

## STAGGERED LINE ARRAYS IN PUSHBROOM CAMERAS: THEORY AND APPLICATION

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### ABSTRACT

Using pushbroom sensors onboard aircrafts or satellites one needs - especially for photogrammetric applications - broad image swaths and high geometrical resolution together. To accomplish both demands staggered line arrays can be used. A staggered line array consists of two identical CCD lines with one shifted half a pixel with respect to the other. Today such sensors are available in the VIS/NIR spectral region with 2 x 12000 pixels (staggered CCD line arrays) and in the mid and thermal infrared with 2 x 512 pixels (staggered MCT line arrays). These sensors will be applied in spaceborne remote sensing missions (small satellite BIRD and International Space Station project FOCUS, both for hot spot detection and evaluation), in experimental airborne systems (with the Infrared Airborne Camera HSRS of the Institute of Space Sensor Technology and Planetary Exploration) and, most important, in the first commercial Airborne Digital Camera (ADC, new name: ADS40) of LH Systems.

The paper presents the theory, some results of the simulation, examples with real imaging systems in the laboratory and on aircrafts.

### 1 INTRODUCTION

#### Spatial Resolution, Point Spread Function, and Sampling Theorem

Spatial resolution depends on the PSF (point spread function) of the whole system. Above all the total PSF up to the sampling device (e.g. the CCD-line) compared to the sampling rate or the Nyquist frequency determines the resolution of the system. This can be illustrated with the example of the star sensor: In case of ideal imaging (with delta peak PSF) the location accuracy is not better than a pixel size. In the case of a wide spreading PSF the star image is smeared over much more than one pixel and can be detected also in the neighbouring pixel. Only under this condition the star position can be detected with subpixel accuracy using an interpolation algorithm.

The investigation of the spatial resolution of an imaging system therefore must consider the PSF (at least from the optics) in relation to the sampling process. Moreover in this consideration the SNR (signal to noise ratio) of the whole system must be included, which can be shown on the following example: A smaller pixel size gives a better spatial resolution but a poorer signal to noise ratio at the same time and vice versa.

In the following the spatial resolution of staggered arrays shall be investigated. To study the resolution the image of one or two delta signal peaks can be investigated. The resolution of the single 12k CCD-line (1), the staggered 2·12k line (2) and the linear 24k line (3) will be compared. For the same focal length the 24k line must have half of the pixel size as the 12k or 2·12k line. If the pixel size for the 24k line is the same as the 12k line, the focal length must be the twice of the 12k line camera. In both cases the radiometric characteristic for the 24k line is the same.

For a single point we define the error of the location of the image as an accuracy measure. In the case of two point accuracy the resolution of these two points is defined by the minimal necessary contrast (according to the Rayleigh-criterion, Jahn, Reulke, 1998)

$$C_{\min} = \frac{\text{MAX}\{I\} - \text{MIN}\{I\}}{\text{MAX}\{I\}} = 0.25. \quad (1)$$

The resolution concept uses the notation “image resolution” or “image quality”. This can lead sometimes to misunderstandings for describing the resolution of small structures defined by a distance (e.g. the distance between two peaks). In this case an increasing resolution is related to a decreasing distance. Therefore we introduce as a measure for the resolution the a Minimum Resolvable Distance (MRD). In the context of a Rayleigh-criterion (equation (1)) based resolution concept, the MRD is the minimum distance of two radiating points to be resolved.

The investigation of the attainable resolution for staggered and non-staggered arrays is the main topic of this paper.

## 1.1 Staggered Arrays

To validate the usefulness of staggered arrays theoretical investigations of the ultimate spatial resolution taking into account the total point spread function (PSF) and Shannon's sampling theorem have been carried through. Staggering means that because of using two shifted sensor lines twice as much sampling points are used as without staggering, i. e. the sampling distance  $\Delta$  of the non-staggered array is divided by two. This often guarantees that the sampling condition can be fulfilled which means that no spatial spectral components above the spatial frequency limit  $2/\Delta$  are present and which often is not fulfilled in the non-staggered case (here no spatial spectral components above  $1/\Delta$  are allowed). Therefore, better spatial resolution is obtained even in the case where the pixel size equals the sampling distance  $\Delta$ . Theoretically, the resolution is as good as for a non-staggered line with half pixel size  $\Delta/2$  if the pixels of the staggered line array have size  $\Delta/2$  too (with pixel distance  $\Delta$ ). Of course, in practice the slightly different viewing angle of both lines of a staggered array can lead to a deterioration because of aircraft motion and attitude fluctuations and a non-flat terrain. Fulfilling the sampling condition further means that no aliasing occurs. This is essential for the image quality in quasi - periodical (textured) image areas. Because the sampling theorem can be applied errorless interpolation between pixels (sub - pixel resolution) is possible. That way precise positions of points, lines, edges and corners can be determined. Furthermore, image restoration methods for enhancing the image quality can be applied more efficiently.

Image simulation with imaging sensor simulation software for staggered and non - staggered sensors developed at the Institute of Space Sensor Technology and Planetary Exploration and experiments in the laboratory confirmed the results of the theory. Airborne experiments with the infrared HSRS sensor system also showed considerable enhancements of the image quality using the staggered mode of operation. It is planned to carry through corresponding investigations also with an prototype of the Airborne Digital Camera using the first time the staggered  $2 \times 12000$  pixels line array in an airborne experiment.

## 2 RESOLUTION AND PSF

### 2.1 Ideal case - neglectable PSF

In the ideal case there is a neglectable influence of the optics compared to the sampling. That means that the width of the PSF is very small (or neglectable) compared with the pixel size. In this case the resolution can be derived by simple geometrical consideration (see Figure 1).

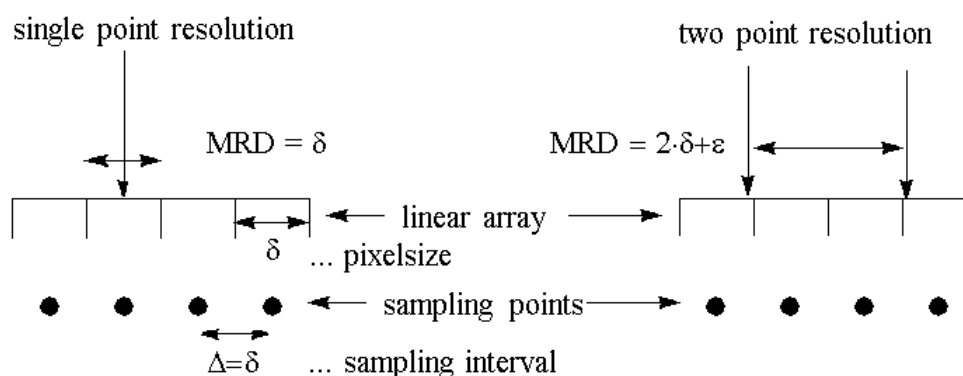


Figure 1. Resolution of linear arrays (neglectable PSF)

In the case of a linear array with neglectable PSF the single point resolution is not better than  $\delta$ , where  $\delta$  is the pixel size. For the two point resolution the criterion (1) can only be fulfilled if the distance between the two points is greater than  $2\delta + \epsilon$  ( $\epsilon$  is a very small value). The same consideration can be carried out for staggered arrays (see Figure 2).

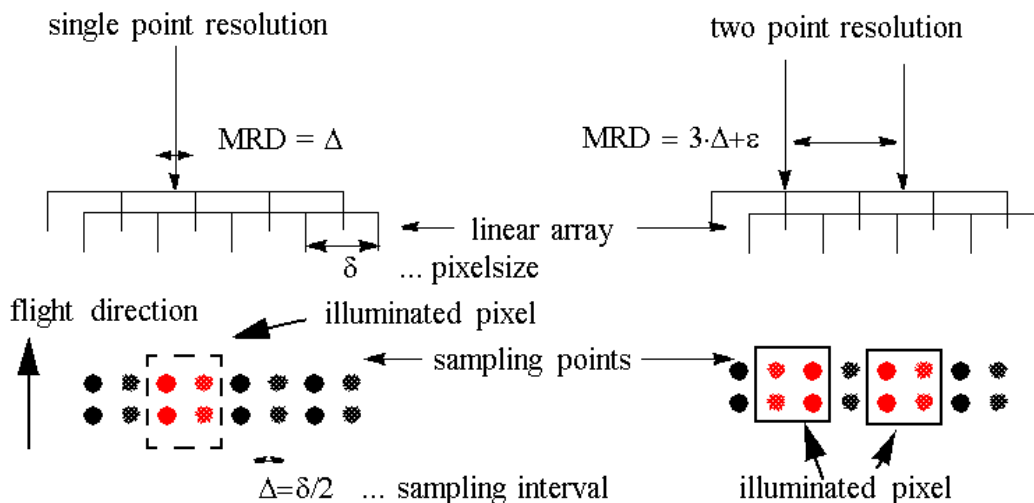


Figure 2. Resolution of staggered arrays (neglectable PSF)

Due to the point like image formation, in the case of staggered arrays each point influences four pixels (two in line direction and two in flight direction). The point can be shifted over a distance of  $\delta/2$  giving the same signal in the four pixels.

For the single point resolution the accuracy of the point determination is  $\Delta = \delta/2$ , and for the two point resolution the MRD is  $3\Delta = 1.5 \cdot \delta$ . As a result, for the resolution of single point imaging we have for CCD-line configurations (1), (2) and (3)

$$MRD(12k) > MRD(2 \cdot 12k_{\text{staggered}}) = MRD(24k) \quad \text{or} \quad \delta > \frac{\delta}{2} = \frac{\delta}{2} \tag{2}$$

and for the two point resolution

$$MRD(12k) > MRD(2 \cdot 12k_{\text{staggered}}) > MRD(24k) \quad \text{or} \quad 2 \cdot \delta > 1.5 \cdot \delta > \delta . \tag{3}$$

For single point consideration the resolution of staggered 12k arrays is equal to 24k arrays. For the double points the resolution improvement due to the staggered arrays is between the both extreme cases of 12k and 24k linear lines. This consideration shows, that in the case of a vanishing PSF the application of a staggered array can improve the resolution.

## 2.2 Real case - real PSF

In this chapter the influence of the PSF on the resolution shall be investigated. The ideal PSF is given by the diffraction limit of the optics. Due to the lens aberration the PSF of a real optics in most cases is worse. Additional effects influencing the total system PSF are the geometry and sensitivity distribution of the detector elements, the motion (in flight direction), the atmosphere and, for higher wavelengths (greater then 700 nm), diffusion effects in the volume of the CCD-element.

This consideration can not be done with a simple geometric illustration. Therefore the sampling process with respect to a special pixel size shall be described. We are looking for the sampling of the optical signal  $I(x,y)$  behind the optics and in front of the focal plane. The sampling is done by rectangular or square detectors with pixel length  $\delta$  arranged in a regular line grid with sampling distance  $\Delta$ . For the staggered arrays is  $\Delta = \delta/2$ .

The signal values  $I_{ij} = I(x_i, y_j)$  ( $x_i = i \cdot \Delta$ ,  $y_j = j \cdot \Delta$ ,  $i, j = 0, \pm 1, \pm 2, \dots$ ) can be obtained by a convolution of  $I(x,y)$  with the geometrical pixel PSF

$$I'(x_i, y_j) = \iint H(x_i - x', y_j - y') \cdot I(x', y') dx' dy' . \tag{4}$$

For clarity the following calculation is done only in one dimension. But the same is valid for the other direction. The pixel PSF for the simplest case (constant sensitivity, no diffusion effects) is

$$H_{pix}(x) = \begin{cases} 1 & \text{if } -\frac{\delta}{2} \leq x \leq \frac{\delta}{2} \\ \delta & \\ 0 & \text{elsewhere} \end{cases} \quad (5)$$

The single or double point source  $\delta(x_0)$ ,  $\delta(x_0+a)$  generates in front of the focal plane an spatially dependent intensity distribution

$$H(x-x_0) \text{ or } H(x-x_0) + H(x-x_0-a). \quad (6)$$

$H(x)$  is the system PSF without the pixel PSF.

The signal in the sampling point  $i \cdot \Delta$  is obtained by integrating the optical signal (6) over the pixel area (applying (4))

$$I(i\Delta) = \int_{-\delta/2}^{\delta/2} [H(x+i\Delta-x_0) + H(x+i\Delta-x_0-a)] dx. \quad (7)$$

Assuming a Gaussian-PSF with a "width" (standard deviation)  $\sigma$  in (7), the integration can be performed using Gauß's probability-integral

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \cdot \int_0^x \exp\left(-\frac{t^2}{2}\right) dt. \quad (8)$$

The result is a pixel dependent intensity distribution:

$$I(i\Delta) = c \cdot \left\{ \Phi\left(i\frac{\Delta}{\sigma} + \frac{\delta}{2\sigma} - \frac{x_0}{\sigma}\right) - \Phi\left(i\frac{\Delta}{\sigma} - \frac{\delta}{2\sigma} - \frac{x_0}{\sigma}\right) \right\} \quad (9)$$

(for single spot) and for double spot:

$$I(i\Delta) = c \cdot \left\{ \Phi\left(i\frac{\Delta}{\sigma} + \frac{\delta}{2\sigma} - \frac{x_0}{\sigma}\right) - \Phi\left(i\frac{\Delta}{\sigma} - \frac{\delta}{2\sigma} - \frac{x_0}{\sigma}\right) + \Phi\left(i\frac{\Delta}{\sigma} + \frac{\delta}{2\sigma} - \frac{x_0+a}{\sigma}\right) - \Phi\left(i\frac{\Delta}{\sigma} - \frac{\delta}{2\sigma} - \frac{x_0+a}{\sigma}\right) \right\} \quad (10)$$

The measured values  $I(i\Delta)$  (10) depend (for a given pixel distance  $\Delta/\sigma$  and a pixel size  $\delta/\sigma$  (related to the PSF width  $\sigma$ )) also on the point position (phase)  $x_0/\sigma$  and, for double point resolution, on the distance  $a/\sigma$  of the light spots. That means that the accuracy definition or the contrast (1) depends on the displacement of the point source relative to the pixel.

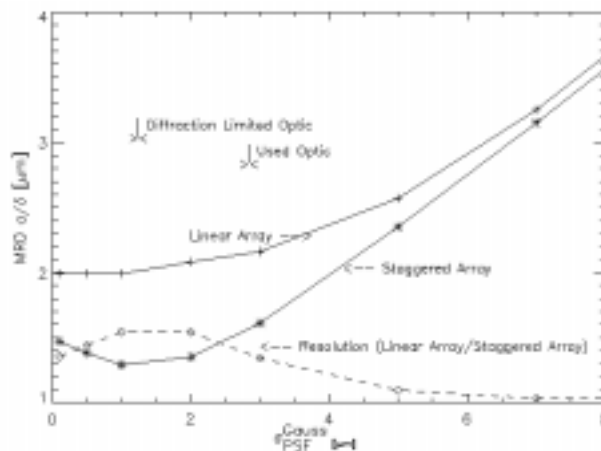


Figure 3. Double point resolution in relation to the optical PSF.

Figure 3 shows the resolution in case of double point resolution (distance between the two spots with respect to the pixel size) as a function of the PSF parameter  $\sigma$ . Here it was assumed that the used optics has a f-number = 4. Then for the diffraction limited optics the equivalent  $\sigma$  is  $1\mu\text{m}$  (see Jahn, Reulke, 1998) and for the real optics we have  $\sigma = 2.6\mu\text{m}$  (calculated from Schlienger, 1996).

In case of neglectable PSF the limit for the double point accuracy (3) is valid. With increasing PSF-width  $\sigma_{PSF}$  especially for the staggered arrays the MRD also increases, while the resolution for single arrays keeps constant. We find the minimal MRD for staggered arrays in case of diffraction limited optics. Increasing  $\sigma_{PSF}$  further the MRD becomes worse. For the used optics the resolution is in the same range as in the ideal case. The comparison of the MRD's of all different line types is shown in Figure 4.

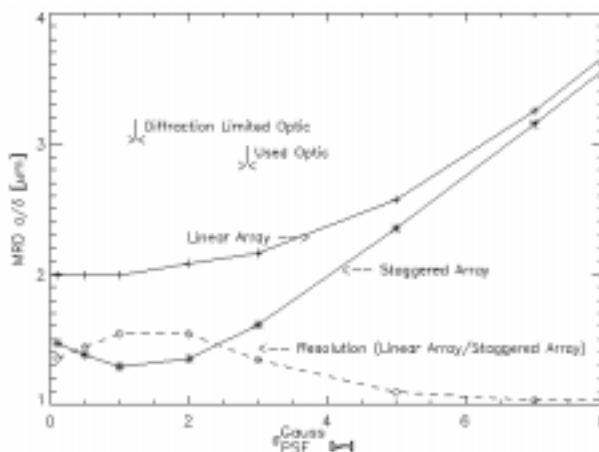


Figure 4. double point resolution for the different line types

The normalized MRD of the three different CCD-lines for the width  $\sigma_{PSF}$  of the used optics is

$$2.1 \cdot \frac{a}{\delta} > 1.6 \cdot \frac{a}{\delta} > 1.4 \cdot \frac{a}{\delta} . \tag{11}$$

The resolution of the staggered line is about 90% of the MRD of the linear 24k line!

The accuracy investigation for the single point can be performed with the signal-calculation (9). Because there is no resolution criterion like the Rayleigh approach, we are looking for the error of the image point determination. For that purpose a simple interpolation algorithm is used.

After calculating the signal, a Gaussian will be fitted to the signal. From this fit, the point position can be derived. The error in the point position is a measure of the resolution. The interpolation is only possible for a  $\sigma_{PSF}$  greater than a minimum value (e.g. the diffraction limit). For smaller values of  $\sigma_{PSF}$  there are not enough points for interpolation (e.g. in the case of vanishing PSF and single arrays only one pixel generates a signal, which does not allow the application of an interpolation algorithm). With the application of the interpolation algorithm the location error  $\epsilon$  or the MRD decrease dramatically. Only in the case of a small PSF the phase dependence generates a small error. The location error  $\epsilon$  is much smaller than a tenth of a pixel. The MRD for the three cases is

$$MRD(12k) > MRD(2 \cdot 12k_{\text{staggered}}) > MRD(24k) \quad \text{or} \quad \epsilon > \frac{\epsilon}{5} > \frac{\epsilon}{25} . \tag{12}$$

For a PSF of the real optical system no interpolation error can be recognized. That means that all information can be retrieved from the signal. This means that the sampling theorem is fulfilled.

The introduction of an interpolation approach is a essential for the resolution considerations. If the sampling theorem is not fulfilled, errors in parameter estimation occur. If the PSF-width increases, the parameters become more accurate. This approach is in contradiction to the classical Rayleigh criterion and will be explained in the following chapter more in detail.

### 3 RESOLUTION AND THE SAMPLING THEOREM

If a continuous signal has a band limited Fourier transform then the signal can be uniquely reconstructed without errors from equally spaced samples  $I(i\Delta)$  ( $-\infty < i < \infty$ ) if the spatial sampling or Nyquist frequency

$$k^{Nyq} = \frac{1}{2\Delta} \quad (13)$$

is bigger than the spatial frequency of band limitation  $k^{b.l.}$  ( $\Delta$  is the sampling distance). In this case from the sampling values  $I_{i,j}$  the whole function  $I(x,y)$  can be reconstructed by application of Shannon's sampling theorem (see e.g. Jahn, Reulke, 1995)

$$I(x, y) = \sum_{i,j} I_{i,j} \cdot \sin c \left[ \frac{\pi}{\Delta} (x - i\Delta) \right] \cdot \sin c \left[ \frac{\pi}{\Delta} (y - j\Delta) \right] \quad (14)$$

For illustration the ADC camera of LH Systems (Eckardt et al., 2000) is considered now. The optics to be used has a  $f\# = 4$ . This corresponds to a spatial frequency of band limitation  $k^{b.l.}$  of a diffraction limited optics of approximately 500 lp/mm (see Jahn, Reulke, 1998). Measurements show that the MTF at 130 lp/mm drops to 10%. The characteristic width of the optical PSF therefore is  $\sigma = 2.6\mu\text{m}$  (Schlienger, 1996). The Nyquist frequency  $k^{Nyq}$  for the linear array is 77 lp/mm and for the staggered array 154 lp/mm (with respect to the pixel size  $6.5\mu\text{m}$ ). In the case of linear array we have  $\text{MTF}(77\text{lp/mm})=45\%$  and  $\text{MTF}(154\text{lp/mm})=5\%$ .

The result of this consideration is that (in case of ADC) for staggered arrays the sampling theorem is valid and all geometrical structures which pass through the optics can be reconstructed. The MRD of staggered arrays in this case is the same as for linear 24k arrays.

## 4 EXPERIMENTS

### 4.1 Visual enhancement of ADC image quality using staggered CCD arrays

To show the visual effects of using staggered and non-staggered arrays and to study the influence of various MTF's a simple image simulation program for ADC image simulation written in IDL was developed. First, a test image showing simple objects and patterns was generated (figure 5). Then the test image was Fourier transformed using an IDL FFT procedure. Various Optical Transfer Functions (OTF) (see Jahn, Reulke, 1995) have been taken into account, especially the geometrical OTF of a pixel, the measured (and approximated) OTF of the optics, and the OTF of pixel shift caused by motion. The total OTF obtained by multiplying all single OTF's has substantial spatial frequency transmission above the Nyquist frequency but nearly zero transmission above the doubled Nyquist frequency. That means that with the non-staggered array substantial undersampling can occur whereas with the staggered array undersampling is negligible.

After having multiplied the Fourier transformed test image with the total OTF the inverse Fourier transform is applied. Figure 6 shows the blurred test image.

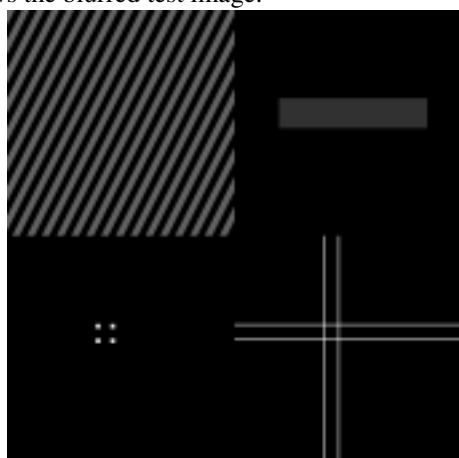


Figure 5. Test image.

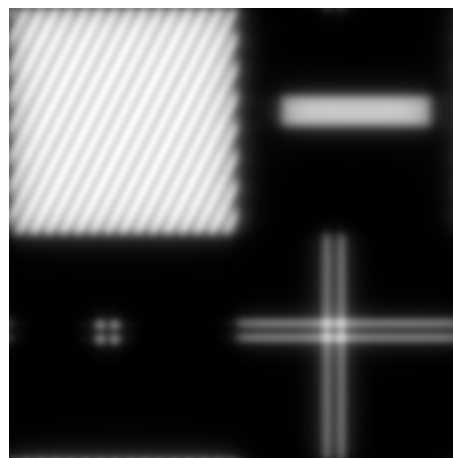


Figure 6. Blurred test image  
spatial frequency of stripe pattern:  $100\text{mm}^{-1}$ , distance of dots and lines:  $10\mu\text{m}$

Then the image is sampled. Two cases are considered:

1. Non-staggered array: The sampling distance is  $\Delta_x = \Delta_y = 6.5\mu\text{m}$ .
2. Staggered array: The sampling distance is  $0.5 \cdot \Delta_x = 0.5 \cdot \Delta_y = 3.25\mu\text{m}$ .

Figures 7 and 8 are zoomed versions of the sampled blurred test image in order to see better. What we see is that aliasing takes place in the striped pattern of the non-staggered sensor (fig. 7): wrong wavelength and wave direction are observed. The crossed line pairs and the dots can be resolved with the staggered array but not (or worse) with the non-staggered array. This demonstrates the superiority of the staggered array over the non-staggered CCD line. Other simulated images confirm that.

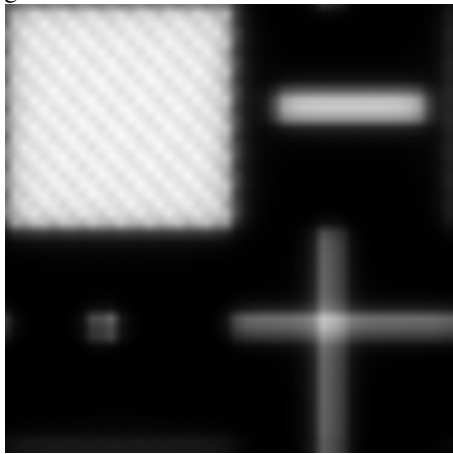


Figure 7. Sampled test image, non-staggered

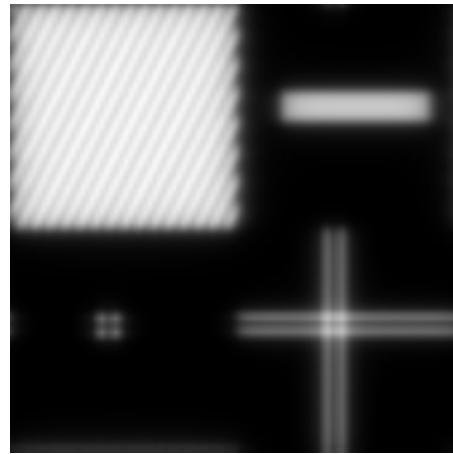


Figure 8. Sampled test image, staggered

## 4.2 Laboratory test

To demonstrate the staggering effect once more, an experimental setup with a CCD camera was made. The aim of this experiment was to show the resolution enhancement of the staggering effect by subsampling an image by half of a pixel in horizontal and vertical direction. The target structure was a bar pattern with variable number of bars per mm.

To show the resolution improvement using staggered arrays, the frequency of band limitation of the optics must be much bigger than the Nyquist frequency of the CCD-sensor. As a result, aliasing effects occur if target structures, which pass the optics, contain higher frequencies than the Nyquist limit.

To show the resolution of the (non-staggered) camera the optics was defocused in order to avoid aliasing. The bar pattern consists of strips from 1 lp/mm to 10 lp/mm (which is seen left and right of the bar pattern). With respect to the magnification of this optical arrangement of about 6 the frequency spectrum in front of the CCD-matrix is between 6 and 60 lp/mm.

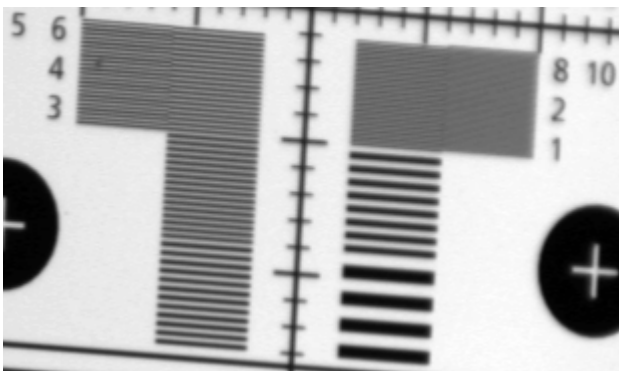


Figure 9. Original defocused non-staggered image

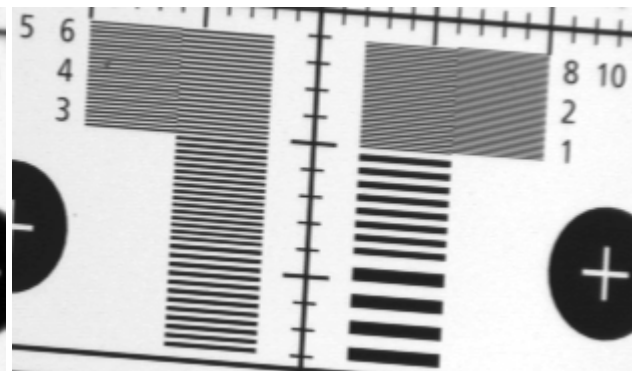


Figure 10. Original focused non-staggered image

Figure 9 shows, that the bar patterns are resolvable in vertical direction up to about 4 lp/mm. Taking into account the magnification this corresponds to about 25lp/mm and is in consistency with the pixel pitch of 20  $\mu\text{m}$ . The difference to the non defocused sharpest possible image is shown in figure 10. The aliasing effects (here especially as Moire patterns and frequency changes) are obvious.

To simulate a staggered array (i.e. to apply subsampling), the CCD matrix was shifted in horizontal and vertical direction by half of a pixel. After measuring, the four generated images have been rearranged in a twice bigger image.

The result is shown in Figure 11. In the resulting image no aliasing is visible. The resolution is improved by a factor of two, because structures up to 8 lp/mm are resolvable.

This laboratory experiment shows very simple and evidently the resolution enhancement by staggering.

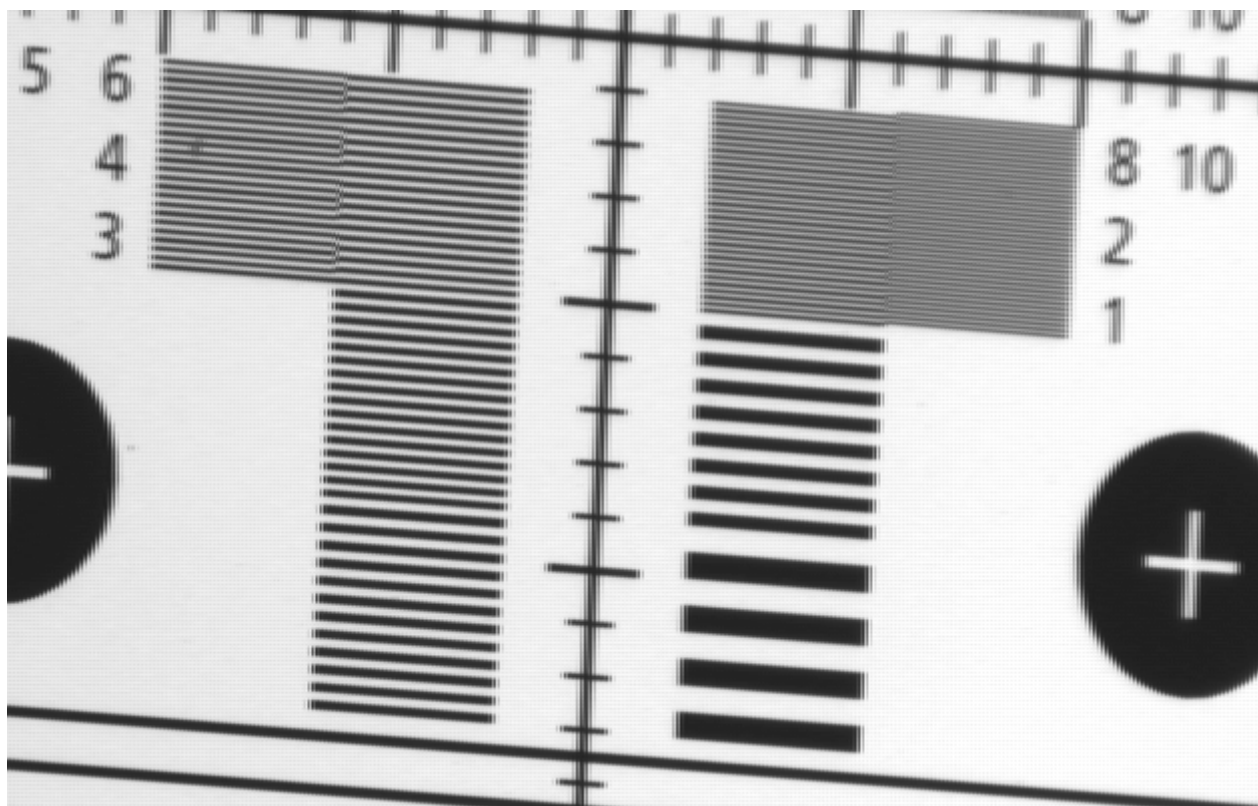


Figure 11. Resulting (staggered) image

#### 4.3 Airborne test with infrared sensor HSRS

HSRS (Hot Spot Recognition Sensor) is a prototype of the IR sensor system on board the small satellite mission BIRD to be launched in 2001. The BIRD sensor system designed for high temperature event detection and evaluation consists of two IR sensors in the mid and thermal IR, respectively, together with a modified WAOSS camera (Oertel et al., 1992). The HSRS Mercury Cadmium Telluride sensors are staggered lines of 2 x 512 pixels each (Skrbek, Lorenz, 1998). With HSRS some airborne experiments have been carried out in order to check the system performance.

Figures 12 and 13 show staggered and non-staggered images of one of those experiments. The aliasing in the non-staggered image is obvious. With staggering no aliasing occurs and a resolution improvement can be observed.

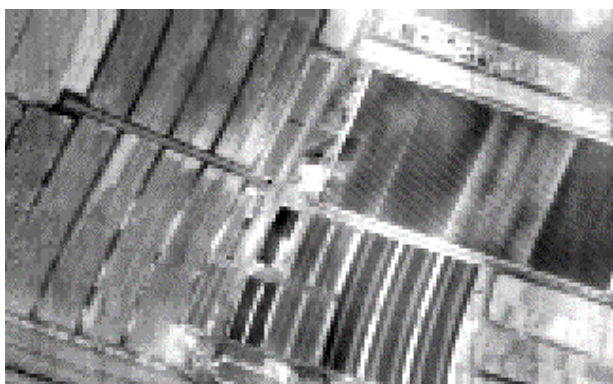


Figure 12. Non-staggered HSRS image

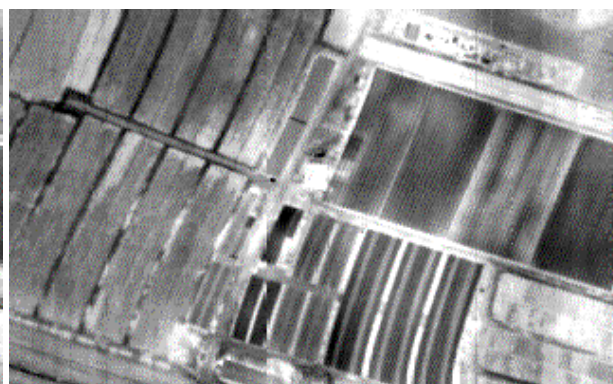


Figure 13. Staggered HSRS image



## 5 CONCLUSIONS

To obtain high resolution digital images with a large swath width, the concept of staggered arrays is very useful. In this paper the theoretical background of image resolution with staggered arrays was presented. A quantitative approach for the resolution of an imaging system depends on the target structure and the resolution definition, which in our approach is related to the Rayleigh-criterion. The resolution was investigated for different target structures with non-staggered and staggered arrays. The results show an enhancement of the resolution with staggered arrays versus non-staggered ones by about a factor of two.

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