COMPARISON OF THE PROJECTIVE BLOCK ADJUSTMENT METHOD VERSUS THE BUNDLE METHOD

Ilkka NIINI
Institute of Photogrammetry and Remote Sensing
Helsinki University of Technology
Finland
ilkka.niini@hut.fi

KEY WORDS: Block Adjustment, Calibration, Orientation, Optimization, Projective Geometry

ABSTRACT

The projective block adjustment method, based on the singular correlation between pairs of images, is compared with a free-network bundle adjustment method. The singular correlation method has two parts, the projective and physical version. The projective version uses the projective parameters and certain constraint equations between the most optimal image pairs, while the second version uses physical parameters which are independent of the image pair combinations and need no constraint equations. No 3-D object unknowns are needed. When the block contains at most four images, the projective and physical versions are consistent, thanks to the addition of two new constraints in the projective version. If there are more than four images, the projective version cannot always use all available data because only a subset of image pairs, and the parameters between them, are taken into the adjustment. On the other hand, the physical version can use all data. Experiments with real data show that the physical results are well comparable with the corresponding free-network bundle results, and the use of the new constraints also makes the projective results better, even though the use of all data still cannot always be guaranteed. In situations, where the projective version can use all available data, the results from the projective version are also similar to the bundle results.

1 INTRODUCTION

The projective block adjustment method is based on the bilinear singular correlation condition $x'^T M x'' = 0$ between the observations of two images. The adjustment method solves the relative geometry of the block in an arbitrary scale, as well as the interior geometry of the images, and optionally, also nonlinear distortions of the images. No 3-D object parameters or approximate values for the orientations are needed, except for the interior orientation parameters. This reduces the system size and computation time compared to the bundle method. The 3-D object model has to be computed after the adjustment by intersecting the projection rays in the object space. This can be done by linear means from the adjusted relative exterior and interior orientations. The method can be divided into two parts, or versions. First, the block is solved by using pure projective parameters. Second, a switch to physical parameters is made, and a new adjustment is performed. The projective version gives approximate values for the physical stage. In the projective version, approximate values for the singular correlation parameters are obtained from a linear solution of the projective singular correlations.

The details of the projective block adjustment method has been presented in earlier articles of the author, e.g. (Niini, 1994), (Niini, 1995), (Niini, 1996), (Niini, 1998), so they are not presented here. Only a short description of two new constraints, which can be used in the projective stage of the method, are presented

The bundle adjustment is widely considered as the most optimal method to make a simultaneous block adjustment. However, it requires approximate values for all orientation parameters, and for the object co-ordinates, which are usually difficult to obtain, especially if there is no object information available.

In this article, the two versions of the new block adjustment method are compared to a free-network bundle method to find out how close their results are to the final bundle adjustment results. This is made with real image data. It is shown that the physical version is well comparable with the free-network bundle method, and even the pure projective version gives good results compared to the bundle results.

2 NEW CONSTRAINTS

The number of projective singular correlation parameters is larger than the true number of physical orientation parameters. The dependencies between the projective parameters have to be resolved with additional constraints between the projective parameters and/or interior orientation parameters.

There are currently four types of additional constraints that are needed in the projective version of the adjustment method, in order to obtain the correct number of independent parameters and equations in the system. The Kruppa equations and the trifocal constraints are already known from earlier studies, e.g. (Niini, 1994) and (Niini, 1998). A new interior orientation constraint has been derived. Also a new constraint which, indeed, connects five singular correlation matrices between any four images together is introduced.

2.1 A new trifocal interior orientation constraint

In a trifocal system, the Kruppa equations (Niini, 1994), derived from the three singular correlation matrices of three image pairs do not take into account the fact that the rotations between these three images i, j, k are dependent through the condition $R_{ij} R_{jk} R_{ik}^T = I$ (=identity matrix). If the interior orientation parameters are computed by using the Kruppa equations only, the relative rotations, when computed, do not necessarily exactly fulfill the consistency condition $R_{ij} R_{jk} R_{ik}^T = I$. This suggests that a new interior orientation constraint is hidden in this condition or in some of its variant. A candidate for this constraint, though a quite complicated one, has been derived from the above condition. It relates the interior orientation parameters of the three images and the singular correlation parameters between these three images, and it guarantees the consistency of the relative rotations of the three images. There is one such constraint per each trifocal plane or triplet of images in a projective block:

$$
\begin{aligned}
&(e_{ji}^T C_j^T C_j e_{jk})(e_{ki}^T C_k^T C_k e_{kj})(e_{jk}^T C_j^T C_j e_{jk})^{\frac{1}{4}} \\
&- \rho_1 (e_{ij}^T C_i^T C_i e_{ik})(e_{ki}^T C_k^T C_k e_{ki})(e_{jk}^T C_j^T C_j e_{jk})^{\frac{5}{4}}(e_{ik}^T C_i^T C_i e_{ik})^{-1} \\
&- \rho_2 (e_{kj}^T C_k^T C_k e_{kj})^{\frac{1}{4}}(e_{ji}^T E_{jk}^T C_j^{-1} C_j^{-T} E_{jk} e_{ji})det(C_j)\sqrt{det(C_j)det(C_k)} = 0.
\end{aligned}
\tag{1}
$$

Above, the interior orientation matrices C are of type

$$
C = \begin{bmatrix} 1 & \beta & -(x_p + \beta y_p) \\ 0 & \alpha & -\alpha y_p \\ 0 & 0 & -c_p \end{bmatrix},
\tag{2}
$$

where x_p, y_p are the principal point co-ordinates, α is affinity (x/y-scale ratio), β is the lack of orthogonality between x- and y-axis, and c_p is the focal length. A scale $\frac{\tau_{ij}^2}{\epsilon_{ij}^2}$ (ratio of the scales of singular correlation matrix and the epipole) is needed because the singular correlation matrices and epipoles are homogeneous quantities. The epipolar matrices have the form

$$
E_{ij} = \begin{bmatrix} 0 & z_e' & -y_e' \\ -z_e' & 0 & x_e' \\ y_e' & -x_e' & 0 \end{bmatrix},
\tag{3}
$$

and the epipole co-ordinate vectors are of form $e_{ij} = [x_e, y_e, z_e]_{ij}^T$.

Additionally, $\rho_1 = \frac{(e_{kj}^T M_{ik}^T M_{ij} e_{jk})(e_{ji}^T M_{jk}^T M_{ik}^T e_{ij})(e_{ik}^T M_{ij}^T M_{jk} e_{ki})}{(e_{kj}^T M_{ik}^T M_{ik} e_{kj})(e_{ji}^T M_{jk}^T M_{jk} e_{ji})(e_{ik}^T M_{ij}^T M_{ij}^T e_{ik})}$ and $\rho_2 = \pm \frac{|M_{jk} e_{ki}|}{|E_{jk} e_{ji}|\sqrt{|ad-bc|}}$. Here, the sign of ρ_2 is the same as the sign of the scale ratio $\frac{\tau_{jk}}{\epsilon_{jk}}$, and it can be determined from two corresponding image co-ordinate vectors x_j, x_k, by using the equality $\lambda E_{jk} x_j = \frac{\tau_{jk}}{\epsilon_{jk}} M_{jk} x_k$. The scalars a, b, c, and d are the four elements of N_{jk} (equation 4). The projection ray scale λ is positive by definition because the object is always in front of the cameras, so the correct sign of $\frac{\tau_{jk}}{\epsilon_{jk}}$ and, hence, ρ_1 can be checked. The sign of ρ_1 is automatically correct from the above formulae. Note that the terms ρ_1 and ρ_2 depend only on the singular correlation parameters.

The epipolar decomposition of the singular correlation matrix is (Thompson, 1968), (Niini, 1998):

$$
M_{ij} = \begin{bmatrix} 1 & -\frac{y_e'}{x_e'} & -\frac{z_e'}{x_e'} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{y_e''}{x_e''} & 1 & 0 \\ -\frac{z_e''}{x_e''} & 0 & 1 \end{bmatrix},
\tag{4}
$$

or, shortly, $M_{ij} = F_{ij}^T N_{ij} F_{ji}$. Matrices F_{ij} and F_{ji} represent shifts to scaled epipoles (co-ordinates x_e', y_e', z_e', x_e'', y_e'', z_e'', of which one can be fixed in each image), and N_{ij} is a transformation matrix between the centred 2-D epipolar pencils, extended to a singular 3×3 format. There are seven parameters in decomposition 4, since one of the elements in the central matrix N_{ij} (the largest one) can be fixed. The decomposition 4 can have other forms, too, depending on the largest epipole co-ordinates. The above form was obtained by scaling the epipoles with the x co-ordinates. This decomposition is used to parameterize the singular correlation matrices in the projective version of the adjustment.

2.2 Constraints between four images

Among any four images and their six possible image pairs, one singular correlation condition can be expressed by using the five other singular correlation matrices. For example, if among four images i, j, k, and l, the pairs ij, ik, il, jl, and kl belong to the image pairs taken in the adjustment then the image points x_j, x_k observed from the missing image pair jk can also be adjusted by using the condition equation

$$(M_{ij}x_j \times M_{ik}x_k)^T M_{il}(M_{jl}^T x_j \times M_{kl}^T x_k) = 0. \tag{5}$$

Above, the operator $\times$ means cross product between vectors. Thanks to the equation 5, all observations in a block of four images can be used with projective parameters. In larger blocks, this constraint can be used for an image pair without its own singular correlation parameters only if the five neighbouring singular correlation matrices are in the adjustment.

2.3 Physical form of singular correlation matrix

The physical decomposition of the singular correlation matrix M, used in the physical version of the adjustment, has a simple form, as follows

$$M = C_i^T R_i^T (B_j - B_i) R_j C_j. \tag{6}$$

R_i and R_j are orthogonal rotation matrices, C_i and C_j are the upper triangular interior orientation matrices, and B_i and B_j are skew symmetric relative base component matrices. In a block adjustment, seven of the orientation parameters have to be fixed in order to fix the datum of the system. No additional constraint equations are needed with this decomposition.

3 NUMBER OF PARAMETERS AND CONSTRAINT EQUATIONS

It is known from projective geometry that there are, in general, $11n - 15$ independent projective parameters in an arbitrary block of n images. This is also the maximum number of free physical orientation parameters in a bundle block when the number of possible non-linear parameters for the lens distortion are not taken into account.

With physically meaningful parameters, there are six exterior orientation parameters for each image in an arbitrary block, three for the position, and three for the rotation. What is the number of free interior orientation parameters? The datum can always be fixed with seven parameters (e.g. position and rotation of the first images, and the scale of the block) or with seven so called inner constraints (Fraser, 1982). This gives $6n - 7$ free parameters in the block. But, because there can be as many as $11n - 15$ free parameters, there has to be at most $(11n - 15) - (6n - 7) = 5n - 8$ free parameters reserved for the interior orientation.

There are at most five linear interior orientation parameters per image (affinity, non-orthogonality, two principal point coordinates, and focal length. This means that, if object constraints cannot be used, eight additional constraints are needed to solve all $5n$ interior orientation parameters, or conversely, at most $5n - 8$ free interior orientation parameters can be solved in an arbitrary block from image information only. Usually, the block is not entirely arbitrary. It is possible that the interior orientation is fully or partially known, or it is the same between all or some images.

The number of equations, constraints, and unknowns of the different adjustment methods are presented in Table 1. This table correspond to an ideal case where all image data can be used in all methods. This is possible, in general, only if all object points can be observed from all images. Therefore, the numbers are maximum values only. Note, that because no 3-D control was assumed here, the point has to be observed from two images before it can be taken into the adjustment.

Method	e	t	c	i	u	r
Projective	$(2n-3)p$	$3(n-2)$	0	$2(2n-3)+(n-2)$	$7(2n-3)+m$	$2np-6n-3p-m+7$
Physical	$(2n-3)p$	0	7	0	$6n+m$	$2np-6n-3p-m+7$
Bundle	$2np$	0	7	0	$6n+m+3p$	$2np-6n-3p-m+7$

Table 1: The characteristics of three adjustment methods. e=number of observation or singular correlation equations, c=number of datum constraints, t=number of trifocal constraints, i=number of interior orientation constraints, u=number of unknowns, and r=e+t+c+i-u (redundancy). n=number of images, p=number of object points, and m=number of interior orientation parameters in the block ($m <= 5n - 8$).

It can be seen from Table 1 that the maximum redundancy is obtainable from the projective version when all possible constraints and data are used. In practice, all object points cannot be observed from all images in an arbitrary block, so it is not likely to have the same redundancy in the projective and bundle methods. However, in the physical adjustment, the same redundancy with the bundle case can be achieved because the equations between pairs of observations can be chosen independently for each point.

4 BLOCK ADJUSTMENT STEPS

The projective block adjustment method is a stepwise method. First, find an optimal combination of image pairs by using a suitable optimization algorithm (Niini, 1998). Second, solve the singular correlation and interior orientation parameters, and possible non-linear parameters (Niini, 1996). Third, solve the rotations of the images. Due to the equation 1, the rotations can now be computed from the adjusted singular correlations and interior orientation parameters without further adjustments. Fourth, solve the positions of the images along with the model co-ordinates. Fifth, change to the physical model (equation 6), and make a new adjustment for all physical parameters simultaneously. The projective stage consist of the steps 1-4, and the physical stage is the fifth step alone. The model is re-computed after the physical stage, too.

The new constraints (equations 1 and 5) are added in the second step of the projective part of the method. The experiments made in this article show that the new constraints make the results from the projective stage better.

4.1 Block optimization in the projective stage

There can be $n(n-1)/2$ image pairs in an arbitrary block. In the optimization stage, the goal is to find the best image pairs among all possible ones so that the inner geometry of the block, in terms of the determinability of the chosen relative projective parameters, would be optimal. It has been shown earlier that, in general, the correct number of independent block parameters is obtained when the block of n images is arranged in the form of $n-2$ image triplets (trifocal planes, or triangles) and $2n-3$ image pairs (Niini, 1998). The optimization is made as a triangular network optimization. The optimization algorithm can be tuned so that the number of observations also affects to the search result, thus, the more observations, the better chances there are to be taken into the adjustment.

The block is solved as a system of $7(2n-3)$ projective parameters, and $m \le 5n-8$ interior orientation parameters, which are constrained with $3(n-2)$ trifocal constraints, with $2(2n-3)$ interior orientation (Kruppa) constraints, and with $n-2$ new trifocal interior orientation constraints. The number of free parameters in the block is then $7(2n-3) - 3(n-2) = 11n - 15$.

One reason for the optimization is also the need to avoid certain degenerate cases of three images inside the block (collinear projection centres, parallel rotation axes, certain symmetric orientations) (Niini, 1998). The coplanarity of the projection centres of three images and an object point is also a dangerous case, unless the point is obseeved from a fourth image, projection centre of which is not in the same plane. Otherwise, if an object point and the projection centres of three or more images lie in the same plane, the $2n_{ip} - 3$ singular correlation equations between the observations of this point are no more independent. In fact, due to the coplanarity, only $n_{ip} - 1$ singular correlation equations are independent for the observations of this point.

4.2 Pointwise optimization in the physical stage

In the physical stage, the adjusted observation pairs can be chosen for each object point individually, so that the correct number of independent equations per object point is obtained. Again, there can be at most $2n_{ip} - 3$ independent equations between the observations of this point when the point has been observed from n_{ip} images. Any additional equation for this point p would be redundant and make the corresponding residual coefficient matrix B singular in the general adjustment system $Ax + Bv = f$. The same optimization algorithm that was used in the general block optimization before the projective stage can be used here, but separately for each point. The datum was fixed by using the seven inner constraints applied to the base components.

4.3 Bundle adjustment

An enhanced collinearity model (Melen, 1994) was used in the bundle adjustment, in order to get the treatment of interior orientation and non-linear lens distortion parameters similar to the one used in the projective block adjustment. The bundle method allows any combination of image observations to be used by requiring only that the observations from different images are consistent with a single 3-D object point. The datum was fixed with seven inner constraints, applied to the image positions only.

5 TESTS WITH REAL DATA

The two versions of the new block adjustment method were compared with a free-network bundle adjustment. Various properties could be compared: the size of the system (number of unknowns), processing time, mean errors of the parameters, root mean square error (RMSE) of the 3-D model, etc. Here, only the system sizes and the RMSE values of the 3-D models were compared.

Figure 1: Real digital images. Upper row: images 1-4 from left to right. Lower row: images 5-8 from left to right.

5.1 Testfield block

The object was a 3-D testfield at Helsinki University of Technology, containing 161 known 3-D points with an accuracy of about 0.03 mm. The camera was a digital camera, Olympus Camedia C-1400L with a zoom lens. The image size was 1280x1024 pixels, and the nominal focal length varied from 1400 to 4000 pixels.

Eight images were taken, images 1-4 with the smallest zoom, and images 5-8 with the largest zoom. The images were taken from clearly different positions and orientations. The images are shown in Figure 1. The unknown interior orientations of the images were partially the same: affinity and non-orthogonality were kept the same, whereas principal point and focal length were kept different between image sets 1-4 and 5-8. In both image sets, five non-linear lens distortion coefficients were taken into account, three for radial distortion, and two for tangential distortion. The effect of the lens parameters was controlled in all adjustment methods with one fictitious observation equation per parameter (Heikkilä, 1992).

Measurements. The images were measured manually with a computer program which had a zoomable measurement window. The measurement accuracy was supposed to be less than half a pixel. 398 image points from 97 different object points could be measured. Approximate values for the singular correlations were computed for all possible image pairs with at least eight corresponding points. This gave 27 image pair combinations, from which 50 image triplets could be formed in this example.

Adjustments. The projective adjustment was performed three times. In the first adjustment (Projective 1), only the observations belonging to the most optimal image pairs were used. In the second adjustment (Projective 2), extra constraints 5 were used where possible, to maximize the number of observations in the adjustment. The extra interior orientation constraints for the trifocal planes (equation 1) were also used in the second projective case.

Because the data and the number of equations were different between different cases, a subset of common data between all adjustments was extracted from the original data, to get more comparable cases. This common data contained 319 image point observations from 77 object points. The third projective case (Projective 3) used this common data, adjusted with all possible constraints.

The physical and bundle adjustments were performed twice, first with all possible data (Physical 1 and Bundle 1) and then with the smaller common data (Physical 2 and Bundle 2). The characteristics of all adjusted cases are in Table 2.

To get the resulting 3-D models from different adjustments comparable, the models were first transformed to the known testfield co-ordinate system by using a seven-parametric similarity transformation. The root mean square error (RMSE) between the transformed co-ordinates and the true co-ordinates was then computed.

Data usage. The data usage is summarized in Table 2. It can be seen from this table that the adjustment based on the projective singular correlation always missed some data (except in the Projective 3 case). There were 97 object points, but in the first projective case all observations of ten object points were totally missed from the adjustment. Similarly, some correlation equations between some image observations were also missed. This can be seen by comparing the redundancy numbers of the cases. For example, in the bundle adjustment, the maximum redundancy of 456 was obtained, whereas the best projective redundancy (in Projective 2) was only 432. Thus, at least 24 useful equations between image observations were missing because the corresponding singular correlation matrices were not taken into the adjustment, or

the constraint 5 could not be formed between the observations. However, it can be noted that the maximum redundancy was obtained and, hence, all image data could be used in the physical adjustment version. In the common data cases, the same redundancy (358) was obtained with all three adjustment methods.

Case	A	B	C	D
Projective 1	87	428	109	319
Projective 2	97	541	109	432
Projective 3	77	467	109	358
Physical 1	97	522	66	456
Physical2	77	424	66	358
Bundle 1	97	813	357	456
Bundle 2	77	655	297	358

Table 2: The characteristics of the testfield cases. A=number of object points in adjustment, B=number of equations, C=number of unknowns, D=redundancy.

5.2 Results

First, the cases with the same data and redundancy were studied (cases Projective 3, Physical 2, Bundle 2). The RMSE between the resulting 3-D models from these cases are shown in Table 3. In these cases, the results were the same up to the used iteration termination limit (here $1 \cdot 10^{-8}$ units). Other adjustment results of these cases were also the same, and, therefore, these results are presented only once in consequent tables 4, 5, 6, and 7, with the title 'Common data cases'. Only the results from the physical and bundle adjustments were expected to be identical, whereas slightly different results were expected from the projective case. However, as the data and redundancy was the same, the final results were also the same. This proves that the computational details (use of constraints, linearization, calculation order, etc.) in different adjustment methods are in coherence, because similar results would not have been possible otherwise. It also demonstrates that the bundle method is not always superior compared to other methods like the ones presented in this article.

Cases	RMSE X	RMSE Y	RMSE Z	RMSE XYZ
Projective 3 - Physical 2	$5.788 \cdot 10^{-10}$	$6.592 \cdot 10^{-10}$	$5.546 \cdot 10^{-10}$	$5.992 \cdot 10^{-10}$
Projective 3 - Bundle 2	$6.833 \cdot 10^{-10}$	$7.033 \cdot 10^{-10}$	$5.606 \cdot 10^{-10}$	$6.521 \cdot 10^{-10}$
Physical 2 - Bundle 2	$1.583 \cdot 10^{-10}$	$7.304 \cdot 10^{-11}$	$1.116 \cdot 10^{-10}$	$1.195 \cdot 10^{-10}$

Table 3: Pairwise difference between the three models (in mm) in cases where the data and redundancy are the same.

The standard error of the adjustments, and the RMSE's between the 3-D models and the known object are shown in Table 4. It shows that the cost of the missing data and equations in the projective case, compared to the bundle results, is relatively small. The best projective RMSE value was 1.0524 mm, whereas the corresponding bundle result was 0.9527 mm, i.e. projective results are worse by a factor of 1.1 only, compared to the bundle results. If the new constraints (equations 1 and 5) were not taken into account (as in case Projective 1), the projective result was about 1.3 times worse than the bundle value (1.2176 mm/0.9527 mm). Thus, these constraints make the results clearly better.

The 3-D model RMSE from the physical adjustment was exactly the same as the corresponsding bundle value (0.9527 mm). This was expected, because the data and redundancy were also the same.

The values of the exterior orientation parameters are not presented here because only the accuracy of the object model was interesting. The final interior orientation and non-linear parameters are presented in tables 5, 6, and 7. They are also same in the physical and bundle cases. The projective results are also very good. For example, the principal point co-ordinates from Projective 2 case differed from the bundle result not more than about one pixel, except the zoomed focal length which was about 14 pixels too short. The results from the first projective case were quite similar, despite the lack of observations.

Case	s_0 (pixels)	RMSE in XYZ (mm)
Projective 1	0.2994	1.2176
Projective 2	0.2925	1.0524
Physical 1	0.2918	0.9527
Bundle 1	0.2918	0.9527
Common data cases	0.2967	1.0580

Table 4: Standard error of adjustment (s_0), and the model accuracy (RMSE).

Case	α	β	x'_p	y'_p	c'_p	x''_p	y''_p	c''_p
Projective 1	0.999775	$-4.220081 \cdot 10^{-3}$	597.817	538.963	1408.079	589.448	540.172	4017.083
Projective 2	0.999414	$-8.12970 \cdot 10^{-5}$	602.658	532.943	1407.311	593.685	537.734	3990.033
Physical 1	0.999441	$-4.019406 \cdot 10^{-5}$	601.903	531.888	1408.079	593.191	537.052	4004.177
Bundle 1	0.999441	$-4.019406 \cdot 10^{-5}$	601.903	531.888	1408.079	593.191	537.052	4004.177
Common data cases	0.999415	$2.17050 \cdot 10^{-8}$	603.000	538.612	1406.692	593.808	547.306	3987.526

Table 5: Adjusted interior orientation parameters. Note: common α, β. Images 1-4: x'_p, y'_p, c'_p. Images 5-8: x''_p, y''_p, c''_p.

Case	k'_1	k'_2	k'_3	p'_1	p'_2
Projective 1	$-1.239 \cdot 10^{-7}$	$5.426 \cdot 10^{-14}$	$4.534 \cdot 10^{-21}$	$-6.216 \cdot 10^{-7}$	$-1.0769 \cdot 10^{-6}$
Projective 2	$-1.234 \cdot 10^{-7}$	$5.318 \cdot 10^{-14}$	$4.627 \cdot 10^{-21}$	$-7.935 \cdot 10^{-7}$	$-1.050 \cdot 10^{-6}$
Physical 1	$-1.224 \cdot 10^{-7}$	$5.238 \cdot 10^{-14}$	$3.862 \cdot 10^{-21}$	$-8.461 \cdot 10^{-7}$	$-1.168 \cdot 10^{-6}$
Bundle 1	$-1.224 \cdot 10^{-7}$	$5.238 \cdot 10^{-14}$	$3.862 \cdot 10^{-21}$	$-8.461 \cdot 10^{-7}$	$-1.168 \cdot 10^{-6}$
Common data cases	$-1.240 \cdot 10^{-7}$	$5.423 \cdot 10^{-14}$	$4.533 \cdot 10^{-21}$	$-5.936 \cdot 10^{-7}$	$-9.135 \cdot 10^{-7}$

Table 6: Adjusted non-linear distortion parameters of images 1-4. Radial distortion coefficients: k'_1, k'_2, k'_3. Tangential distortion coefficients: p'_1, p'_2.

Case	k''_1	k''_2	k''_3	p''_1	p''_2
Projective 1	$3.625 \cdot 10^{-8}$	$2.275 \cdot 10^{-15}$	$2.971 \cdot 10^{-22}$	$-5.099 \cdot 10^{-8}$	$1.517 \cdot 10^{-7}$
Projective 2	$3.423 \cdot 10^{-8}$	$1.017 \cdot 10^{-14}$	$1.221 \cdot 10^{-21}$	$-9.046 \cdot 10^{-9}$	$5.984 \cdot 10^{-8}$
Physical 1	$3.459 \cdot 10^{-8}$	$9.784 \cdot 10^{-15}$	$1.075 \cdot 10^{-21}$	$-4.097 \cdot 10^{-9}$	$6.941 \cdot 10^{-8}$
Bundle 1	$3.459 \cdot 10^{-8}$	$9.784 \cdot 10^{-15}$	$1.075 \cdot 10^{-21}$	$-4.097 \cdot 10^{-9}$	$6.941 \cdot 10^{-8}$
Common data cases	$3.623 \cdot 10^{-8}$	$5.127 \cdot 10^{-15}$	$3.601 \cdot 10^{-22}$	$4.981 \cdot 10^{-7}$	$7.779 \cdot 10^{-8}$

Table 7: Adjusted non-linear distortion parameters of images 5-8. Radial distortion coefficients: k''_1, k''_2, k''_3. Tangential distortion coefficients: p''_1, p''_2.

6 CONCLUSIONS

In this article, the two stages of the projective block adjustment methods based on singular correlation were compared to the free-network bundle adjustment method. The bundle method requires the 3-D object unknowns and their approximate values in the adjustment. The new method does not contain 3-D object unknowns, nor requires approximate values for the exterior orientations. The 3-D model can be intersected from the adjusted orientation parameters.

The projective version of the new adjustment method cannot necessarily use all available data because the block parameters are the singular correlation parameters between certain, optimal image pairs only. To make the projective system stronger, two new constraints were used. The first one constraints the determination of the interior orientation parameters, and the second one allows us to use some of the possibly missed data. Finally, the physical version of the method enhances the results from the projective stage by re-adjusting all data.

The projective and physical version of the method were compared to the free-network bundle method. Examples with real images show that the final results from the physical version are well comparable with the results of a corresponding bundle adjustment. This can make a further adjustment with the bundle method unnecessary. Of course, the bundle method can be used already after the projective stage.

The results from the projective stage are quite good, too, in this example. Even if all possible image observations could not be used, the RMSE values of the obtained 3-D model were not worse than 1.1 times the bundle results. It was also demonstrated that if exactly the same data could be used in the three adjustments, the results were exactly the same.

In the new block adjustment method, the outliers in the data can be a problem because the computation of singular correlation matrices is fragile in the presence of outliers. Additionally, hidden outliers can pass the first, projective stage of the adjustment, and cause problems in the physical stage. In future, a proper method to clean any real data from the outliers in advance or on-line has to be developed. For example, the random sampling consensus could be used. See (Torr and Zisserman, 1998).

REFERENCES

Fraser, C. (1982). Optimization of Precision in Close-Range Photogrammetry. *The Photogrammetric Engineering and Remote Sensing*, 48(4):561–570.

Heikkilä, J. (1992). Systemaattisten kuvavirheiden kompensointi sädekimppublokkitasoituksessa (Compensating systematic image errors in the bundle block adjustment, in Finnish). Master's thesis, Helsinki University of Technology, Finland.

Melen, T. (1994). Geometrical Modelling and Calibration of Video Cameras for Underwater Navigation. ITK Technical Report 94-103-W. Dissertation, University of Technology, Trondheim, Norway.

Niini, I. (1994). Relative Orientation of Multiple Images Using Projective Singular Correlation. Licentiate Thesis, Helsinki University of Technology, Finland.

Niini, I. (1995). Experiences in Building Reconstruction Using Digitized Video Images. In Grün and Kahmen, editors, *Optical 3-D Measurement Techniques III*, pages 295–304. Wichmann, Karlsruhe, Germany.

Niini, I. (1996). Orthogonal 3-D Reconstruction Using Video Images. In *International Archives of Photogrammetry and Remote Sensing*, volume 31, part B3, pages 581–584. Vienna, Austria.

Niini, I. (1998). On the Algorithmic Aspects of the Projective Block Adjustment. In *International Archives of Photogrammetry and Remote Sensing*, volume 32, part 3/1, pages 81–86. Columbus, Ohio, USA.

Thompson, E. H. (1968). The Projective Theory of Relative Orientation. *Photogrammetria*, 23(2):67–75.

Torr, P. and Zisserman, A. (1998). Robust Detection of Degenerate Configurations while Estimating the Fundamental Matrix. *Computer Vision and Image Understanding*, 71(3):312–333.