L_{11}^{2}} = 0$$

$$v_{4} = y_{0} - \frac{L_{5}L_{9} + L_{6}L_{10} + L_{7}L_{11}}{L_{9}^{2} + L_{10}^{2} + L_{11}^{2}} = 0$$

$$v_{5} = -c_{x}^{2} - x_{0}^{2} + \frac{L_{1}^{2} + L_{2}^{2} + L_{3}^{2}}{L_{9}^{2} + L_{10}^{2} + L_{11}^{2}} = 0$$

$$v_{6} = -c_{y}^{2} - y_{0}^{2} + \frac{L_{5}^{2} + L_{6}^{2} + L_{7}^{2}}{L_{9}^{2} + L_{10}^{2} + L_{11}^{2}} = 0$$
(12)

These four more conditional equations (Crosilla, Guerra and Visintini, 1993) report relationships joining  $\mathbf{x}_0$ ,  $\mathbf{y}_0$ ,  $\mathbf{c}_{\mathbf{x}}$  and  $\mathbf{c}_{\mathbf{y}}$  to some functions of the  $\mathbf{L}_j$  parameters; they can be applied if the internal orientation parameters are exactly known. On the contrary, if the internal orientation parameters are not exactly known, they can be considered as sthocastic values characterized by a certain dispersion.

Conditional equations such as (12) are particularly useful for the solution of the numerical problem, since they make it more robust also in case the values of  $\mathbf{x}_0$ ,  $\mathbf{y}_0$ ,  $\mathbf{c}_{\mathbf{x}}$  and  $\mathbf{c}_{\mathbf{y}}$  are very approximate. As a value of  $\mathbf{x}_0$  and  $\mathbf{y}_0$  the coordinates of the image centers can be used with enough precision, while  $\mathbf{c}_{\mathbf{x}}$  and  $\mathbf{c}_{\mathbf{y}}$  can be substituted by a unique value given by the nominal focal length of the objective multiplied by the photo enlargement. In this way a sort of probability ellipses is identified in correspondence to the image center: the coefficient of radial distortion  $\mathbf{K}_1$  can be better estimated since it refers to a point of central simmetry defined with enough precision.

#### 3.2. Correct weighting of the observational equations

The problem of the correct weighting of the observational equations, both cartographic and photogrammetric, represents a very important aspect within the practical application of model (8).

The method used is the so-called "danish method" (Krarup, Juhl and Kubik, 1980) which assigns a variable weight p<sub>i</sub> at each iteration, function of the correspondent standardized residual vs<sub>i</sub>, according to the following expression:

$$p_i(vs_i) = e^{-0.05 vs_i^k}$$
 (13)

where:

k is a coefficient equal to 4.4 in the first three iterations and equal to 3 in the following ones.

The values so computed are used to find the diagonal matrix  $\mathbf{Q}$  of the expression  $\mathbf{Q_{ll}} = \mathbf{BQ_{ss}B^T} + \mathbf{Q}$ .

This method, however, should be used with a particural care since it can cause divergent solutions. The analitycal model (8) is in fact correct only if the functions are linear or linearizable around a very approximative value. In this regard the approximative values of the unknowns are in general very far from the true values, for the reason described in the next paragraph 3.3. The increments from such value estimated in the first iterations are therefore significantly different from zero, and for this reason the coefficient matrices A and B should contain also the partial derivates of heigher order.

Since the program implements only terms of first order, the estimation of the residual  $\mathbf{v}$  by the relationship (6)  $\tilde{\mathbf{v}} = \mathbf{l} - \mathbf{A}\hat{\mathbf{x}} - \mathbf{B}\tilde{\mathbf{s}}$  is wrong, because  $\mathbf{v}$  is also function of the heigher order terms not considered at all.

To solve this problem a constant diagonal matrix  $\mathbf{Q} = \gamma \mathbf{I}$ , where  $\gamma = 0.1$ , is applied for the first 7 iterations, and the danish method is applied for the following ones.

#### 3.3. Numerical problems

The analitycal model reported in (8) makes it possible to solve simultaneously two different problems: i.e. the cartographic georeference and the non-conventional photogrammetric survey. Furthermore for each phase a sort of specific problems are also solved. These are the plane transformations between the different map units and map layers and, for the photogrammetric survey, the updating of particulars not already reported in cartography. From the computational point of view a big difficulty is given by the fact that for the most of the unknown values to be estimated or predicted their approximative values are not available.

For this purpose and for each group of unknowns contained in the subvectors  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$  and s different operational strategies have been identified.

The working method followed considers the geometrical meaning of the different unknowns, and consists of a cascade process for the computation of the approximate values.

The estimation of the parameters of an homologous transformation and the planimetric coordinates of the cartographic tie points, contained in the subvector  $\mathbf{x}_1$ , allows to estimate the planimetric coordinates of the cartographic-photogrammetric tie points  $\mathbf{x}_4$ . These values together with the orthometric height  $\mathbf{x}_{2_2}$  make it possible to compute the approximate values of the DLT parameteres contained in the subvector  $\mathbf{x}_{2_4}$ .

The increments of coordinates contained in the vector s and relative to cartographic and photogrammetric control points refer to approximate values which are more close to reality because they could come, for instance, from GPS measurements.

#### 3.4. Use of GPS in cartography

Since the beginning, GPS has also been tried to determine the coordinates of the photogrammetric check points (Ackermann, 1991). For this purpose it is useful to recall some of the aspects jointed to the instrumentation, the errors of the methods, the productivity and the cartographic applications.

The instruments available today are technically better than those of some years ago, the receivers have a better signal to noise ratio and are less sensitive to systematic errors. This has permitted the obtaining of greater and greater precision, especially in the differential use of the code. Furthermore, the completion of the constellation has permitted, with a single code, that is, with receivers that cost of a few thousands dollars, the obtaining of sub metrical precision. These precisions are already sufficient for the support and updating of middle and small scale maps (1:10.000+1:100.000) but not yet for maps of larger scale.

The use of the single code has the advantage of furnishing low cost coordinates that are free of momentary losses of signal, however, long sessions of measurements are necessary to obtain differential sub metrical precision because of multi-path errors (Yola and Kleusberg, 1991).

The precision and the productivity increases with the use of single frequency receivers which, at the moment, have a cost that can be compared with that of a good total station. It is not necessary to use two frequencies for the support or the construction of large scale maps (1:1.000÷1:5.000) up to a

distance of 10÷15 km. In this extent the precision furnished by the construction companies varies from ±(1cm+2ppm) for the static modality to ±(2cm+2ppm) for the stop-and-go method.

The most productive methods, in increasing order, are: the pseudo-static method which forces the reoccupation of site after about an hour, the stop-and-go method and the continuous kinematic method.

The kinematic method can also be used for positioning in real time (RTK) with the use of transmitting station, a receiver and two radio modem (Allison, Griffioen and Talbot, 1994).

The stop-and-go method forces the presence of a good constellation (at least 5 satellites). All the methods are suitable for a maximum distance of 10+15 km.

The use of the single L1 frequency introduces "ionodeformations" on a ground network, which as a first approximation, are assimilable to a scale factor, even though it is possible to recover 90% of the ionospheric delay with the divergence method in the C/A & L1 receivers.

The use of the predicted ephemeris, which are indispensable for the RTK positioning technique, causes an error which is reflected on the baseline components, but only slightly on the slope distances.

These and other errors increases for long distances of the fixed receiver, then is not possible to use data from a fixed station national service over the 15 km distance, but instead it is necessary to use a couple of L1 receivers.

#### 3.5. Transformation problems

GPS cartographic surveying permits updating of a GIS database with new points or obtain the GPS coordinates from the already surveyed points. In this second case the proposed technique allows one both to obtain the transformation parameters from among the GPS coordinates, cartographic coordinates and orthometric heights and to improve the global precision of the map. This latter requirement is very useful if one uses the digital version of cartographic products or cadastral maps.

It is known that the precision for long distances is poor for these maps, while instead in this case the GPS system furnishes the greatest precision.

To obtain the  $(\varphi,\lambda,h)_n$  coordinates of a set of points referring to a local ellipsoid (n) by means of the  $(\varphi,\lambda,h)_w$  of the same points referring to another ellipsoid (w) that is differently oriented, the well-known Molodensky formula is useful. Let's imagine that the first system is the national system, in which the cartography is referred, and the WGS84 geocentric system is the second. It is known in Italy, from experience, that the variations due to passage are of the order 100-200 m in planimetry and of about 50 m in altimetry. One can write (Pierozzi, 1989):

$$\begin{vmatrix} \phi_{n} \\ \lambda_{n} \\ h_{n} \end{vmatrix} = \begin{vmatrix} \phi_{w} \\ \lambda_{w} \\ h_{w} \end{vmatrix} + \mathbf{A}(\delta X_{0}, \delta Y_{0}, \delta Z_{0}, \delta \epsilon_{X}, \delta \epsilon_{Y}, \delta \epsilon_{Z}, \delta a, \delta \alpha)^{T} = \mathbf{A} \delta^{T}$$
(14)

The first six components of the  $\delta$  vector are the shift and rotation between the two systems, the last two are the semiaxis and the ellipsoidal flattering variations. The hypothesis that no scale factors exist between the systems has been assumed in the formula. The terms of matrix A are known non-linear functions of  $(\phi, \lambda, h)_w$ , the  $\delta$  terms are "a priori" unknowns of the problem (apart from terms  $\delta a$  and  $\delta \alpha$ ). When, in a certain area, a certain number of points are available in both systems, equation (14) is useful to obtain by a least square solution these

six parameters to use for the other GPS points inside the same sample.

It is possible to show, in the topographic field, i.e. in a surrounding about 10 km, that equations (14) can be simplified by finding a linear relationship between the  $(X_G\,,Y_G\,,Z_G)$  coordinate system and the (X,Y,Z) cartographic coordinate system.

 $\dot{X}_G$  and  $Y_G$  are the coordinates obtained from the cartographic transformation of the  $\phi_G$ ,  $\lambda_G$  values of the GPS network, and  $h_G$  is the ellipsoidal height of the GPS point.

The (X,Y) cartographic system is that of the Gauss projection, the Z coordinate is the orthometric height of the point. The maximum errors of this approximation are of few ppm in planimetry, also for great height differences of the ground. A second hypothesis, which is reasonable in the topographic field is that the geoid undulation can be modelled with a plain of the type (Sguerso and Radicioni, 1992):

$$N_n = N_0 + \gamma \phi_w + \delta \lambda_w \tag{15}$$

One can finally write:

$$\begin{vmatrix} \mathbf{Y} \\ \mathbf{X} \\ \mathbf{Z} \end{vmatrix} = \begin{vmatrix} \mathbf{Y}_0 \\ \mathbf{X}_0 \\ \mathbf{Z}_0 \end{vmatrix} + \begin{vmatrix} \mathbf{p} \mathbf{Y}_{G} + \mathbf{q} \mathbf{X}_{G} \\ \mathbf{r} \mathbf{Y}_{G} + \mathbf{s} \mathbf{X}_{G} \\ \mathbf{t} \mathbf{Y}_{G} + \mathbf{u} \mathbf{X}_{G} + \mathbf{h}_{G} \end{vmatrix}$$
(16)

Because of the not precise absolute positioning, the coordinates used in equation (16) are not exactly the WGS coordinates but generic GPS coordinates ( $\phi_G$ ,  $\lambda_G$ ,  $h_G$ ), shifted with respects to the first, maximum values of 100+200 m for the whole network. Nine unknowns appear in this linear equations that can solved sharing the planimetric from the altimetric problem or by solving the couple problem.

#### 4. PRACTICAL EXPERIMENT

The photogrammetric method proposed in this paper, considering the last version of the software FOTO3D (Visintini, 1993) that implements the analytical model (7), is characterized by low cost and a kind of applicability that can be considered almost in "real time". It is then possible to produce and update an economical GIS geometrical bidimensional database.

The method is highly productive since it requires just some amateur pictures taken in the field and no kind of surveying measurements if the georeferencing process is performed using as control points the cartographic grid. It is only necessary to locate (at least) two vertical signals on two different carthographic-photographic tie points.

If this is not the case, control points can be determined for instance by GPS measurements according to what already reported in 3.4.

The method proposed has been applied to a part of the unit n. 087044 of the Technical Mapping of the Friuli-Venezia Giulia Italian region in scale 1:5.000, obtained by aerial photogrammetric survey in 1977. The map projection is the conformal Italian Gauss-Boaga which considers the international ellipsoid oriented at Mt. Mario (Roma).

The cartographic unit has been acquired in digital form by an Epson GT-9000 scanner having a resolution of 600 dpi (real dimension of the pixel on the terrain 21.1 cm).

A quick analysis of the map content has put in evidence the absence of some buildings, built later than the original flight. For this reason some pictures of these lacking particulars (see Figure 1 and 2) have been captured on the terrain by a Pentax P30 reflex camera with an objective focal length of 28 mm.

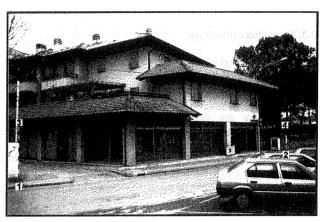


Figure 1: First non-metric image of the building to be updated in cartography



Figure 2: Second non-metric image of the building to be updated in cartography

The image coordinates of tie and control points have been acquired by a Calcomp Drawing Board II digitizer. Furthermore, the FOTO3D software has been applied to georeference the cartographic unit and to determine the external orientation parameters of some images, obtaining in this way a digital version of the map and its updating.

Points 1, 2, 3 and 4 (see Figure 1 and 2) are the cartographic-photogrammetric tie points used to join both methods and to fix a weak datum to the photogrammetric survey. The approximate values of their planimetric coordinates have been computed by a preliminary georeference procedure, while their approximate height has been assumed by a point where height is reported on the map and by two vertical signals visible in the photos.

Furthermore 26 unknown points have been used. These have been chosen with an homogeneous distribution on the image.

The obtained results seem to be particularly promising and are

The obtained results seem to be particularly promising and are summarized in the following points:

- it was possible to orient two images without any topographic measurements;
- the numerical instability, proper of the DLT, relative to the orientation with control points belonging to a plane, has been overcome with success;
- the planimetric accuracy of unknown points is characterized by a relative accuracy of almost 4 cm: for 16 points along the same vertical (in groups of 2) the difference of planimetric coordinates obtained is equal to 39 mm.
- the estimated height value for the unknown points is characterized by a relative accuracy of ~ 3 cm: for 16 points of

constant height (in groups of 2 or 3) the difference of height obtained is equal to 24 mm;

The mean values of the standard deviations for the estimated coordinates of 33 points computed with FOTO3D are reported in the following table:

	st.dev.X	st.dev.Y	st.dev.Z	st.dev.TOT
mean	21	12	9	26
rmse	±24	±15	±11	±30

Table 3: Mean values obtained in the practical experiment (in millimeters)

These values are still better then those reported in the above statements: recall that the table values are estimated and should be considered, with a probability of 99%, a ± 3-st.dev. values. In any case, the obtained precision is much higher than that required in the technical specifications of a digital mapping in scale 1:5.000.

#### 5. CONCLUSIONS

The method proposed for the production and updating of low cost digital cartography, to be used as the geometric database of a GIS, proves to be particularly interesting, both from a theoretical and from a practical point of view.

The adopted analytical model (mixed linear model) allows the simultaneous estimation and prediction of several groups of unknowns, so that the different reliabilities of different observations (cartographic, photogrammetric, GPS) can be considered in the definition of a unique "datum".

The processes of georeferencing the cartographic units and updating the map content are performed simultaneously, exploiting the correlation created by cartographic-photogrammetric tie points.

The instrumentation required for the application of this method is extremely simple: as field operations a single-frequency GPS receiver (two ones in case a fixed station is too far) and an amateur camera are enough; then a PC for calculations, a scanner, a digitizer and the FOTO3D software are required.

In this way, an up-to-date geometric database, having a satisfactory precision, can be produced at low cost and in short time, using an existing cartography and amateur photographs, as was shown in the above example.

Through GPS control it is possible to update cartographic areas with no cartographic particulars visible in the images, to reduce deformations of the cartographic support and to join, if necessary, several layers into a unique reference.

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