MEMBERSHIP FUNCTIONS FOR FUZZY SET REPRESENTATION OF GEOGRAPHIC FEATURES

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ABSTRACT

A conceptual framework for geographic feature representation has been developed by applying data modelling abstraction levels to spatial, thematic, and temporal dimensions of geographic phenomena and structuring attributes and relationships for each dimension to build a features knowledgebase. Many aspects of the framework are currently implemented in geographic information system (GIS) software directly as classical mathematical sets. However, the strength of the framework appears to be in its ability to include multiple representations through feature-based or object-oriented approaches. The theory of fuzzy sets provides a basis for multiple representation of non-partitioned geographic features with indeterminate boundaries. Representation of geographic phenomena as fuzzy features in a GIS requires the development and implementation of fuzzy operators including buffering; overlay supporting union, intersection, and complement; and boundary. These GIS processing operators have been developed (Katinsky, 1994; Usery, 1996a) and implemented as additional functions of the commercial GIS software package, Imagine from ERDAS, Inc.

Using fuzzy set representation within the developed framework and applying the fuzzy set GIS operators requires the development of specific fuzzy set membership functions for individual geographic features from multiple data sources. The fuzzy set function may model the indeterminacy of spatial position, thematic attributes, or temporal attributes. This paper presents examples of fuzzy set membership functions with either the spatial or thematic dimensions represented. For example, a hill may be represented as a fuzzy set by using the elevation values from a digital elevation model (DEM) in the fuzzy membership function while a weed patch in an agricultural field can be modelled by using spatial position as the membership function in a digitized aerial image.

1. INTRODUCTION

The representation of geographic phenomena in computer-based processing systems has evolved from purely geometric symbolization of points, lines, and areas for mapped entities to the inclusion of thematic attributes associated with the geometric objects in relational databases of current commercial software packages. Recent research in feature-based approaches has led to the development of a comprehensive framework for representing geographic phenomena and provides a mechanism to include both thematic and temporal attributes and relationships in addition to the spatial or geometric models of current systems (Usery, 1993; 1996a; 1996b). This framework allows a variety of representations for single features including multiple geometries from differing data sources.

One approach to representing geographic features with indeterminate boundaries uses fuzzy set theory (Altman, 1994; Katinsky, 1994; Usery, 1996a). While this approach has been demonstrated to capture the indeterminacy of particular features, the membership function of the fuzzy set remains problematic for specific feature types. For example, one may represent the thematic dimension of a feature as a fuzzy set (Burrough, 1989; Wang, 1990). Alternatively, a fuzzy boundary based on spatial position may better capture the feature's indeterminacy (Katinsky, 1994). Temporal attributes could also be modelled as fuzzy sets. The outstanding problem is one of determining an appropriate membership function for the specific feature of interest.

It is the purpose of this paper to present examples of geographic features with specific fuzzy set membership functions from multiple data sources. Section 2 briefly describes the conceptual framework for geographic features which allows multiple representations from multiple data sources. Section 3 draws from Katinsky (1994) and briefly describes the fuzzy operators, union, intersection, buffering, and boundary which are necessary for processing fuzzy features in a GIS. Section 3 also provides specific examples of fuzzy set membership functions based on data source and the feature dimension, spatial or thematic, which is the controlling parameter of the fuzzy set function. A final section draws conclusions and provides a basis for further research toward developing a features knowledgebase to support geographic analysis.

2. A FRAMEWORK FOR GEOGRAPHIC FEATURES

Since all geographic phenomena can be modelled along the three dimensions of space, theme, and time (Berry, 1964; Sinton, 1978) and each phenomenon possesses unique characteristics and interactions with other phenomena, feature representation must include attributes of individual features and relationships among features (Usery, 1994a). The data modelling levels of concept, model, and structure for each of the three feature dimensions are documented elsewhere within a conceptual framework which supports multiple representations (Usery, 1996a; 1996b). That framework is constructed with the geographic feature as the real world entity with its object representation including attributes of space, theme, and time. Thus, the space-dominant model of current software packages and current GIS is avoided and the actual locational coordinates and topology of a geographic phenomenon become attributes and relationships of the feature in a manner similar to the thematic and temporal attributes and relationships.

The strength of this representational framework is to allow geometry of the feature to vary with the data source. For example, a terrain feature, such as a hill, can be represented in a DEM as a set of pixel values defined in a Boolean operation. Alternatively, the geometry of the hill may be defined as a fuzzy set with some pixels possessing partial membership values in the hill feature. A third geometric representation from the same source is to draw a vector line around the spatial extent of the hill, again defining a Boolean set of pixels but this time only using the vector line as the hill boundary rather than the actual elevation values of the pixels as in the first case. If one examines the hill from a different data source, a raster scanned topographic map or digital raster graphic (DRG) for example, the geometry is likely to vary from the DEM representation. Finally, if various scales of topographic maps are used, then the representational geometry of the hill changes with scale. The feature-based framework allows all of these representations of the hill to be equally valid, simultaneously available, and any one of them may be used for analytical purposes. For a more detailed presentation of the framework, see Usery (1994a; 1994b; 1996a; 1996b).

3. FUZZY GIS OPERATORS AND FUZZY FEATURES

Representation of geographic phenomena as fuzzy features in a GIS requires operators which perform standard GIS functions, such as overlay and buffering, on the fuzzy representations. Katinsky (1994) developed a suite of fuzzy operators including fuzzy overlay with union, intersection, and complement, fuzzy spatial buffering, and fuzzy boundary. The mathematical model and GIS data model for these operations are detailed in Katinsky (1994). An implementation of these functions as an extension of the Imagine GIS and image processing software has been developed (ERDAS, 1995). The examples below are developed from the Katinsky mathematical theory and the Imagine implementation.

Using spatial position to determine the fuzzy extent of a geographic feature requires the following definitions:

◇ **Definition 1:** Given a universe, V, of objects, a **fuzzy set** $A^* \subset V$ is a mapping, denoted f_{A^*} from V to the unit interval, $[0,1]$ where $f_{A^*}(x)$ is the **membership value** of x in A^* for any $x \in V$.

◇ **Definition 2:** A **map space** V is a bounded subset of R^2.

◇ **Definition 3:** A **fuzzy feature** is a fuzzy set whose universe is a map space.

◇ **Example 1:** Let $V = [0, 100\text{m}]^2$ be a map space and let $A^* \subset V$ be a fuzzy feature representing a weed patch in a cotton field centered at (15m,20m). Define the feature with the following membership function:

$$f_A{*}(v) = \begin{cases} 1 & : \|v - (10,20)\| < 10 \\ \dfrac{10 - \|v - (15,20)\|}{10} & : 10 < \|v - (10,20)\| < 20 \\ 0 & : 20 \leq \|v - (10,20)\| \end{cases} \quad (1)$$

where the dual vertical bars indicate the Euclidean distance between the points.

The example defines any location within 10 metres as definitely within the weed patch. Any location farther than 20 metres from the center of the weed patch is definitely outside the feature. Locations greater than 10 metres and less than 20 metres have membership values which linearly relate to distance from the weed patch center. While this is a simplistic model using a linear interpolation formula, it illustrates the concept of a fuzzy feature defined by the spatial feature dimension.

Note that once the spatial extent of the fuzzy feature is defined using spatial position as the defining parameter in the fuzzy set function, that function can be used with any data set which has corresponding geometry. For example, the formula above can be used to define the weed patch in a DEM and generate the elevations over which the weed patch occurs. Similarly, the same formula can be used to define the pixels in a Landsat Thematic Mapper image to determine the spectral reflectance in multiple image bands for the weed patch with a possibility value associated with each pixel position indicating the strength of that pixel as a part of the weed feature.

An example defined on the basis of thematic attribute data uses a similar spatial extent formula; however, the membership function determining the spatial extent uses the thematic attribute as the defining parameter of the fuzzy set function. An example with elevation values defining a hill is presented in Usery (1996a). Following is an example defining the extent of moisture based on measured rainfall.

◇ **Example 2:** Let $V = [0, 1000]^2$ be a map space and let $A^* \subset V$ be a fuzzy feature representing rainfall amount with a peak value of 5 cm. Define the feature with the following membership function:

$$f_A*(v_r) = \begin{cases} 1 & : f(v_r) \geq 3.0 \\ \frac{f(v_r)-2.0}{1.0} & : 2.0 < f(v_r) < 3.0 \\ 0 & : f(v_r) \leq 2.0 \end{cases} \quad (2)$$

where $f(v_r)$ = rainfall in centimeters of the pixel in a raster image of rainfall.

Both of these examples have been developed on the basis of a raster geometric image in which the pixel values can be assigned real number values in the closed interval [0,1].

4. CONCLUSIONS

The development of representational methods for fuzzy features offers the potential to improve the accuracy of analytical results from GIS analysis. While the examples presented are simplistic cases relying on spatial and thematic attributes of geographic datasets in raster formats, more complex fuzzy set functions can be formed in a similar manner by changing the fuzzy interpolation functions from linear to polynomial and other non-linear functions. The use of spatial position as the defining parameter of fuzziness allows the same function to be used in a variety of image representations of the same feature. Use of thematic values as the defining parameter necessarily limits the function to a single image type. The use of a comprehensive framework which supports multiple representations and multiple data sources appears to hold promise for applications requiring analysis of many datasets.

While the current trend toward feature-based implementations (sometimes using object-oriented computer science approaches) of GIS holds promise for better user interfaces and the shielding of users from the complexities of geometric manipulation of geographic features, the strength of these approaches appear to be in allowing multiple representations within a GIS for a single geographical entity. Fuzzy sets offer one approach to representing geographic features with indeterminate boundaries with spatial, thematic, and in the future, temporal attributes and relationships defining the extent of the fuzzy set function.

Future research should include development of specific applications to test the fuzzy set representations in the feature-based approach. Among such applications, representation of agricultural phenomena with fuzzy spatial extents and fuzzy functions defined by thematic parameters such as phosphorus, potassium, nitrogen, soil ph, and other attributes show promise.

5. REFERENCES

Altman, D., 1994. "Fuzzy Set Theoretic Approaches for Handling Imprecision in Spatial Analysis," *International Journal of Geographical Information Systems,* Vol. 8, pp. 271-289.

Berry, B.J.L., 1964. Approaches to Spatial Analysis: A Regional Synthesis. *Annals of the Association of American Geographers*, Vol. 54, pp. 2-11.

Burrough, P.A., 1989. "Fuzzy Mathematical Methods for Soil Survey and Land Evaluation," *Journal of Soil Science*, Volume 40, pp. 477-492.

ERDAS, 1995. *ERDAS Imagine 8.2 User Manuals*, ERDAS, Inc., Atlanta, Georgia.

Katinsky, M., 1994. *Fuzzy Set Modelling in Geographic Information Systems*, Unpublished Master's Thesis, University of Wisconsin-Madison, 46 p.

Sinton, D., 1978. "The Inherent Structure of Information as a Constraint to Analysis: Mapped Thematic Data as a Case Study," In G. Dutton, (ed), *Harvard Papers on Geographic Information Systems, Volume 6*, Addison Wesley, Reading, MA.

Usery, E.L., 1993. "Category Theory and the Structure of Features in GIS," *Cartography and GIS*, Vol. 20, No. 1, pp. 5-12.

Usery, E.L., 1994a. "Display of Geographic Features from Multiple Image and Map Databases," *International Archives of Photogrammetry, Volume 30-4, Commission IV Symposium on Mapping and Geographic Information Systems,* Athens, Georgia, pp. 1-9.

Usery, E.L., 1994b. "Implementation Constructs for Raster Features," *Proceedings, ASPRS/ACSM Annual Convention*, Reno, Nevada, pp. 661-670.

Usery, E.L., 1996a. "A Conceptual Framework and Fuzzy Set Implementation for Geographic Features," In Burrough, P.A. and A.U. Frank, (eds.), 1996. *Geographic Objects with Indeterminate Boundaries*, Taylor and Francis, Ltd, London, In Press.

Usery, E.L., 1996b. "A Feature-Based Geographic Information System Model," *Photogrammetric Engineering and Remote Sensing*, In Press.

Wang, F., 1990. "Improving Remote Sensing Image Analysis through Fuzzy Information Representation," *Photogrammetric Engineering and Remote Sensing*, Vol. 56, No. 8, pp. 1163-1168.