AIRCRAFT POSITION AND ATTITUDE DETERMINATION BY GPS AND INS

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Direct Georeferencing, exterior orientation, GPS/INS integration, airborne position and attitude accuracies.

ABSTRACT

The parameters of exterior orientation for airborne imaging sensors, i.e. position and attitude of the sensor at each exposure, can be obtained by integrating GPS and INS. By matching the accuracy of the external orientation parameters to the accuracy required on the ground, photo control is not needed any more to estimate parameters of exterior orientation. This greatly reduces the requirements for ground control which can now be configured to optimize camera calibration and transformation to a local coordinate system, where needed. The full potential of this approach for non-conventional airborne sensors, such as imaging scanners and digital frame cameras, is only now being explored, although some of the underlying ideas have been partially applied for several years in GPS-aided block triangulation. The paper briefly reviews the principle of airborne georeferencing and its implementation by using an inertial navigation system (INS) integrated with differential GPS. The position and attitude performance of INS and GPS are discussed and INS/GPS integration strategies are analysed. Results show that the present accuracy of INS/GPS integration is sufficient for many of the current and emerging mapping and resource applications.

1. INTRODUCTION

During the last three decades, airborne photogrammetry, when applied to mapping, has been performed in a single mode of operation: aerotriangulation with block adjustment of either bundles or stereo models. This mode was well justified by operational constraints. Because ground control was usually scarce, the geometrical strength of the bundle had to be used to the fullest. By creating homogeneity within the photogrammetric block, smooth residual errors could be expected which could be well appoximated by simple interpolation procedures between a few date points.

With the advent of reliable methods of kinematic GPS positioning, the interpolation component of the process was considerably improved. By being able to position the projective center of each exposure with high absolute accuracy, the translational components of the block configuration were strengthened, and position biases, scale factors, and drifts in latitude and longitude could be estimated with high accuracy, independent of existing ground control. In addition, camera calibration became much easier provided a few ground control points were available either in the area or close to it. The orientation component was indirectly strengthened because coordinate constraints between exposure stations also constrained the relative orientation between adjacent bundles. The fact that this introduced a high correlation between translation and orientation components seemed not to be critical in practice because the geometrical strength of the individual bundle was comparable in accuracy to the derived orientation changes. Thus, GPS-aided aerotriangulation in block adjustment mode has emerged as the optimal procedure for those applications requiring high-precision optical cameras and area coverage, see for instance Ackermann (1994), Hothem et al. (1994), Lukas (1994), for details.

The paper, therefore, addresses applications where these conditions are not satisfied, i.e. either situations where sensors other than high-precision optical cameras areflown or strip or model coverage rather than block coverage is required. In these applications, external orientation becomes as important a parameter as external position has become in GPS-aided photogrammetry. The reason for this is that either the sensors used do not have the same geometrical strength as high-precision aerial cameras or that the photo coverage is such that the geometrical strength of the bundle for relative orientation is not sufficient. In the first case, digital frame cameras and line scanners come to mind. In the second case, highway and powerline design, coastal mapping, and pipeline maintenance could be mentioned. Since each individual image is now georeferenced, i.e. the parameters of interior and exterior orientation in each image are known, ground control is not necessary to derive these parameters. This provides considerable flexibility for post-mission modelling which now can be done with georeferenced images as the basic unit. The accuracy of the method obviously depends on the accuracy with which the georeferencing parameters can be determined. This question will be analyzed in the following.

2. GEOREFERENCING OF AIRBORNE SENSORS

Georeferencing of airborne sensors is treated in some detail in Schwarz et al. (1993), A brief review of the major concepts will be given here to provide a framework for the following discussion. Georeferencing describes a series of transformations necessary to obtain coordinates in a chosen mapping system (m) from the output of a remote sensing device in the body frame (b) of the aircraft. The important parameters for this transformation are depicted in Fig. 1.

This paper is an updated and abbreviated version of Schwarz (1995) published in 'Fritsch/Hobbie (eds.) Photogrammetric Week '95, Wichmann Verlag'

The mathematical model corresponding to this figure is

$$\Delta\, \mathbf{r}^m(t) = \mathbf{r}^m(t) + s\mathbf{R}^m{}_b(t)\mathbf{p}^b, \qquad (1)$$

where

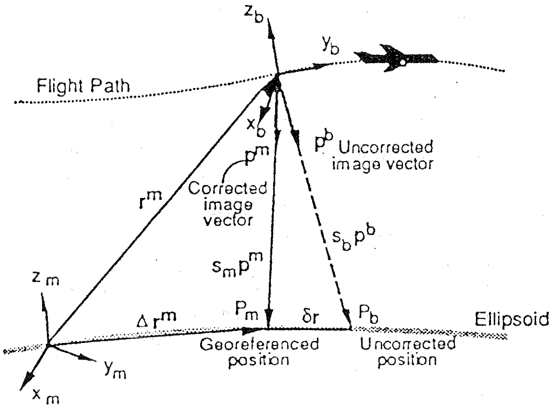

Figure 1: Georeferencing of Airborne Sensing Data

$\Delta\,\mathbf{r}^m$	is the position vector of an image object in the chosen mapping frame;
$\mathbf{r}^m$	is the coordinate vector from the origin of the mapping frame to the centre of the position sensor on the airplane, given in the m-frame
$\mathbf{R}^m{}_b$	is the three-dimensional transformation matrix which rotates the aircraft body frame (b-frame) into the mapping frame;
s	is a scale factor which depends on the distance between camera and object;
$\mathbf{p}^b$	is the vector of image coordinates given in the b-frame.

Equation (1) is only a first approximation of the actual situation. Since the three sensors for positioning, attitude determination, and imaging are at different locations in the aircraft, another set of transformations is necessary to relate all sensors to the origin and the axes of the chosen body frame in the aircraft. The parameters of these transformations can be considered fixed for one mission and are obtained through a pre-flight calibration, for details see Schwarz et al. (1994).

The resulting modelling equations are

$$\Delta\, \mathbf{r}^m(t) = \mathbf{r}^m(t) + \mathbf{R}^m{}_b(t)\,\{\, s\, d\mathbf{R}^b{}_c\, (\mathbf{p}^c + d\mathbf{r}^c) + d\mathbf{r}^b \,\} \qquad (2)$$

The additional vectors and matrices in equation (2) are as follows:

$d\mathbf{R}^b{}_c$	is the transformation matrix which rotates the camera frame (c-frame) into the body frame;
$\mathbf{p}^c$	is the imaging vector in the c-frame;
dr^b	is the translation vector between the centre of the attitude sensor and the perspective centre of the camera; and
$d\mathbf{r}^c$	is the translation vector between the centre of the positioning sensor and the perspective centre of the camera.

The camera frame is defined as having its origin in the perspective centre of the camera, its z-axis along the vector between the perspective centre and the principal point of the photograph, and its (x,y)-axes in the plane of the photograph with origin at the principal point. The corresponding image vector is therefore of the form

$$p^c = \begin{pmatrix} x - x_p \\ y - y_p \\ -f \end{pmatrix} \qquad (3)$$

In case of pushbroom scanners and CCD frame imagers, the second vector component is replaced by

$$y^c = (y - y_P) / ky$$

where ky accounts for the non-squareness of the CCD pixels.

It should be noted that the origins of the position and attitude sensors are not identical. Furthermore, the vectors $\mathbf{r}^m$ and $\Delta\,\mathbf{r}^m$, as well as the rotation matrix $\mathbf{R}^m{}_b$ are time dependent quantities while the vectors p^c and dr^b, as well as the matrix $dR^b{}_c$ are not.. This implies that the aircraft is considered as a rigid body whose rotational and translational dynamics is adequately described by changes in $\Delta\mathbf{r}^m$ and $\mathbf{R}^m{}_b$. This means that the translational and rotational dynamics at the three sensor locations is uniform, in other words, differential rotations and translations between the three locations have not been modelled. It also means that the origin and orientation of the three sensor systems can be considered fixed for the duration of the flight. These are valid assumptions in most cases but may not always be true.

The quantities $\Delta\mathbf{r}^m$, $\mathbf{R}^m{}_b$ and p^c in equation (2) are determined by measurement, the first two in real time, the third in post mission. The quantities $d\mathbf{R}^b{}_c$ and $d\mathbf{r}^b$ are determined by calibration, either before or during the mission. To determine $d\mathbf{R}^b{}_c$ by calibration, a minimum of three well determined ground control points are required, while $d\mathbf{r}^b$ can be obtained by direct measurement on the ground. The scale factor s changes with flying altitude of the aircraft above ground. It can, therefore, either be approximated by assuming a constant flying altitude, calibrated by introducing a digital terrain model, or determined by measurement, using either stereo techniques or an auxiliary device such as a laser scanner. For precise georeferencing, the latter technique is the most interesting to be investigated because it provides all necessary measurements form the same airborne platform. Thus, datum problems can be avoided. For a more detailed discussion of calibration issues, see Schwarz et al. (1993).

3. DETERMINATION OF GEOREFERENCING PARAMETERS BY GPS AND INS

For the georeferencing process, the parameters $\Delta\,\mathbf{r}^m$ and $\mathbf{R}^m{}_b$ are obviously of prime importance. They are usually determined by combining the output of an inertial measuring unit (IMU) with that of one or several receivers of the Global Positioning System (GPS). Although other observables can be used, they are less suited for the task at hand and will not be treated here. To obtain position and orientation as functions of time, they are modelled as functions of the time-dependent IMU and GPS observables. The resulting model is a system of first-order differential equations in which $\Delta \mathbf{r}^m$ and $\mathbf{R}^m{}_b$ are variables. In engineering applications, such a system is often called a state vector model. It is the model underlying Kalman filtering and is therefore well-suited for both system integration and optimal real-time estimation. The advantage of using a state vector model lies in the possibility of imposing smoothness conditions on the solution by the definition of covariances for the state vector elements and of the spectral densities.

The models for determining georeferencing parameters from either IMU or GPS observables will not be presented in detail. A brief discussion is given in Schwarz et al. (1993), while Wei and Schwarz (1990) should be consulted for details on the IMU model and Schwarz et al. (1989) for details on the GPS model. What follows is a brief descriptive account of the salient features of each system and the reasons for integrating them.

A strapdown IMU outputs three components of the specific force vector and three components of the angular velocity vector in the body frame system. To use these observables to derive position, velocity, and attitude in an earth-fixed coordinate system, the attitude between the measurement frame and the earth-fixed frame must be determined as a function of time. This is accomplished by determining the initial attitude in the so-called alignment procedure, by correcting the measured angular velocities for earth rotation and by then integrating them in the earth-fixed frame. Since attitude is now known as a function of time, the specific force measurements can be transformed into the earth-fixed frame. By subtracting gravity from the transformed measurements, vehicle acceleration is obtained. By integrating acceleration once with respect to time, velocity is obtained, by integrating twice, position is obtained. The earth-fixed frame is in principle arbitrary but is usually chosen either as a local geodetic frame or a geocentric Cartesian frame. Because of the integrations involved in the process, initial errors, caused for instance by sensor biases, grow quickly with time. Thus, a free inertial system will show systematic errors in position, velocity, and attitude which oscillate with a period of 84 minutes, the so-called Schuler period. The presence of these errors is the major reason why integration with GPS is advantageous and why it results in a far superior determination of the georeferencing parameters.

GPS observables are either of the pseudorange type or of the carrier phase type. Models to transform the resulting range equations into positions and velocities are well-known, see, for instance, Wells et al. (1986). In the process, orbital models as well as atmospheric models are needed and the Earth rotation rate is again assumed to be known. By locating one receiver at a known master station and referencing the moving receiver to it, major errors in the GPS measurement can be eliminated. These differential procedures can be applied to pseudorange measurements as well as to carrier phase measurements and will be assumed as the modus operandi in the following. In the typical case of one ground receiver and one moving receiver, only the translational vector $\mathbf{r}^m(t)$ can be determined because one antenna does not fix a vector within the rigid body and thus the determination of rotational parameters is not possible. Three body-mounted antennas, preferably orthogonal to one another, are the minimum requirement for the determination of $\mathbf{R}^m{}_b(t)$. The attitude matrix in the body frame is obtained by using double differentiated carrier phase measurements, see El-Mowafy (1994) for details. The distance between antennas must be considered as constant and accurately known and a proper initialization of the $\mathbf{R}^m{}_b$-matrix is required while the system is stationary.

Thus, the georeferencing parameters, position and attitude, can be obtained from either an IMU or a multiple receiver GPS system. The stand-alone accuracy of GPS and INS is given in Tables 1 and 2. Table 1 shows the positioning and attitude determination capability of GPS for different observables and processing methods. In general, these results are achievable in post-mission mode. The table shows that all required positioning accuracies can be met but that operational constraints may be necessary to satisfy the requirements of high accuracy applications. For details under which circumstances these results are achievable, see Cannon and Lachapelle (1992), Lachapelle et al. (1994), Shi and Cannon (19949. In general, the relative accuracy over short time spans, say one minute, are better than the numbers quoted.

Model	Separation	Accuracy	Mode
Pseudorange point positioning (single rev) using precise orbits & clock corrections		100 m horizontal 150 m in height	real time
		1 - 2 m horizontal 2 - 4 m in height	post mission
Smoothed pseudorange	10 km	0.5 - 3 m horizontal 0.8 - 4 m in height	post mission (or real time)
Pseudorange differential positioning	500 km	3 - 7 m horizontal 4 - 8 m in height	
Carrier phase differential positioning	10 km	3 - 20 cm horizontal 5 - 30 cm in height	post mission
	50 km	15 - 30 cm horizon. 20 - 40 cm in height	
	200 km (with precise orbits)	15 - 30 cm horizon. 20 - 40 cm in height	
Attitude determination	1 m	18 - 30 arcminutes	post mission (or real time)
	5 m	4 - 6 arcminutes	
	10 m	2 - 3 arcminutes	

Table 1: GPS Positioning and Attitude Accuracies.

Table 2 shows position, velocity, and attitude performance of intertial navigation systems (INS) for two different accuracy classes. Because INS errors are a function of time, they are quoted for different time intervals. Most of the time-dependent errors follow a systematic pattern and can therefore be greatly reduced by appropriate update measurements. The residual noise level for a navigation-grade INS after GPS-updating will usually be close to the value given for the one second interval. For a low performance system the noise level will be above the one second value.

Error in	System Accuracy Class			
	nav. grade		low accuracy	
Attitude	pitch & roll;	azimuth	pitch & roll;	azimuth
1 h	10" - 30"	60" - 180"	0.5 - 1.0	1° - 3°
1 min	5" - 10"	15" - 20"	0.1 - 0.3	0.2° - 0.5°
1 s	3" - 5"	3" - 20"	0.01 - 0.02	.02° - 0.05°
Velocity				
1 h	0.5 - 1.0 m/s		200 - 300 ,/s	
1 min	0.03 - 0.10 m/s		1 - 2 m/s	
1 s	0.001 - 0.003 m/s		0.002 - 0.005 m/s	
Position				
1 h	500 - 1000 m		200 - 300 km	
1 min	0.3 - 1.0 m		30 - 50 m	
1 s	0.02 - 0.05 m		0.3 - 0.5 m	

Tab. 2: Current INS Performance.

4. ACCURACY ACHIEVABLE BY AN INTEGRATED INS/GPS

As is obvious from Tables 1 and 2, the stand-alone accuracy of each system will not give the highest possible accuracy. INS will have superior orientation accuracy, GPS superior position accuracy. Thus, an integration of the two systems will result in an optimal solution which will also provide much needed redundancy.

GPS positioning using differential carrier phase is superior in accuracy as long as no cycle slips occur. GPS relative positions are, therefore, ideally suited as INS updates and resolve the problem of systematic error growth in the IMU trajectory. On the other hand, the IMU-derived attitude is usually superior to that obtained from a GPS multi-antenna system. In addition, IMU-derived position differences are very accurate in the short term and can thus be used to detect and eliminate cycle slips and to bridge loss of lock periods. Because of the high data rate, they provide a much smoother interpolation than GPS. Integration of the two data streams via a Kalman filter thus provides results which are superior in accuracy, reliability, and homogeneity.

The following figures will illustrate the orientation accuracy currently achievable with an integrated INS/GPS. Position accuracy is not discussed in the same detail because it is largely dependent on GPS accuracy for which Table 1 and the references given there can be consulted.

Fig. 2a shows the attitude output of a navigation-grade INS in static mode over a period of about 25 minutes. The systematic error is smooth and reaches a minimum after about 21 minutes. It is clearly part of the Schuler-type oscillation. The noise about this trend is very small and shows white noise characteristics. After eliminating the trend, the noise pattern in Fig. 2b results. It shows the system attitude noise which is at the level of 3 arcseconds (RMS) and represents the attitude accuracy achievable under ideal conditions. Fig. 2c shows the noise of the same system mounted in an aircraft with the engines switched on. As before, the trend has been eliminated. In this case, the low noise level of the lab test cannot be maintained. The noise is now between 15 and

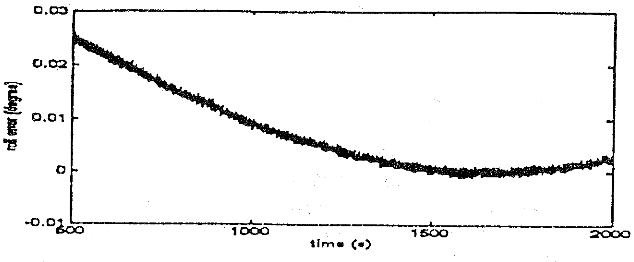

Figure 2a: Total roll error, static case, lab

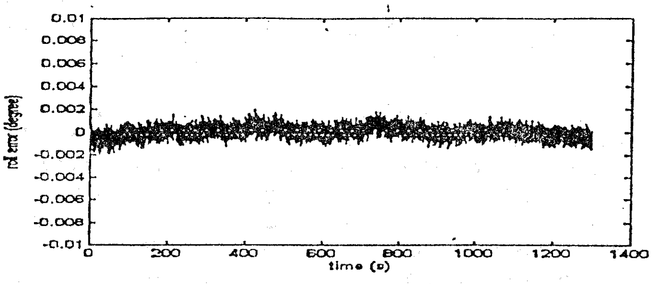

Figure 2b: Roll noise, static case, lab (σ =3")

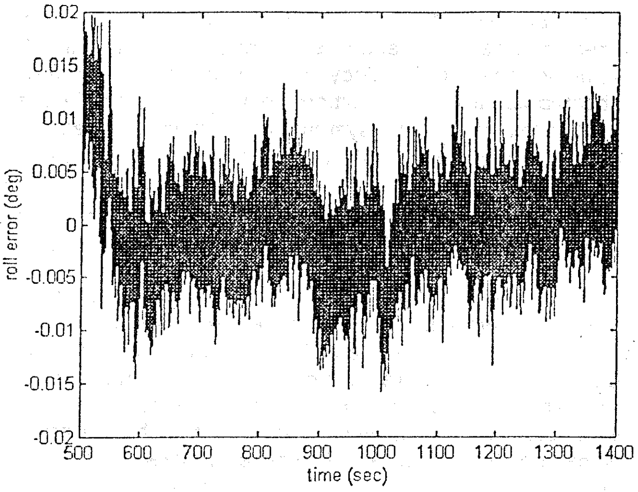

Figure 2c: Roll error, static case, aircraft, engine switched on (σ = 18")

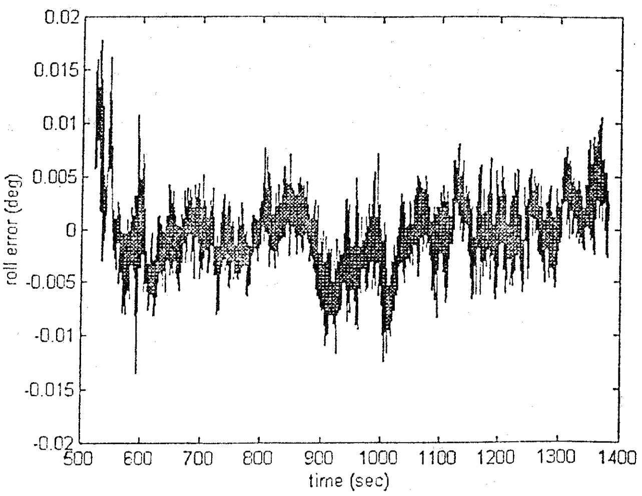

Figure 2d: Roll error, static case, aircraft, engine on, vibration filter (11")

20 arcseconds and is due to aircraft vibration and wind gusts. If the vibrational effects above the aircraft dynamics of 10 Hz are eliminated, the noise drops to 11 arcseconds in pitch and roll, see Fig. 2d. Adding error components due to the interaction of synchronization errors and aircraft dynamics, the pitch and roll noise to be expected under dynamic conditions will be about 15 - 20 arcseconds.

To achieve this noise level, systematic errors due to Schuler-type oscillations in attitude have to be completely eliminated by 1HZ GPS position and velocity updates. Figure 3 shows again a lab experiment where such updates have been used for the roll component.

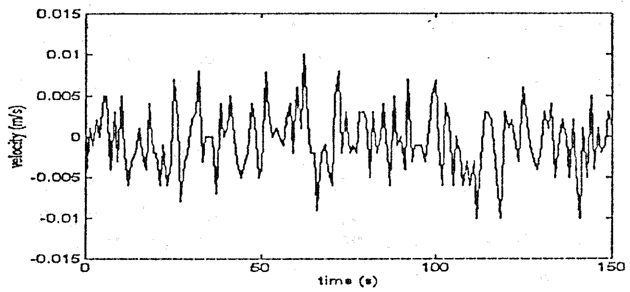

Figure 3: Roll error, static case, lab, GPS velocity updates (σ = 12")

The residual error of 12" seems to indicate that Schuler-type oscillations can indeed be eliminated. However, a closer analysis reveals that this is not the case for all three attitude components. To show this, consider the relationship between INS acceleration errors δ v, misalignment errors ε, accelerometer biases b, and specific force components f. They are of the form

$$\begin{aligned} \delta\dot{v}_e &= f_u\varepsilon_n - f_n\varepsilon_u + b_e \\ \delta\dot{v}_n &= -f_u\varepsilon_e + f_e\varepsilon_u + b_n \\ \delta\dot{v}_u &= f_n\varepsilon_e - f_e\varepsilon_u + b_u \end{aligned} \qquad (4)$$

where the subscripts n, e, u denote north, east, and upward. The upward component of specific force contains the effect of gravity and is thus always considerably larger than the north and east components. In a photogrammetric flight, constant velocity along a straight line is the preferred operational environment. Thus, f_n and f_e are rarely larger than 0.1 m/s^2, while f_u is always about 10 m/s^2.

Rewriting the second equation with respect to ε_u and ε_n results in

$$\begin{aligned} \varepsilon_u &= \frac{f_u\varepsilon_n - \delta\dot{v}_e + b_e}{f_n} \\ \varepsilon_n &= \frac{f_n\varepsilon_u + \delta\dot{v}_e + b_e}{f_u} \end{aligned} \qquad (5)$$

Assuming a flight to the north, these equations show the heading and roll errors as functions of specific force, misalignment, acceleration error, and accelerometer bias. Assuming the same magnitude of errors in both cases and introducing typical error sizes, the following conclusions can be drawn. Due to the size of the specific force component in the denominator, the determination of roll (and pitch) will be more accurate than the determination of heading. Actually, the better the constant velocity condition is maintained, the better pitch and roll will be determined and the poorer the estimation of heading will be. Only when the aircraft manoeuvres in such a way that major horizontal accelerations are introduced, will the heading accuracy be improved. It can therefore be concluded that GPS updates are sufficient to eliminate pitch and roll oscillations to the level of INS attitude noise, but that similar results cannot be achieved in heading without a regular pattern of large horizontal aircraft accelerations. For a more detailed discussion of these interrelationships, see Schwarz and Wei (1994) and Zhang (1995).

Besides Schuler-type oscillations which are caused by the interaction of small initial errors with the natural frequency of the INS, additional errors can be expected which are due to the interaction of small systematic sensor errors and aircraft dynamics. These errors are difficult to isolate in a controlled experiment. Their order of magnitude can be assessed, however, by conducting airborne tests where the attitude parameters determined by the onboard GPS/INS are compared to attitude parameters independently determined from accurate ground control by photogrammetric methods. Targeted control points in the photographs are used to estimate camera attitude at flying height by a bundle adjustment. Two such tests will be briefly discussed.

In both cases the system consists of an LTN 90/100, a navigation-grade INS, which was integrated with two Ashtech Z12 receivers, one on the airplane and one at the master station. A Zeiss LMK aerial camera was used to obtain a block of photographs of a test field with dense GPS control. The attitude estimated from this control was compared to the attitude obtained from the integrated INS/GPS. For details of this test, see Skaloud (1995). The attitude accuracy determined from the ground control is typically at the level of 4-5 arcseconds in each component, about twice as accurate as expected from the INS under the best circumstances. The comparison with the INS-derived attitude indicates that the differences are between 15 and 35 arcseconds, which was expected in this case because no vibration filtering has been applied. Thus, the differences represent largely the INS noise under operational conditions plus some additional errors due to residual Schuler oscillations and aircraft dynamics. However, due to constraints in this specific test, the time period shown is only two minutes. A large effect of errors due to residual Schuler-type oscillations and aircraft dynamics could therefore not be expected. Thus for time intervals of a few minutes an attitude noise level of 20 - 30 arcseconds can be maintained.

To investigate whether this is also true for time periods typically encountered in production flights, a small test field with accurate GPS ground control was overflown nine times from different directions, accumulating a total flight time of about one hour. 77 photographs out of a total of 168 were selected for the block adjustment. The resulting parameters of exterior orientation determined by photogrammetric means had standard deviations of about 3 cm in each of the coordinates and 7 arcseconds in each of the orientation parameters. The discrepancies between these parameters and those determined by GPS/INS were much larger. The root-mean-square (rms) discrepancies were 15 cm in horizontal position, 20 cm in vertical position, 1 arc minute in azimuth, and two arc minutes in roll and pitch. When these parameters directly determined by GPS/INS were used for image georeferencing without the use of ground control, the rms discrepancies on control points were 0.3 m horizontally and 0.5 m vertically. This is sufficient for most mapping and resource applications. For more details, see Skaloud et al (1996).

These results are quite unexpected and need further analysis. The error pattern for roll and azimuth seems to indicate a major influence of errors due to aircraft dynamics, while the pitch error shows a linear drift with time. The fact that, in contradiction to theory, the azimuth performance is better than that in roll and pitch, might be due to the frequent 360° turns, necessary to return to the test area. However, these results should be considered preliminary until more detailed investigations have been done. They show that it is very difficult to reach an attitude noise level of 20 - 30 arcseconds over an extended period of time using current hardware.

5. CONCLUSIONS

Georeferencing of airborne imaging sensors has the potential of considerably extending current photogrammetric applications and greatly simplifying the use of digital imaging sensors. It also adds flexibility to the use of current high-precision aerial cameras and considerably reduces the need for accurate ground control.

The integration of inertial and GPS satellite techniques currently offers the best potential for implementing georeferencing systems at different levels of accuracy and for combining them with existing and future airborne imaging sensors. Major advantages are the high data rates of the inertial measuring unit, compactness which allows direct mounting on the sensor head, and uniform high accuracy due to continuous GPS updating.

Theoretical studies and test results, analyzed in this paper, indicate that current systems are capable of georeferencing airborne imaging sensors for mapping and resource applications with an accuracy of about 0.5 m (rms). Higher accuracy can be achieved if the long-term stability of the attitude sensors can be improved to a noise level of about 20".

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